INSTRUCTOR'S SOLUTIONS MANUAL, VOLUMES 1, 2, 3 AND 4

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to accompany

FUNDAMENTALS OF PHYSICS

EIGHTH EDITION

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Preface

This book includes the solutions to the PROBLEMS sections of the 8th edition of **Fundamentals of Physics** by Halliday, Resnick and Walker. We have not included solutions or discussions that pertain to the Questions sections. These solutions have been typed using Microsoft Word and MathType™ equation editor. The solution files are available on the Instructor's Companion Website (www.wiley.com/college/halliday). Additional information regarding MathType™ can be found at www.mathtype.com

The author has put great time and effort into writing high quality solutions. He welcomes comments and suggestions from readers, please also report any errors that you may find. The author's email address is: senben_liao@yahoo.com

Note to adopters regarding the Instructor's Solutions Manual:

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Chapter 1

1. The metric prefixes (micro, pico, nano, …) are given for ready reference on the inside front cover of the textbook (see also Table 1–2).

(a) Since 1 km = 1×10^3 m and 1 m = 1×10^6 μm,

$$1\,\text{km} = 10^3\ \text{m} = \left(10^3\ \text{m}\right)\left(10^6\ \mu\text{m/m}\right) = 10^9\ \mu\text{m}.$$

The given measurement is 1.0 km (two significant figures), which implies our result should be written as 1.0×10^9 μm.

(b) We calculate the number of microns in 1 centimeter. Since 1 cm = 10^{-2} m,

$$1\,\text{cm} = 10^{-2}\ \text{m} = \left(10^{-2}\text{m}\right)\left(10^6\ \mu\text{m/m}\right) = 10^4\ \mu\text{m}.$$

We conclude that the fraction of one centimeter equal to 1.0 μm is 1.0×10^{-4}.

(c) Since 1 yd = (3 ft)(0.3048 m/ft) = 0.9144 m,

$$1.0\,\text{yd} = \left(0.91\text{m}\right)\left(10^6\ \mu\text{m/m}\right) = 9.1 \times 10^5\ \mu\text{m}.$$

2. (a) Using the conversion factors 1 inch = 2.54 cm exactly and 6 picas = 1 inch, we obtain

$$0.80 \text{ cm} = (0.80 \text{ cm})\left(\frac{1 \text{ inch}}{2.54 \text{ cm}}\right)\left(\frac{6 \text{ picas}}{1 \text{ inch}}\right) \approx 1.9 \text{ picas}.$$

(b) With 12 points = 1 pica, we have

$$0.80 \text{ cm} = (0.80 \text{ cm})\left(\frac{1 \text{ inch}}{2.54 \text{ cm}}\right)\left(\frac{6 \text{ picas}}{1 \text{ inch}}\right)\left(\frac{12 \text{ points}}{1 \text{ pica}}\right) \approx 23 \text{ points}.$$

3. Using the given conversion factors, we find

(a) the distance *d in rods* to be

$$d = 4.0 \text{ furlongs} = \frac{(4.0 \text{ furlongs})(201.168 \text{ m/furlong})}{5.0292 \text{ m/rod}} = 160 \text{ rods},$$

(b) and that distance *in chains* to be

$$d = \frac{(4.0 \text{ furlongs})(201.168 \text{ m/furlong})}{20.117 \text{ m/chain}} = 40 \text{ chains}.$$

4. The conversion factors $1 \text{ gry} = 1/10 \text{ line}$, $1 \text{ line} = 1/12 \text{ inch}$ and 1 point = 1/72 inch imply that

$$1 \text{ gry} = (1/10)(1/12)(72 \text{ points}) = 0.60 \text{ point}.$$

Thus, $1 \text{ gry}^2 = (0.60 \text{ point})^2 = 0.36 \text{ point}^2$, which means that $0.50 \text{ gry}^2 = 0.18 \text{ point}^2$.

5. Various geometric formulas are given in Appendix E.

(a) Expressing the radius of the Earth as

$$R = \left(6.37 \times 10^{6}\ \text{m}\right)\left(10^{-3}\ \text{km/m}\right) = 6.37 \times 10^{3}\ \text{km},$$

its circumference is $s = 2\pi R = 2\pi(6.37 \times 10^{3}\ \text{km}) = 4.00 \times 10^{4}\ \text{km}$.

(b) The surface area of Earth is $A = 4\pi R^2 = 4\pi\left(6.37 \times 10^{3}\ \text{km}\right)^2 = 5.10 \times 10^{8}\ \text{km}^2$.

(c) The volume of Earth is $V = \dfrac{4\pi}{3}R^3 = \dfrac{4\pi}{3}\left(6.37 \times 10^{3}\ \text{km}\right)^3 = 1.08 \times 10^{12}\ \text{km}^3$.

6. From Figure 1.6, we see that 212 S is equivalent to 258 W and 212 – 32 = 180 S is equivalent to 216 – 60 = 156 Z. The information allows us to convert S to W or Z.

(a) In units of W, we have

$$50.0 \text{ S} = (50.0 \text{ S})\left(\frac{258 \text{ W}}{212 \text{ S}}\right) = 60.8 \text{ W}$$

(b) In units of Z, we have

$$50.0 \text{ S} = (50.0 \text{ S})\left(\frac{156 \text{ Z}}{180 \text{ S}}\right) = 43.3 \text{ Z}$$

7. The volume of ice is given by the product of the semicircular surface area and the thickness. The area of the semicircle is $A = \pi r^2/2$, where r is the radius. Therefore, the volume is

$$V = \frac{\pi}{2} r^2 z$$

where z is the ice thickness. Since there are 10^3 m in 1 km and 10^2 cm in 1 m, we have

$$r = (2000\,\text{km})\left(\frac{10^3\ \text{m}}{1\,\text{km}}\right)\left(\frac{10^2\ \text{cm}}{1\,\text{m}}\right) = 2000\times 10^5\ \text{cm}.$$

In these units, the thickness becomes

$$z = 3000\,\text{m} = (3000\,\text{m})\left(\frac{10^2\ \text{cm}}{1\,\text{m}}\right) = 3000\times 10^2\ \text{cm}$$

which yields $V = \frac{\pi}{2}\left(2000\times 10^5\ \text{cm}\right)^2\left(3000\times 10^2\ \text{cm}\right) = 1.9\times 10^{22}\ \text{cm}^3.$

8. We make use of Table 1-6.

(a) We look at the first ("cahiz") column: 1 fanega is equivalent to what amount of cahiz? We note from the already completed part of the table that 1 cahiz equals a dozen fanega. Thus, 1 fanega = $\frac{1}{12}$ cahiz, or 8.33×10^{-2} cahiz. Similarly, "1 cahiz = 48 cuartilla" (in the already completed part) implies that 1 cuartilla = $\frac{1}{48}$ cahiz, or 2.08×10^{-2} cahiz. Continuing in this way, the remaining entries in the first column are 6.94×10^{-3} and 3.47×10^{-3}.

(b) In the second ("fanega") column, we similarly find 0.250, 8.33×10^{-2}, and 4.17×10^{-2} for the last three entries.

(c) In the third ("cuartilla") column, we obtain 0.333 and 0.167 for the last two entries.

(d) Finally, in the fourth ("almude") column, we get $\frac{1}{2}$ = 0.500 for the last entry.

(e) Since the conversion table indicates that 1 almude is equivalent to 2 medios, our amount of 7.00 almudes must be equal to 14.0 medios.

(f) Using the value (1 almude = 6.94×10^{-3} cahiz) found in part (a), we conclude that 7.00 almudes is equivalent to 4.86×10^{-2} cahiz.

(g) Since each decimeter is 0.1 meter, then 55.501 cubic decimeters is equal to 0.055501 m^3 or 55501 cm^3. Thus, 7.00 almudes = $\frac{7.00}{12}$ fanega = $\frac{7.00}{12}(55501\ cm^3) = 3.24 \times 10^4\ cm^3$.

9. We use the conversion factors found in Appendix D.

$$1 \text{ acre} \cdot \text{ft} = (43{,}560 \text{ ft}^2) \cdot \text{ft} = 43{,}560 \text{ ft}^3$$

Since 2 in. = (1/6) ft, the volume of water that fell during the storm is

$$V = (26 \text{ km}^2)(1/6 \text{ ft}) = (26 \text{ km}^2)(3281 \text{ft/km})^2(1/6 \text{ ft}) = 4.66 \times 10^7 \text{ ft}^3.$$

Thus,

$$V = \frac{4.66 \times 10^7 \text{ ft}^3}{4.3560 \times 10^4 \text{ ft}^3 / \text{acre} \cdot \text{ft}} = 1.1 \times 10^3 \text{ acre} \cdot \text{ft}.$$

10. A day is equivalent to 86400 seconds and a meter is equivalent to a million micrometers, so

$$\frac{(3.7\,\mathrm{m})(10^6\,\mu\mathrm{m/m})}{(14\,\mathrm{day})(86400\,\mathrm{s/day})} = 3.1\ \mu\mathrm{m/s}.$$

11. A week is 7 days, each of which has 24 hours, and an hour is equivalent to 3600 seconds. Thus, two weeks (a fortnight) is 1209600 s. By definition of the micro prefix, this is roughly 1.21×10^{12} μs.

12. The metric prefixes (micro (μ), pico, nano, …) are given for ready reference on the inside front cover of the textbook (also, Table 1–2).

(a) $$1\ \mu\text{century} = \left(10^{-6}\ \text{century}\right)\left(\frac{100\ \text{y}}{1\ \text{century}}\right)\left(\frac{365\ \text{day}}{1\ \text{y}}\right)\left(\frac{24\ \text{h}}{1\ \text{day}}\right)\left(\frac{60\ \text{min}}{1\ \text{h}}\right) = 52.6\ \text{min}.$$

(b) The percent difference is therefore

$$\frac{52.6\ \text{min} - 50\ \text{min}}{52.6\ \text{min}} = 4.9\%.$$

13. (a) Presuming that a French decimal day is equivalent to a regular day, then the ratio of weeks is simply 10/7 or (to 3 significant figures) 1.43.

(b) In a regular day, there are 86400 seconds, but in the French system described in the problem, there would be 10^5 seconds. The ratio is therefore 0.864.

14. We denote the pulsar rotation rate *f* (for frequency).

$$f = \frac{1 \text{ rotation}}{1.55780644887275 \times 10^{-3} \text{ s}}$$

(a) Multiplying *f* by the time-interval t = 7.00 days (which is equivalent to 604800 s, if we ignore *significant figure* considerations for a moment), we obtain the number of rotations:

$$N = \left(\frac{1 \text{ rotation}}{1.55780644887275 \times 10^{-3} \text{ s}} \right) (604800 \text{ s}) = 388238218.4$$

which should now be rounded to 3.88×10^8 rotations since the time-interval was specified in the problem to three significant figures.

(b) We note that the problem specifies the *exact* number of pulsar revolutions (one million). In this case, our unknown is *t*, and an equation similar to the one we set up in part (a) takes the form $N = ft$, or

$$1 \times 10^6 = \left(\frac{1 \text{ rotation}}{1.55780644887275 \times 10^{-3} \text{ s}} \right) t$$

which yields the result t = 1557.80644887275 s (though students who do this calculation on their calculator might not obtain those last several digits).

(c) Careful reading of the problem shows that the time-uncertainty *per revolution* is $\pm 3 \times 10^{-17}$ s . We therefore expect that as a result of one million revolutions, the uncertainty should be $(\pm 3 \times 10^{-17})(1 \times 10^6) = \pm 3 \times 10^{-11}$ s .

15. The time on any of these clocks is a straight-line function of that on another, with slopes $\neq$ 1 and y-intercepts $\neq$ 0. From the data in the figure we deduce

$$t_C = \frac{2}{7}t_B + \frac{594}{7}, \quad t_B = \frac{33}{40}t_A - \frac{662}{5}.$$

These are used in obtaining the following results.

(a) We find

$$t'_B - t_B = \frac{33}{40}\left(t'_A - t_A\right) = 495 \text{ s}$$

when $t'_A - t_A = 600$ s.

(b) We obtain $t'_C - t_C = \frac{2}{7}\left(t'_B - t_B\right) = \frac{2}{7}\left(495\right) = 141$ s.

(c) Clock B reads $t_B = (33/40)(400) - (662/5) \approx 198$ s when clock A reads $t_A = 400$ s.

(d) From $t_C = 15 = (2/7)t_B + (594/7)$, we get $t_B \approx -245$ s.

16. Since a change of longitude equal to $360°$ corresponds to a 24 hour change, then one expects to change longitude by $360°/24 = 15°$ before resetting one's watch by 1.0 h.

17. None of the clocks advance by exactly 24 h in a 24-h period but this is not the most important criterion for judging their quality for measuring time intervals. What is important is that the clock advance by the same amount in each 24-h period. The clock reading can then easily be adjusted to give the correct interval. If the clock reading jumps around from one 24-h period to another, it cannot be corrected since it would impossible to tell what the correction should be. The following gives the corrections (in seconds) that must be applied to the reading on each clock for each 24-h period. The entries were determined by subtracting the clock reading at the end of the interval from the clock reading at the beginning.

CLOCK	Sun. -Mon.	Mon. -Tues.	Tues. -Wed.	Wed. -Thurs.	Thurs. -Fri.	Fri. -Sat.
A	–16	–16	–15	–17	–15	–15
B	–3	+5	–10	+5	+6	–7
C	–58	–58	–58	–58	–58	–58
D	+67	+67	+67	+67	+67	+67
E	+70	+55	+2	+20	+10	+10

Clocks C and D are both good timekeepers in the sense that each is consistent in its daily drift (relative to WWF time); thus, C and D are easily made "perfect" with simple and predictable corrections. The correction for clock C is less than the correction for clock D, so we judge clock C to be the best and clock D to be the next best. The correction that must be applied to clock A is in the range from 15 s to 17s. For clock B it is the range from -5 s to +10 s, for clock E it is in the range from -70 s to -2 s. After C and D, A has the smallest range of correction, B has the next smallest range, and E has the greatest range. From best to worst, the ranking of the clocks is C, D, A, B, E.

18. The last day of the 20 centuries is longer than the first day by

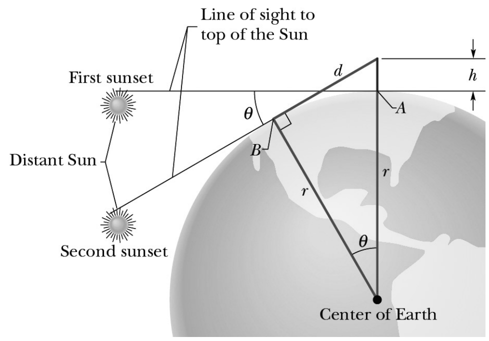
$$(20 \text{ century})\,(0.001 \text{ s/century}) = 0.02 \text{ s}.$$

The average day during the 20 centuries is (0 + 0.02)/2 = 0.01 s longer than the first day. Since the increase occurs uniformly, the cumulative effect T is

$$\begin{aligned} T &= (\text{average increase in length of a day})(\text{number of days}) \\ &= \left(\frac{0.01 \text{ s}}{\text{day}}\right)\left(\frac{365.25 \text{ day}}{\text{y}}\right)(2000 \text{ y}) \\ &= 7305 \text{ s} \end{aligned}$$

or roughly two hours.

19. When the Sun first disappears while lying down, your line of sight to the top of the Sun is tangent to the Earth's surface at point A shown in the figure. As you stand, elevating your eyes by a height h, the line of sight to the Sun is tangent to the Earth's surface at point B.



Let d be the distance from point B to your eyes. From Pythagorean theorem, we have

$$d^2 + r^2 = (r+h)^2 = r^2 + 2rh + h^2$$

or $d^2 = 2rh + h^2$, where r is the radius of the Earth. Since $r \gg h$, the second term can be dropped, leading to $d^2 \approx 2rh$. Now the angle between the two radii to the two tangent points A and B is θ, which is also the angle through which the Sun moves about Earth during the time interval t = 11.1 s. The value of θ can be obtained by using

$$\frac{\theta}{360°} = \frac{t}{24 \text{ h}}.$$

This yields

$$\theta = \frac{(360°)(11.1 \text{ s})}{(24 \text{ h})(60 \text{ min/h})(60 \text{ s/min})} = 0.04625°.$$

Using $d = r\tan\theta$, we have $d^2 = r^2\tan^2\theta = 2rh$, or

$$r = \frac{2h}{\tan^2\theta}$$

Using the above value for θ and h = 1.7 m, we have $r = 5.2 \times 10^6$ m.

20. The density of gold is

$$\rho = \frac{m}{V} = \frac{19.32\ \text{g}}{1\ \text{cm}^3} = 19.32\ \text{g/cm}^3.$$

(a) We take the volume of the leaf to be its area A multiplied by its thickness z. With density $\rho = 19.32\ \text{g/cm}^3$ and mass $m = 27.63$ g, the volume of the leaf is found to be

$$V = \frac{m}{\rho} = 1.430\ \text{cm}^3.$$

We convert the volume to SI units:

$$V = \left(1.430\ \text{cm}^3\right)\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^3 = 1.430 \times 10^{-6}\ \text{m}^3.$$

Since $V = Az$ with $z = 1 \times 10^{-6}$ m (metric prefixes can be found in Table 1–2), we obtain

$$A = \frac{1.430 \times 10^{-6}\ \text{m}^3}{1 \times 10^{-6}\ \text{m}} = 1.430\ \text{m}^2.$$

(b) The volume of a cylinder of length ℓ is $V = A\ell$ where the cross-section area is that of a circle: $A = \pi r^2$. Therefore, with $r = 2.500 \times 10^{-6}$ m and $V = 1.430 \times 10^{-6}\ \text{m}^3$, we obtain

$$\ell = \frac{V}{\pi r^2} = 7.284 \times 10^4\ \text{m} = 72.84\ \text{km}.$$

21. We introduce the notion of density:

$$\rho = \frac{m}{V}$$

and convert to SI units: 1 g = 1×10^{-3} kg.

(a) For volume conversion, we find 1 cm^3 = $(1 \times 10^{-2}\text{m})^3 = 1 \times 10^{-6}\text{m}^3$. Thus, the density in kg/m^3 is

$$1\ \text{g/cm}^3 = \left(\frac{1\ \text{g}}{\text{cm}^3}\right)\left(\frac{10^{-3}\ \text{kg}}{\text{g}}\right)\left(\frac{\text{cm}^3}{10^{-6}\ \text{m}^3}\right) = 1 \times 10^3\ \text{kg/m}^3 .$$

Thus, the mass of a cubic meter of water is 1000 kg.

(b) We divide the mass of the water by the time taken to drain it. The mass is found from $M = \rho V$ (the product of the volume of water and its density):

$$M = \left(5700\ \text{m}^3\right)\left(1 \times 10^3\ \text{kg/m}^3\right) = 5.70 \times 10^6\ \text{kg}.$$

The time is t = (10h)(3600 s/h) = 3.6×10^4 s, so the *mass flow rate R* is

$$R = \frac{M}{t} = \frac{5.70 \times 10^6\ \text{kg}}{3.6 \times 10^4\ \text{s}} = 158\ \text{kg/s}.$$

22. (a) We find the volume in cubic centimeters

$$193 \text{ gal} = (193 \text{ gal})\left(\frac{231 \text{ in}^3}{1 \text{ gal}}\right)\left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right)^3 = 7.31 \times 10^5 \text{ cm}^3$$

and subtract this from 1×10^6 cm^3 to obtain 2.69×10^5 cm^3. The conversion gal $\rightarrow$ in^3 is given in Appendix D (immediately below the table of Volume conversions).

(b) The volume found in part (a) is converted (by dividing by (100 cm/m)^3) to 0.731 m^3, which corresponds to a mass of

$$\left(1000 \text{ kg/m}^3\right)\left(0.731 \text{ m}^2\right) = 731 \text{ kg}$$

using the density given in the problem statement. At a rate of 0.0018 kg/min, this can be filled in

$$\frac{731 \text{kg}}{0.0018 \text{ kg/min}} = 4.06 \times 10^5 \text{ min} = 0.77 \text{ y}$$

after dividing by the number of minutes in a year (365 days)(24 h/day) (60 min/h).

23. If M_E is the mass of Earth, m is the average mass of an atom in Earth, and N is the number of atoms, then $M_E = Nm$ or $N = M_E/m$. We convert mass m to kilograms using Appendix D ($1\text{ u} = 1.661 \times 10^{-27}$ kg). Thus,

$$N = \frac{M_E}{m} = \frac{5.98 \times 10^{24}\ \text{kg}}{(40\ \text{u})\left(1.661 \times 10^{-27}\ \text{kg/u}\right)} = 9.0 \times 10^{49}.$$

24. (a) The volume of the cloud is $(3000 \text{ m})\pi(1000 \text{ m})^2 = 9.4 \times 10^9 \text{ m}^3$. Since each cubic meter of the cloud contains from 50×10^6 to 500×10^6 water drops, then we conclude that the entire cloud contains from 4.7×10^{18} to 4.7×10^{19} drops. Since the volume of each drop is $\frac{4}{3}\pi(10 \times 10^{-6} \text{ m})^3 = 4.2 \times 10^{-15} \text{ m}^3$, then the total volume of water in a cloud is from 2×10^3 to 2×10^4 m^3.

(b) Using the fact that $1 \text{ L} = 1\times10^3 \text{cm}^3 = 1\times10^{-3} \text{m}^3$, the amount of water estimated in part (a) would fill from 2×10^6 to 2×10^7 bottles.

(c) At 1000 kg for every cubic meter, the mass of water is from two million to twenty million kilograms. The coincidence in numbers between the results of parts (b) and (c) of this problem is due to the fact that each liter has a mass of one kilogram when water is at its normal density (under standard conditions).

25. We introduce the notion of density, $\rho = m/V$, and convert to SI units: 1000 g = 1 kg, and 100 cm = 1 m.

(a) The density ρ of a sample of iron is

$$\rho = \left(7.87\ \text{g/cm}^3\right)\left(\frac{1\ \text{kg}}{1000\ \text{g}}\right)\left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^3 = 7870\ \text{kg/m}^3.$$

If we ignore the empty spaces between the close-packed spheres, then the density of an individual iron atom will be the same as the density of any iron sample. That is, if M is the mass and V is the volume of an atom, then

$$V = \frac{M}{\rho} = \frac{9.27 \times 10^{-26}\ \text{kg}}{7.87 \times 10^{3}\ \text{kg/m}^3} = 1.18 \times 10^{-29}\ \text{m}^3.$$

(b) We set $V = 4\pi R^3/3$, where R is the radius of an atom (Appendix E contains several geometry formulas). Solving for R, we find

$$R = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(\frac{3\left(1.18 \times 10^{-29}\ \text{m}^3\right)}{4\pi}\right)^{1/3} = 1.41 \times 10^{-10}\ \text{m}.$$

The center-to-center distance between atoms is twice the radius, or 2.82×10^{-10} m.

26. If we estimate the "typical" large domestic cat mass as 10 kg, and the "typical" atom (in the cat) as 10 u $\approx 2 \times 10^{-26}$ kg, then there are roughly (10 kg)/(2×10^{-26} kg) $\approx 5 \times 10^{26}$ atoms. This is close to being a factor of a thousand greater than Avogradro's number. Thus this is roughly a kilomole of atoms.

27. According to Appendix D, a nautical mile is 1.852 km, so 24.5 nautical miles would be 45.374 km. Also, according to Appendix D, a mile is 1.609 km, so 24.5 miles is 39.4205 km. The difference is 5.95 km.

28. The metric prefixes (micro (μ), pico, nano, …) are given for ready reference on the inside front cover of the textbook (see also Table 1–2). The surface area A of each grain of sand of radius r = 50 μm = 50 × 10^{-6} m is given by $A = 4\pi(50 \times 10^{-6})^2 = 3.14 \times 10^{-8}$ m^2 (Appendix E contains a variety of geometry formulas). We introduce the notion of density, $\rho = m/V$, so that the mass can be found from $m = \rho V$, where ρ = 2600 kg/m^3. Thus, using $V = 4\pi r^3/3$, the mass of each grain is

$$m = \rho V = \rho\left(\frac{4\pi r^3}{3}\right) = \left(2600\ \frac{\text{kg}}{\text{m}^3}\right)\frac{4\pi\left(50\times10^{-6}\ \text{m}\right)^3}{3} = 1.36\times10^{-9}\ \text{kg}.$$

We observe that (because a cube has six equal faces) the indicated surface area is 6 m^2. The number of spheres (the grains of sand) N that have a total surface area of 6 m^2 is given by

$$N = \frac{6\ \text{m}^2}{3.14\times10^{-8}\ \text{m}^2} = 1.91\times10^{8}.$$

Therefore, the total mass M is $M = Nm = \left(1.91\times10^{8}\right)\left(1.36\times10^{-9}\ \text{kg}\right) = 0.260$ kg.

29. The volume of the section is (2500 m)(800 m)(2.0 m) = 4.0×10^6 m^3. Letting "*d*" stand for the thickness of the mud after it has (uniformly) distributed in the valley, then its volume there would be (400 m)(400 m)*d*. Requiring these two volumes to be equal, we can solve for *d*. Thus, *d* = 25 m. The volume of a small part of the mud over a patch of area of 4.0 m^2 is (4.0)*d* = 100 m^3. Since each cubic meter corresponds to a mass of 1900 kg (stated in the problem), then the mass of that small part of the mud is 1.9×10^5 kg.

30. To solve the problem, we note that the first derivative of the function with respect to time gives the rate. Setting the rate to zero gives the time at which an extreme value of the variable mass occurs; here that extreme value is a maximum.

(a) Differentiating $m(t) = 5.00t^{0.8} - 3.00t + 20.00$ with respect to t gives

$$\frac{dm}{dt} = 4.00t^{-0.2} - 3.00.$$

The water mass is the greatest when $dm/dt = 0,$ or at $t = (4.00/3.00)^{1/0.2} = 4.21\,\text{s}.$

(b) At $t = 4.21\,\text{s},$ the water mass is

$$m(t = 4.21\,\text{s}) = 5.00(4.21)^{0.8} - 3.00(4.21) + 20.00 = 23.2\,\text{g}.$$

(c) The rate of mass change at $t = 2.00\,\text{s}$ is

$$\begin{aligned}\left.\frac{dm}{dt}\right|_{t=2.00\,\text{s}} &= \left[4.00(2.00)^{-0.2} - 3.00\right]\text{g/s} = 0.48\ \text{g/s} = 0.48\frac{\text{g}}{\text{s}}\cdot\frac{1\,\text{kg}}{1000\ \text{g}}\cdot\frac{60\ \text{s}}{1\ \text{min}} \\ &= 2.89\times10^{-2}\ \text{kg/min}.\end{aligned}$$

(d) Similarly, the rate of mass change at $t = 5.00\,\text{s}$ is

$$\begin{aligned}\left.\frac{dm}{dt}\right|_{t=2.00\,\text{s}} &= \left[4.00(5.00)^{-0.2} - 3.00\right]\text{g/s} = -0.101\ \text{g/s} = -0.101\frac{\text{g}}{\text{s}}\cdot\frac{1\,\text{kg}}{1000\ \text{g}}\cdot\frac{60\ \text{s}}{1\ \text{min}} \\ &= -6.05\times10^{-3}\ \text{kg/min}.\end{aligned}$$

31. The mass density of the candy is

$$\rho = \frac{m}{V} = \frac{0.0200 \text{ g}}{50.0 \text{ mm}^3} = 4.00\times10^{-4} \text{ g/mm}^3 = 4.00\times10^{-4} \text{ kg/cm}^3.$$

If we neglect the volume of the empty spaces between the candies, then the total mass of the candies in the container when filled to height h is $M = \rho A h$, where $A = (14.0 \text{ cm})(17.0 \text{ cm}) = 238 \text{ cm}^2$ is the base area of the container that remains unchanged. Thus, the rate of mass change is given by

$$\begin{aligned}\frac{dM}{dt} = \frac{d(\rho A h)}{dt} = \rho A \frac{dh}{dt} &= (4.00\times10^{-4} \text{ kg/cm}^3)(238 \text{ cm}^2)(0.250 \text{ cm/s}) \\ &= 0.0238 \text{ kg/s} = 1.43 \text{ kg/min}.\end{aligned}$$

32. Table 7 can be completed as follows:

(a) It should be clear that the first column (under "wey") is the reciprocal of the first row – so that $\frac{9}{10} = 0.900$, $\frac{3}{40} = 7.50 \times 10^{-2}$, and so forth. Thus, 1 pottle = 1.56×10^{-3} wey and 1 gill = 8.32×10^{-6} wey are the last two entries in the first column.

(b) In the second column (under "chaldron"), clearly we have 1 chaldron = 1 caldron (that is, the entries along the "diagonal" in the table must be 1's). To find out how many chaldron are equal to one bag, we note that 1 wey = 10/9 chaldron = 40/3 bag so that $\frac{1}{12}$ chaldron = 1 bag. Thus, the next entry in that second column is $\frac{1}{12} = 8.33 \times 10^{-2}$. Similarly, 1 pottle = 1.74×10^{-3} chaldron and 1 gill = 9.24×10^{-6} chaldron.

(c) In the third column (under "bag"), we have 1 chaldron = 12.0 bag, 1 bag = 1 bag, 1 pottle = 2.08×10^{-2} bag, and 1 gill = 1.11×10^{-4} bag.

(d) In the fourth column (under "pottle"), we find 1 chaldron = 576 pottle, 1 bag = 48 pottle, 1 pottle = 1 pottle, and 1 gill = 5.32×10^{-3} pottle.

(e) In the last column (under "gill"), we obtain 1 chaldron = 1.08×10^{5} gill, 1 bag = 9.02×10^{3} gill, 1 pottle = 188 gill, and, of course, 1 gill = 1 gill.

(f) Using the information from part (c), 1.5 chaldron = (1.5)(12.0) = 18.0 bag. And since each bag is 0.1091 m^3 we conclude 1.5 chaldron = (18.0)(0.1091) = 1.96 m^3.

33. The first two conversions are easy enough that a *formal* conversion is not especially called for, but in the interest of *practice makes perfect* we go ahead and proceed formally:

(a) $11 \text{ tuffets} = (11 \text{ tuffets})\left(\dfrac{2 \text{ peck}}{1 \text{ tuffet}}\right) = 22 \text{ pecks}$.

(b) $11 \text{ tuffets} = (11 \text{ tuffets})\left(\dfrac{0.50 \text{ Imperial bushel}}{1 \text{ tuffet}}\right) = 5.5 \text{ Imperial bushels}$.

(c) $11 \text{ tuffets} = (5.5 \text{ Imperial bushel})\left(\dfrac{36.3687 \text{ L}}{1 \text{ Imperial bushel}}\right) \approx 200 \text{ L}$.

34. (a) Using the fact that the area A of a rectangle is (width) × (length), we find

$$\begin{aligned} A_{\text{total}} &= (3.00\,\text{acre}) + (25.0\,\text{perch})(4.00\,\text{perch}) \\ &= (3.00\text{ acre})\left(\frac{(40\text{ perch})(4\text{ perch})}{1\,\text{acre}}\right) + 100\,\text{perch}^2 \\ &= 580\text{ perch}^2. \end{aligned}$$

We multiply this by the perch^2 → rood conversion factor (1 rood/40 perch^2) to obtain the answer: A_{total} = 14.5 roods.

(b) We convert our intermediate result in part (a):

$$A_{\text{total}} = (580\text{ perch}^2)\left(\frac{16.5\,\text{ft}}{1\,\text{perch}}\right)^2 = 1.58 \times 10^5\text{ ft}^2.$$

Now, we use the feet → meters conversion given in Appendix D to obtain

$$A_{\text{total}} = (1.58 \times 10^5\text{ ft}^2)\left(\frac{1\,\text{m}}{3.281\,\text{ft}}\right)^2 = 1.47 \times 10^4\text{ m}^2.$$

35. (a) Dividing 750 miles by the expected "40 miles per gallon" leads the tourist to believe that the car should need 18.8 gallons (in the U.S.) for the trip.

(b) Dividing the two numbers given (to high precision) in the problem (and rounding off) gives the conversion between U.K. and U.S. gallons. The U.K. gallon is larger than the U.S gallon by a factor of 1.2. Applying this to the result of part (a), we find the answer for part (b) is 22.5 gallons.

36. The customer expects a volume $V_1 = 20 \times 7056$ in^3 and receives $V_2 = 20 \times 5826$ in^3, the difference being $\Delta V = V_1 - V_2 = 24600\ \text{in}^3$, or

$$\Delta V = \left(24600\ \text{in}^3\right)\left(\frac{2.54\,\text{cm}}{1\ \text{inch}}\right)^3\left(\frac{1\,\text{L}}{1000\ \text{cm}^3}\right) = 403\,\text{L}$$

where Appendix D has been used.

37. (a) Using Appendix D, we have 1 ft = 0.3048 m, 1 gal = 231 in.^{3}, and 1 in.^{3} = 1.639×10^{-2} L. From the latter two items, we find that 1 gal = 3.79 L. Thus, the quantity 460 ft^2/gal becomes

$$460\ \text{ft}^2/\text{gal} = \left(\frac{460\ \text{ft}^2}{\text{gal}}\right)\left(\frac{1\ \text{m}}{3.28\ \text{ft}}\right)^2\left(\frac{1\ \text{gal}}{3.79\ \text{L}}\right) = 11.3\ \text{m}^2/\text{L}.$$

(b) Also, since 1 m^3 is equivalent to 1000 L, our result from part (a) becomes

$$11.3\ \text{m}^2/\text{L} = \left(\frac{11.3\ \text{m}^2}{\text{L}}\right)\left(\frac{1000\,\text{L}}{1\ \text{m}^3}\right) = 1.13 \times 10^4\ \text{m}^{-1}.$$

(c) The inverse of the original quantity is $(460\ \text{ft}^2/\text{gal})^{-1} = 2.17 \times 10^{-3}$ gal/ft^2.

(d) The answer in (c) represents the volume of the paint (in gallons) needed to cover a square foot of area. From this, we could also figure the paint thickness [it turns out to be about a tenth of a millimeter, as one sees by taking the reciprocal of the answer in part (b)].

38. The total volume V of the real house is that of a triangular prism (of height h = 3.0 m and base area $A = 20 \times 12 = 240\ \text{m}^2$) in addition to a rectangular box (height h' = 6.0 m and same base). Therefore,

$$V = \frac{1}{2}hA + h'A = \left(\frac{h}{2} + h'\right)A = 1800\ \text{m}^3.$$

(a) Each dimension is reduced by a factor of 1/12, and we find

$$V_{\text{doll}} = \left(1800\ \text{m}^3\right)\left(\frac{1}{12}\right)^3 \approx 1.0\ \text{m}^3.$$

(b) In this case, each dimension (relative to the real house) is reduced by a factor of 1/144. Therefore,

$$V_{\text{miniature}} = \left(1800\ \text{m}^3\right)\left(\frac{1}{144}\right)^3 \approx 6.0 \times 10^{-4}\ \text{m}^3.$$

39. Using the (exact) conversion 2.54 cm = 1 in. we find that 1 ft = (12)(2.54)/100 = 0.3048 m (which also can be found in Appendix D). The volume of a cord of wood is $8 \times 4 \times 4 = 128$ ft^3, which we convert (multiplying by 0.3048^3) to 3.6 m^3. Therefore, one cubic meter of wood corresponds to $1/3.6 \approx 0.3$ cord.

40. (a) In atomic mass units, the mass of one molecule is (16 + 1 + 1)u = 18 u. Using Eq. 1–9, we find

$$18\,\text{u} = (18\,\text{u})\left(\frac{1.6605402\times 10^{-27}\,\text{kg}}{1\,\text{u}}\right) = 3.0\times 10^{-26}\,\text{kg}.$$

(b) We divide the total mass by the mass of each molecule and obtain the (approximate) number of water molecules:

$$N \approx \frac{1.4\times 10^{21}}{3.0\times 10^{-26}} \approx 5\times 10^{46}.$$

41. (a) The difference between the total amounts in "freight" and "displacement" tons, $(8 - 7)(73) = 73$ barrels bulk, represents the extra M&M's that are shipped. Using the conversions in the problem, this is equivalent to $(73)(0.1415)(28.378) = 293$ U.S. bushels.

(b) The difference between the total amounts in "register" and "displacement" tons, $(20 - 7)(73) = 949$ barrels bulk, represents the extra M&M's are shipped. Using the conversions in the problem, this is equivalent to $(949)(0.1415)(28.378) = 3.81 \times 10^3$ U.S. bushels.

42. (a) The receptacle is a volume of (40 cm)(40 cm)(30 cm) = 48000 cm^3 = 48 L = (48)(16)/11.356 = 67.63 standard bottles, which is a little more than 3 nebuchadnezzars (the largest bottle indicated). The remainder, 7.63 standard bottles, is just a little less than 1 methuselah. Thus, the answer to part (a) is 3 nebuchadnezzars and 1 methuselah.

(b) Since 1 methuselah.= 8 standard bottles, then the extra amount is 8 − 7.63 = 0.37 standard bottle.

(c) Using the conversion factor 16 standard bottles = 11.356 L, we have

$$0.37 \text{ standard bottle} = (0.37 \text{ standard bottle})\left(\frac{11.356 \text{ L}}{16 \text{ standard bottles}}\right) = 0.26 \text{ L}.$$

43. The volume of one unit is 1 $cm^3 = 1 \times 10^{-6}$ m^3, so the volume of a mole of them is 6.02×10^{23} $cm^3 = 6.02 \times 10^{17}$ m^3. The cube root of this number gives the edge length: $8.4\times10^5\ m^3$. This is equivalent to roughly 8×10^2 kilometers.

44. Equation 1-9 gives (to very high precision!) the conversion from atomic mass units to kilograms. Since this problem deals with the ratio of total mass (1.0 kg) divided by the mass of one atom (1.0 u, but converted to kilograms), then the computation reduces to simply taking the reciprocal of the number given in Eq. 1-9 and rounding off appropriately. Thus, the answer is 6.0×10^{26}.

45. We convert meters to astronomical units, and seconds to minutes, using

$$\begin{aligned} 1000\ \text{m} &= 1\ \text{km} \\ 1\ \text{AU} &= 1.50 \times 10^8\ \text{km} \\ 60\ \text{s} &= 1\ \text{min}. \end{aligned}$$

Thus, 3.0×10^8 m/s becomes

$$\left(\frac{3.0 \times 10^8\ \text{m}}{\text{s}}\right)\left(\frac{1\ \text{km}}{1000\ \text{m}}\right)\left(\frac{\text{AU}}{1.50 \times 10^8\ \text{km}}\right)\left(\frac{60\ \text{s}}{\text{min}}\right) = 0.12\ \text{AU/min}.$$

46. The volume of the water that fell is

$$\begin{aligned} V &= \left(26 \text{ km}^2\right)(2.0 \text{ in.}) = \left(26 \text{ km}^2\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)^2 (2.0 \text{ in.})\left(\frac{0.0254 \text{ m}}{1 \text{ in.}}\right) \\ &= \left(26 \times 10^6 \text{ m}^2\right)(0.0508 \text{ m}) \\ &= 1.3 \times 10^6 \text{ m}^3. \end{aligned}$$

We write the mass-per-unit-volume (density) of the water as:

$$\rho = \frac{m}{V} = 1 \times 10^3 \text{ kg/m}^3.$$

The mass of the water that fell is therefore given by $m = \rho V$:

$$m = \left(1 \times 10^3 \text{ kg/m}^3\right)\left(1.3 \times 10^6 \text{ m}^3\right) = 1.3 \times 10^9 \text{ kg}.$$

47. A million milligrams comprise a kilogram, so 2.3 kg/week is 2.3×10^6 mg/week. Figuring 7 days a week, 24 hours per day, 3600 second per hour, we find 604800 seconds are equivalent to one week. Thus, $(2.3 \times 10^6$ mg/week)/(604800 s/week) = 3.8 mg/s.

48. The mass of the pig is 3.108 slugs, or (3.108)(14.59) = 45.346 kg. Referring now to the corn, a U.S. bushel is 35.238 liters. Thus, a value of 1 for the *corn-hog ratio* would be equivalent to 35.238/45.346 = 0.7766 in the indicated metric units. Therefore, a value of 5.7 for the *ratio* corresponds to 5.7(0.777) ≈ 4.4 in the indicated metric units.

49. Two jalapeño peppers have spiciness = 8000 SHU, and this amount multiplied by 400 (the number of people) is 3.2×10^6 SHU, which is roughly ten times the SHU value for a single habanero pepper. More precisely, 10.7 habanero peppers will provide that total required SHU value.

50. The volume removed in one year is

$$V = (75 \times 10^4 \text{ m}^2)(26 \text{ m}) \approx 2 \times 10^7 \text{ m}^3$$

which we convert to cubic kilometers: $V = \left(2 \times 10^7 \text{ m}^3\right)\left(\dfrac{1 \text{ km}}{1000 \text{ m}}\right)^3 = 0.020 \text{ km}^3$.

51. The number of seconds in a year is 3.156×10^7. This is listed in Appendix D and results from the product

$$(365.25 \text{ day/y})\ (24 \text{ h/day})\ (60 \text{ min/h})\ (60 \text{ s/min}).$$

(a) The number of shakes in a second is 10^8; therefore, there are indeed more shakes per second than there are seconds per year.

(b) Denoting the age of the universe as 1 u-day (or 86400 u-sec), then the time during which humans have existed is given by

$$\frac{10^6}{10^{10}} = 10^{-4} \text{ u-day},$$

which may also be expressed as $\left(10^{-4} \text{ u-day}\right)\left(\dfrac{86400 \text{ u-sec}}{1 \text{ u-day}}\right) = 8.6 \text{ u-sec}.$

52. Abbreviating wapentake as "wp" and assuming a hide to be 110 acres, we set up the ratio 25 wp/11 barn along with appropriate conversion factors:

$$\frac{(25\ \text{wp})\left(\frac{100\ \text{hide}}{1\ \text{wp}}\right)\left(\frac{110\ \text{acre}}{1\ \text{hide}}\right)\left(\frac{4047\ \text{m}^2}{1\ \text{acre}}\right)}{(11\ \text{barn})\left(\frac{1\times 10^{-28}\ \text{m}^2}{1\ \text{barn}}\right)} \approx 1\times 10^{36}.$$

53. (a) Squaring the relation 1 ken = 1.97 m, and setting up the ratio, we obtain

$$\frac{1\ \text{ken}^2}{1\ \text{m}^2} = \frac{1.97^2\ \text{m}^2}{1\ \text{m}^2} = 3.88.$$

(b) Similarly, we find

$$\frac{1\ \text{ken}^3}{1\ \text{m}^3} = \frac{1.97^3\ \text{m}^3}{1\ \text{m}^3} = 7.65.$$

(c) The volume of a cylinder is the circular area of its base multiplied by its height. Thus,

$$\pi r^2\, h = \pi\left(3.00\right)^2\left(5.50\right) = 156\ \text{ken}^3.$$

(d) If we multiply this by the result of part (b), we determine the volume in cubic meters: $(155.5)(7.65) = 1.19 \times 10^3\ \text{m}^3$.

54. The mass in kilograms is

$$(28.9 \text{ piculs})\left(\frac{100\,\text{gin}}{1\,\text{picul}}\right)\left(\frac{16\,\text{tahil}}{1\,\text{gin}}\right)\left(\frac{10\,\text{chee}}{1\,\text{tahil}}\right)\left(\frac{10\text{ hoon}}{1\text{ chee}}\right)\left(\frac{0.3779\,\text{g}}{1\,\text{hoon}}\right)$$

which yields 1.747×10^6 g or roughly 1.75×10^3 kg.

55. In the simplest approach, we set up a ratio for the total increase in *horizontal depth x* (where $\Delta x = 0.05$ m is the increase in horizontal depth per step)

$$x = N_{\text{steps}}\,\Delta x = \left(\frac{4.57}{0.19}\right)(0.05\text{ m}) = 1.2\text{ m}.$$

However, we can approach this more carefully by noting that if there are $N = 4.57/.19 \approx 24$ rises then under normal circumstances we would expect $N - 1 = 23$ runs (horizontal pieces) in that staircase. This would yield (23)(0.05 m) = 1.15 m, which - to two significant figures - agrees with our first result.

56. Since one atomic mass unit is $1\,\text{u} = 1.66\times10^{-24}$ g (see Appendix D), the mass of one mole of atoms is about $m = (1.66\times10^{-24}\ \text{g})(6.02\times10^{23}) = 1\,\text{g}$. On the other hand, the mass of one mole of atoms in the common Eastern mole is

$$m' = \frac{75\ \text{g}}{7.5} = 10\ \text{g}$$

Therefore, in atomic mass units, the average mass of one atom in the common Eastern mole is

$$\frac{m'}{N_A} = \frac{10\ \text{g}}{6.02\times10^{23}} = 1.66\times10^{-23}\ \text{g} = 10\ \text{u}.$$

57. (a) When θ is measured in radians, it is equal to the arc length s divided by the radius R. For a very large radius circle and small value of θ, such as we deal with in Fig. 1–9, the arc may be approximated as the straight line-segment of length 1 AU. First, we convert θ = 1 arcsecond to radians:

$$(1\text{ arcsecond})\left(\frac{1\text{ arcminute}}{60\text{ arcsecond}}\right)\left(\frac{1°}{60\text{ arcminute}}\right)\left(\frac{2\pi\text{ radian}}{360°}\right)$$

which yields $\theta = 4.85 \times 10^{-6}$ rad. Therefore, one parsec is

$$R_o = \frac{s}{\theta} = \frac{1\text{ AU}}{4.85 \times 10^{-6}} = 2.06 \times 10^{5}\ \text{AU}.$$

Now we use this to convert R = 1 AU to parsecs:

$$R = (1\text{ AU})\left(\frac{1\text{ pc}}{2.06 \times 10^{5}\ \text{AU}}\right) = 4.9 \times 10^{-6}\ \text{pc}.$$

(b) Also, since it is straightforward to figure the number of seconds in a year (about 3.16×10^7 s), and (for constant speeds) distance = speed × time, we have

$$1\,\text{ly} = (186{,}000\,\text{mi/s})\,(3.16 \times 10^{7}\,\text{s})\ 5.9 \times 10^{12}\ \text{mi}$$

which we convert to AU by dividing by 92.6×10^6 (given in the problem statement), obtaining 6.3×10^4 AU. Inverting, the result is 1 AU = $1/6.3 \times 10^4 = 1.6 \times 10^{-5}$ ly.

58. The volume of the filled container is 24000 cm^3 = 24 liters, which (using the conversion given in the problem) is equivalent to 50.7 pints (U.S). The expected number is therefore in the range from 1317 to 1927 Atlantic oysters. Instead, the number received is in the range from 406 to 609 Pacific oysters. This represents a shortage in the range of roughly 700 to 1500 oysters (the answer to the problem). Note that the minimum value in our answer corresponds to the minimum Atlantic minus the maximum Pacific, and the maximum value corresponds to the maximum Atlantic minus the minimum Pacific.

59. (a) For the minimum (43 cm) case, 9 cubit converts as follows:

$$9\,\text{cubit} = (9\,\text{cubit})\left(\frac{0.43\,\text{m}}{1\,\text{cubit}}\right) = 3.9\,\text{m}.$$

And for the maximum (43 cm) case we obtain

$$9\,\text{cubit} = (9\,\text{cubit})\left(\frac{0.53\,\text{m}}{1\,\text{cubit}}\right) = 4.8\,\text{m}.$$

(b) Similarly, with 0.43 m $\rightarrow$ 430 mm and 0.53 m $\rightarrow$ 530 mm, we find 3.9×10^3 mm and 4.8×10^3 mm, respectively.

(c) We can convert length and diameter first and then compute the volume, or first compute the volume and then convert. We proceed using the latter approach (where d is diameter and ℓ is length).

$$V_{\text{cylinder, min}} = \frac{\pi}{4}\ell d^2 = 28\ \text{cubit}^3 = \left(28\ \text{cubit}^3\right)\left(\frac{0.43\,\text{m}}{1\ \text{cubit}}\right)^3 = 2.2\ \text{m}^3.$$

Similarly, with 0.43 m replaced by 0.53 m, we obtain $V_{\text{cylinder, max}} = 4.2\ \text{m}^3$.

60. (a) We reduce the stock amount to British teaspoons:

$$\begin{aligned}
1 \text{ breakfastcup} &= 2 \times 8 \times 2 \times 2 = 64 \text{ teaspoons} \\
1 \text{ teacup} &= 8 \times 2 \times 2 = 32 \text{ teaspoons} \\
6 \text{ tablespoons} &= 6 \times 2 \times 2 = 24 \text{ teaspoons} \\
1 \text{ dessertspoon} &= 2 \text{ teaspoons}
\end{aligned}$$

which totals to 122 British teaspoons, or 122 U.S. teaspoons since liquid measure is being used. Now with one U.S cup equal to 48 teaspoons, upon dividing $122/48 \approx 2.54$, we find this amount corresponds to 2.5 U.S. cups plus a remainder of precisely 2 teaspoons. In other words,

$$122 \text{ U.S. teaspoons} = 2.5 \text{ U.S. cups} + 2 \text{ U.S. teaspoons.}$$

(b) For the nettle tops, one-half quart is still one-half quart.

(c) For the rice, one British tablespoon is 4 British teaspoons which (since dry-goods measure is being used) corresponds to 2 U.S. teaspoons.

(d) A British saltspoon is $\frac{1}{2}$ British teaspoon which corresponds (since dry-goods measure is again being used) to 1 U.S. teaspoon.

Chapter 2

Chapter 2

1. We use Eq. 2-2 and Eq. 2-3. During a time t_c when the velocity remains a positive constant, speed is equivalent to velocity, and distance is equivalent to displacement, with $\Delta x = v\, t_c$.

(a) During the first part of the motion, the displacement is Δx_1 = 40 km and the time interval is

$$t_1 = \frac{(40\text{ km})}{(30\text{ km/h})} = 1.33\text{ h}.$$

During the second part the displacement is Δx_2 = 40 km and the time interval is

$$t_2 = \frac{(40\text{ km})}{(60\text{ km/h})} = 0.67\text{ h}.$$

Both displacements are in the same direction, so the total displacement is

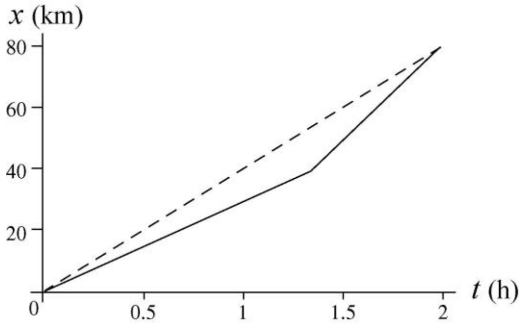
$$\Delta x = \Delta x_1 + \Delta x_2 = 40\text{ km} + 40\text{ km} = 80\text{ km}.$$

The total time for the trip is $t = t_1 + t_2$ = 2.00 h. Consequently, the average velocity is

$$v_{\text{avg}} = \frac{(80\text{ km})}{(2.0\text{ h})} = 40\text{ km/h}.$$

(b) In this example, the numerical result for the average speed is the same as the average velocity 40 km/h.

(c) As shown below, the graph consists of two contiguous line segments, the first having a slope of 30 km/h and connecting the origin to (t_1, x_1) = (1.33 h, 40 km) and the second having a slope of 60 km/h and connecting (t_1, x_1) to (t, x) = (2.00 h, 80 km). From the graphical point of view, the slope of the dashed line drawn from the origin to (t, x) represents the average velocity.



2. Average speed, as opposed to average velocity, relates to the total distance, as opposed to the net displacement. The distance D up the hill is, of course, the same as the distance down the hill, and since the speed is constant (during each stage of the motion) we have speed = D/t. Thus, the average speed is

$$\frac{D_{\text{up}} + D_{\text{down}}}{t_{\text{up}} + t_{\text{down}}} = \frac{2D}{\frac{D}{v_{\text{up}}} + \frac{D}{v_{\text{down}}}}$$

which, after canceling D and plugging in v_{up} = 40 km/h and v_{down} = 60 km/h, yields 48 km/h for the average speed.

3. The speed (assumed constant) is v = (90 km/h)(1000 m/km) ⁄ (3600 s/h) = 25 m/s. Thus, in 0.50 s, the car travels (0.50 s)(25 m/s) ≈ 13 m.

4. Huber's speed is

$$v_0 = (200\text{ m})/(6.509\text{ s}) = 30.72\text{ m/s} = 110.6\text{ km/h},$$

where we have used the conversion factor 1 m/s = 3.6 km/h. Since Whittingham beat Huber by 19.0 km/h, his speed is v_1=(110.6 km/h + 19.0 km/h)=129.6 km/h, or 36 m/s (1 km/h = 0.2778 m/s). Thus, the time through a distance of 200 m for Whittingham is

$$\Delta t = \frac{\Delta x}{v_1} = \frac{200\text{ m}}{36\text{ m/s}} = 5.554\text{ s}.$$

5. Using $x = 3t - 4t^2 + t^3$ with SI units understood is efficient (and is the approach we will use), but if we wished to make the units explicit we would write

$$x = (3\text{ m/s})t - (4\text{ m/s}^2)t^2 + (1\text{ m/s}^3)t^3.$$

We will quote our answers to one or two significant figures, and not try to follow the significant figure rules rigorously.

(a) Plugging in t = 1 s yields x = 3 – 4 + 1 = 0.

(b) With t = 2 s we get $x = 3(2) - 4(2)^2 + (2)^3 = -2$ m.

(c) With t = 3 s we have x = 0 m.

(d) Plugging in t = 4 s gives x = 12 m.

For later reference, we also note that the position at t = 0 is x = 0.

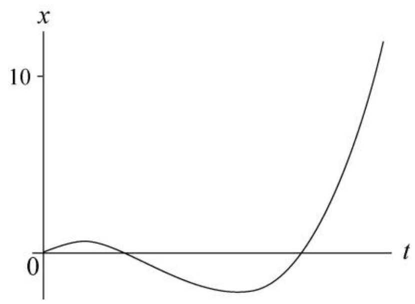
(e) The position at t = 0 is subtracted from the position at t = 4 s to find the displacement Δx = 12 m.

(f) The position at t = 2 s is subtracted from the position at t = 4 s to give the displacement Δx = 14 m. Eq. 2-2, then, leads to

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{14 \text{ m}}{2 \text{ s}} = 7 \text{ m/s}.$$

(g) The horizontal axis is $0 \le t \le 4$ with SI units understood.

Not shown is a straight line drawn from the point at $(t, x) = (2, -2)$ to the highest point shown (at $t = 4$ s) which would represent the answer for part (f).



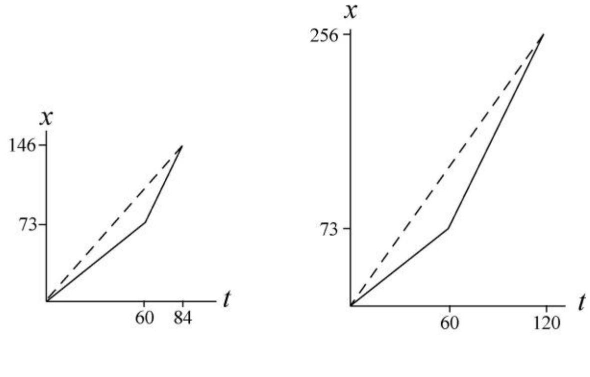
6. (a) Using the fact that time = distance/velocity while the velocity is constant, we find

$$v_{\text{avg}} = \frac{73.2 \text{ m} + 73.2 \text{ m}}{\frac{73.2 \text{ m}}{1.22 \text{ m/s}} + \frac{73.2 \text{ m}}{3.05 \text{ m}}} = 1.74 \text{ m/s}.$$

(b) Using the fact that distance = vt while the velocity v is constant, we find

$$v_{\text{avg}} = \frac{(1.22 \text{ m/s})(60 \text{ s}) + (3.05 \text{ m/s})(60 \text{ s})}{120 \text{ s}} = 2.14 \text{ m/s}.$$

(c) The graphs are shown below (with meters and seconds understood). The first consists of two (solid) line segments, the first having a slope of 1.22 and the second having a slope of 3.05. The slope of the dashed line represents the average velocity (in both graphs). The second graph also consists of two (solid) line segments, having the same slopes as before — the main difference (compared to the first graph) being that the stage involving higher-speed motion lasts much longer.



7. Converting to seconds, the running times are t_1 = 147.95 s and t_2 = 148.15 s, respectively. If the runners were equally fast, then

$$s_{\text{avg}_1} = s_{\text{avg}_2} \quad \Rightarrow \quad \frac{L_1}{t_1} = \frac{L_2}{t_2}.$$

From this we obtain

$$L_2 - L_1 = \left(\frac{t_2}{t_1} - 1\right) L_1 = \left(\frac{148.15}{147.95} - 1\right) L_1 = 0.00135 L_1 \approx 1.4 \text{ m}$$

where we set $L_1 \approx 1000$ m in the last step. Thus, if L_1 and L_2 are no different than about 1.4 m, then runner 1 is indeed faster than runner 2. However, if L_1 is shorter than L_2 by more than 1.4 m, then runner 2 would actually be faster.

8. Let v_w be the speed of the wind and v_c be the speed of the car.

(a) Suppose during time interval t_1, the car moves in the same direction as the wind. Then its effective speed is $v_{eff,1} = v_c + v_w$, and the distance traveled is $d = v_{eff,1} t_1 = (v_c + v_w) t_1$. On the other hand, for the return trip during time interval t_2, the car moves in the opposite direction of the wind and the effective speed would be $v_{eff,2} = v_c - v_w$. The distance traveled is $d = v_{eff,2} t_2 = (v_c - v_w) t_2$. The two expressions can be rewritten as

$$v_c + v_w = \frac{d}{t_1} \quad \text{and} \quad v_c - v_w = \frac{d}{t_2}$$

Adding the two equations and dividing by two, we obtain $v_c = \frac{1}{2}\left(\frac{d}{t_1} + \frac{d}{t_2}\right)$. Thus, method 1 gives the car's speed v_c in windless situation.

(b) If method 2 is used, the result would be

$$v_c' = \frac{d}{(t_1 + t_2)/2} = \frac{2d}{t_1 + t_2} = \frac{2d}{\dfrac{d}{v_c + v_w} + \dfrac{d}{v_c - v_w}} = \frac{v_c^2 - v_w^2}{v_c} = v_c\left[1 - \left(\frac{v_w}{v_c}\right)^2\right].$$

The fractional difference would be

$$\frac{v_c - v_c'}{v_c} = \left(\frac{v_w}{v_c}\right)^2 = (0.0240)^2 = 5.76 \times 10^{-4}.$$

9. The values used in the problem statement make it easy to see that the first part of the trip (at 100 km/h) takes 1 hour, and the second part (at 40 km/h) also takes 1 hour. Expressed in decimal form, the time left is 1.25 hour, and the distance that remains is 160 km. Thus, a speed v = (160 km)/(1.25 h) = 128 km/h is needed.

10. The amount of time it takes for each person to move a distance L with speed v_s is $\Delta t = L/v_s$. With each additional person, the depth increases by one body depth d

(a) The rate of increase of the layer of people is

$$R = \frac{d}{\Delta t} = \frac{d}{L/v_s} = \frac{dv_s}{L} = \frac{(0.25\text{ m})(3.50\text{ m/s})}{1.75\text{ m}} = 0.50\text{ m/s}$$

(b) The amount of time required to reach a depth of $D = 5.0\text{ m}$ is

$$t = \frac{D}{R} = \frac{5.0\text{ m}}{0.50\text{ m/s}} = 10\text{ s}$$

11. Recognizing that the gap between the trains is closing at a constant rate of 60 km/h, the total time which elapses before they crash is t = (60 km)/(60 km/h) = 1.0 h. During this time, the bird travels a distance of $x = vt$ = (60 km/h)(1.0 h) = 60 km.

12. (a) Let the fast and the slow cars be separated by a distance d at t = 0. If during the time interval $t = L/v_s = (12.0\text{ m})/(5.0\text{ m/s}) = 2.40\text{ s}$ in which the slow car has moved a distance of $L = 12.0\text{ m}$, the fast car moves a distance of $vt = d + L$ to join the line of slow cars, then the shock wave would remain stationary. The condition implies a separation of

$$d = vt - L = (25\text{ m/s})(2.4\text{ s}) - 12.0\text{ m} = 48.0\text{ m}.$$

(b) Let the initial separation at $t = 0$ be $d = 96.0\text{ m}$. At a later time t, the slow and the fast cars have traveled $x = v_s t$ and the fast car joins the line by moving a distance $d + x$. From

$$t = \frac{x}{v_s} = \frac{d + x}{v},$$

we get

$$x = \frac{v_s}{v - v_s} d = \frac{5.00\text{ m/s}}{25.0\text{ m/s} - 5.00\text{ m/s}}(96.0\text{ m}) = 24.0\text{ m},$$

which in turn gives $t = (24.0\text{ m})/(5.00\text{ m/s}) = 4.80\text{ s}$. Since the rear of the slow-car pack has moved a distance of $\Delta x = x - L = 24.0\text{ m} - 12.0\text{ m} = 12.0\text{ m}$ downstream, the speed of the rear of the slow-car pack, or equivalently, the speed of the shock wave, is

$$v_{\text{shock}} = \frac{\Delta x}{t} = \frac{12.0\text{ m}}{4.80\text{ s}} = 2.50\text{ m/s}.$$

(c) Since $x > L$, the direction of the shock wave is downstream.

13. (a) Denoting the travel time and distance from San Antonio to Houston as T and D,

respectively, the average speed is

$$s_{\text{avg1}} = \frac{D}{T} = \frac{(55\text{ km/h})(T/2) + (90\text{ km/h})(T/2)}{T} = 72.5\text{ km/h}$$

which should be rounded to 73 km/h.

(b) Using the fact that time = distance/speed while the speed is constant, we find

$$s_{\text{avg2}} = \frac{D}{T} = \frac{D}{\frac{D/2}{55\text{ km/h}} + \frac{D/2}{90\text{ km/h}}} = 68.3\text{ km/h}$$

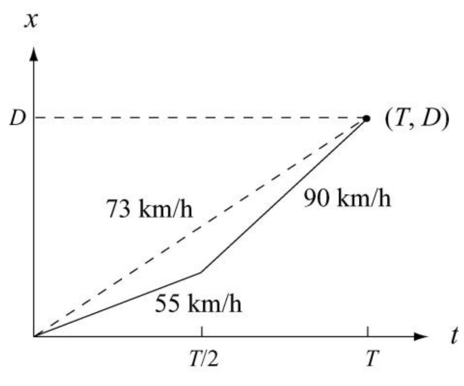
which should be rounded to 68 km/h.

(c) The total distance traveled ($2D$) must not be confused with the net displacement (zero). We obtain for the two-way trip

$$s_{\text{avg}} = \frac{2D}{\frac{D}{72.5\text{ km/h}} + \frac{D}{68.3\text{ km/h}}} = 70\text{ km/h}.$$

(d) Since the net displacement vanishes, the average velocity for the trip in its entirety is zero.

(e) In asking for a *sketch*, the problem is allowing the student to arbitrarily set the distance D (the intent is *not* to make the student go to an Atlas to look it up); the student can just as easily arbitrarily set T instead of D, as will be clear in the following discussion. We briefly describe the graph (with kilometers-per-hour understood for the slopes): two contiguous line segments, the first having a slope of 55 and connecting the origin to $(t_1, x_1) = (T/2, 55T/2)$ and the second having a slope of 90 and connecting (t_1, x_1) to (T, D) where $D = (55 + 90)T/2$. The average velocity, from the graphical point of view, is the slope of a line drawn from the origin to (T, D). The graph (not drawn to scale) is depicted below:



14. We use the functional notation $x(t)$, $v(t)$ and $a(t)$ in this solution, where the latter two quantities are obtained by differentiation:

$$v(t) = \frac{dx(t)}{dt} = -12t \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -12$$

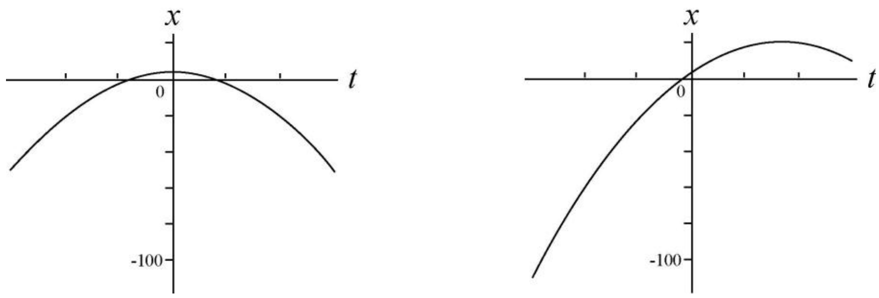
with SI units understood.

(a) From $v(t) = 0$ we find it is (momentarily) at rest at $t = 0$.

(b) We obtain $x(0) = 4.0$ m

(c) and (d) Requiring $x(t) = 0$ in the expression $x(t) = 4.0 - 6.0t^2$ leads to $t = \pm 0.82$ s for the times when the particle can be found passing through the origin.

(e) We show both the asked-for graph (on the left) as well as the "shifted" graph which is relevant to part (f). In both cases, the time axis is given by $-3 \le t \le 3$ (SI units understood).



(f) We arrived at the graph on the right (shown above) by adding $20t$ to the $x(t)$ expression.

(g) Examining where the slopes of the graphs become zero, it is clear that the shift causes the $v = 0$ point to correspond to a larger value of x (the top of the second curve shown in part (e) is higher than that of the first).

15. We use Eq. 2-4. to solve the problem.

(a) The velocity of the particle is

$$v = \frac{dx}{dt} = \frac{d}{dt}\ (4 - 12t + 3t^2) \ = -12 + 6t.$$

Thus, at $t = 1$ s, the velocity is $v = (-12 + (6)(1)) = -6$ m/s.

(b) Since $v < 0$, it is moving in the negative x direction at $t = 1$ s.

(c) At $t = 1$ s, the *speed* is $|v| = 6$ m/s.

(d) For $0 < t < 2$ s, $|v|$ decreases until it vanishes. For $2 < t < 3$ s, $|v|$ increases from zero to the value it had in part (c). Then, $|v|$ is larger than that value for $t > 3$ s.

(e) Yes, since v smoothly changes from negative values (consider the $t = 1$ result) to positive (note that as $t \to +\infty$, we have $v \to +\infty$). One can check that $v = 0$ when $t = 2$ s.

(f) No. In fact, from $v = -12 + 6t$, we know that $v > 0$ for $t > 2$ s.

16. Using the general property $\frac{d}{dx}\exp(bx) = b\exp(bx)$, we write

$$v = \frac{dx}{dt} = \left(\frac{d(19t)}{dt}\right) e^{-t} + (19t) \cdot \left(\frac{de^{-t}}{dt}\right).$$

If a concern develops about the appearance of an argument of the exponential ($-t$) apparently having units, then an explicit factor of $1/T$ where $T = 1$ second can be inserted and carried through the computation (which does not change our answer). The result of this differentiation is

$$v = 16(1 - t)e^{-t}$$

with t and v in SI units (s and m/s, respectively). We see that this function is zero when $t = 1$ s. Now that we know *when* it stops, we find out *where* it stops by plugging our result $t = 1$ into the given function $x = 16te^{-t}$ with x in meters. Therefore, we find $x = 5.9$ m.

17. We use Eq. 2-2 for average velocity and Eq. 2-4 for instantaneous velocity, and work with distances in centimeters and times in seconds.

(a) We plug into the given equation for x for $t = 2.00$ s and $t = 3.00$ s and obtain $x_2 = 21.75$ cm and $x_3 = 50.25$ cm, respectively. The average velocity during the time interval $2.00 \le t \le 3.00$ s is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{50.25\text{ cm} - 21.75\text{ cm}}{3.00\text{ s} - 2.00\text{ s}}$$

which yields $v_{\text{avg}} = 28.5$ cm/s.

(b) The instantaneous velocity is $v = \frac{dx}{dt} = 4.5t^2$, which, at time $t = 2.00$ s, yields $v = (4.5)(2.00)^2 = 18.0$ cm/s.

(c) At $t = 3.00$ s, the instantaneous velocity is $v = (4.5)(3.00)^2 = 40.5$ cm/s.

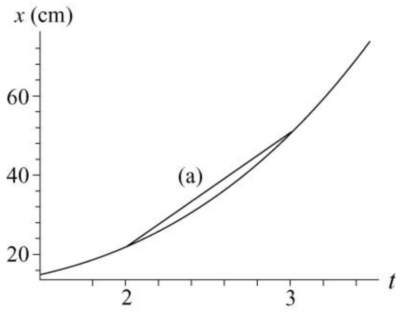
(d) At $t = 2.50$ s, the instantaneous velocity is $v = (4.5)(2.50)^2 = 28.1$ cm/s.

(e) Let t_m stand for the moment when the particle is midway between x_2 and x_3 (that is, when the particle is at $x_m = (x_2 + x_3)/2 = 36$ cm). Therefore,

$$x_m = 9.75 + 1.5t_m^3 \quad \Rightarrow \quad t_m = 2.596$$

in seconds. Thus, the instantaneous speed at this time is $v = 4.5(2.596)^2 = 30.3$ cm/s.

(f) The answer to part (a) is given by the slope of the straight line between $t = 2$ and $t = 3$ in this x-vs-t plot. The answers to parts (b), (c), (d) and (e) correspond to the slopes of tangent lines (not shown but easily imagined) to the curve at the appropriate points.



18. We use the functional notation $x(t)$, $v(t)$ and $a(t)$ and find the latter two quantities by differentiating:

$$v(t) = \frac{dx(t)}{dt} = -15t^2 + 20 \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -30t$$

with SI units understood. These expressions are used in the parts that follow.

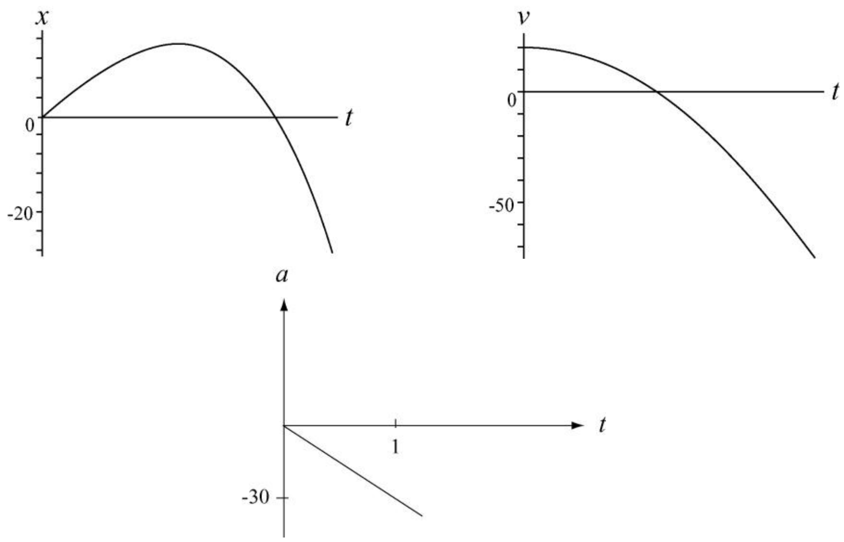
(a) From $0 = -15t^2 + 20$, we see that the only positive value of t for which the particle is (momentarily) stopped is $t = \sqrt{20/15} = 1.2$ s.

(b) From $0 = -30t$, we find $a(0) = 0$ (that is, it vanishes at $t = 0$).

(c) It is clear that $a(t) = -30t$ is negative for $t > 0$

(d) The acceleration $a(t) = -30t$ is positive for $t < 0$.

(e) The graphs are shown below. SI units are understood.



19. We represent its initial direction of motion as the $+x$ direction, so that $v_0 = +18$ m/s

and v = –30 m/s (when t = 2.4 s). Using Eq. 2-7 (or Eq. 2-11, suitably interpreted) we find

$$a_{\text{avg}} = \frac{(-30\ \text{m/s}) - (+1\ \text{m/s})}{2.4\ \text{s}} = -20\ \text{m/s}^2$$

which indicates that the average acceleration has magnitude 20 m/s^2 and is in the opposite direction to the particle's initial velocity.

20. (a) Taking derivatives of $x(t) = 12t^2 - 2t^3$ we obtain the velocity and the acceleration functions:

$$v(t) = 24t - 6t^2 \quad \text{and} \quad a(t) = 24 - 12t$$

with length in meters and time in seconds. Plugging in the value t = 3 yields $x(3) = 54\ \text{m}$.

(b) Similarly, plugging in the value t = 3 yields $v(3)$ = 18 m/s.

(c) For t = 3, $a(3)$ = –12 m/s^2.

(d) At the maximum x, we must have v = 0; eliminating the t = 0 root, the velocity equation reveals t = 24/6 = 4 s for the time of maximum x. Plugging t = 4 into the equation for x leads to x = 64 m for the largest x value reached by the particle.

(e) From (d), we see that the x reaches its maximum at t = 4.0 s.

(f) A maximum v requires a = 0, which occurs when t = 24/12 = 2.0 s. This, inserted into the velocity equation, gives v_{max} = 24 m/s.

(g) From (f), we see that the maximum of v occurs at t = 24/12 = 2.0 s.

(h) In part (e), the particle was (momentarily) motionless at t = 4 s. The acceleration at that time is readily found to be 24 – 12(4) = –24 m/s^2.

(i) The *average velocity* is defined by Eq. 2-2, so we see that the values of x at t = 0 and t = 3 s are needed; these are, respectively, x = 0 and x = 54 m (found in part (a)). Thus,

$$v_{\text{avg}} = \frac{54-0}{3-0} = 18\ \text{m/s} \quad .$$

21. In this solution, we make use of the notation $x(t)$ for the value of x at a particular t. The notations $v(t)$ and $a(t)$ have similar meanings.

(a) Since the unit of ct^2 is that of length, the unit of c must be that of length/time^2, or m/s^2 in the SI system.

(b) Since bt^3 has a unit of length, b must have a unit of length/time^3, or m/s^3.

(c) When the particle reaches its maximum (or its minimum) coordinate its velocity is zero. Since the velocity is given by $v = dx/dt = 2ct - 3bt^2$, $v = 0$ occurs for $t = 0$ and for

$$t = \frac{2c}{3b} = \frac{2(3.0\ \text{m/s}^2)}{3(2.0\ \text{m/s}^3)} = 1.0\ \text{s}\ .$$

For $t = 0$, $x = x_0 = 0$ and for $t = 1.0$ s, $x = 1.0\ \text{m} > x_0$. Since we seek the maximum, we reject the first root ($t = 0$) and accept the second ($t = 1$s).

(d) In the first 4 s the particle moves from the origin to $x = 1.0$ m, turns around, and goes back to

$$x(4\ \text{s}) = (3.0\ \text{m/s}^2)(4.0\ \text{s})^2 - (2.0\ \text{m/s}^3)(4.0\ \text{s})^3 = -80\ \text{m}\ .$$

The total path length it travels is 1.0 m + 1.0 m + 80 m = 82 m.

(e) Its displacement is $\Delta x = x_2 - x_1$, where $x_1 = 0$ and $x_2 = -80$ m. Thus, $\Delta x = -80\ \text{m}$.

The velocity is given by $v = 2ct - 3bt^2 = (6.0\ \text{m/s}^2)t - (6.0\ \text{m/s}^3)t^2$.

(f) Plugging in $t = 1$ s, we obtain

$$v(1\ \text{s}) = (6.0\ \text{m/s}^2)(1.0\ \text{s}) - (6.0\ \text{m/s}^3)(1.0\ \text{s})^2 = 0.$$

(g) Similarly, $v(2\ \text{s}) = (6.0\ \text{m/s}^2)(2.0\ \text{s}) - (6.0\ \text{m/s}^3)(2.0\ \text{s})^2 = -12\text{m/s}$.

(h) $v(3\ \text{s}) = (6.0\ \text{m/s}^2)(3.0\ \text{s}) - (6.0\ \text{m/s}^3)(3.0\ \text{s})^2 = -36\ \text{m/s}$.

(i) $v(4\ \text{s}) = (6.0\ \text{m/s}^2)(4.0\ \text{s}) - (6.0\ \text{m/s}^3)(4.0\ \text{s})^2 = -72\ \text{m/s}$.

The acceleration is given by $a = dv/dt = 2c - 6b = 6.0\ \text{m/s}^2 - (12.0\ \text{m/s}^3)t$.

(j) Plugging in $t = 1$ s, we obtain

$$a(1\ \text{s}) = 6.0\ \text{m/s}^2 - (12.0\ \text{m/s}^3)(1.0\ \text{s}) = -6.0\ \text{m/s}^2.$$

(k) $a(2\ \text{s}) = 6.0\ \text{m/s}^2 - (12.0\ \text{m/s}^3)(2.0\ \text{s}) = -18\ \text{m/s}^2$.

(l) $a(3\ \text{s}) = 6.0\ \text{m/s}^2 - (12.0\ \text{m/s}^3)(3.0\ \text{s}) = -30\ \text{m/s}^2$.

(m) $a(4\ \text{s}) = 6.0\ \text{m/s}^2 - (12.0\ \text{m/s}^3)(4.0\ \text{s}) = -42\ \text{m/s}^2$.

22. We use Eq. 2-2 (average velocity) and Eq. 2-7 (average acceleration). Regarding our coordinate choices, the initial position of the man is taken as the origin and his direction of motion during 5 min $\leq t \leq$ 10 min is taken to be the positive x direction. We also use the fact that $\Delta x = v\Delta t'$ when the velocity is constant during a time interval $\Delta t'$.

(a) The entire interval considered is $\Delta t = 8 - 2 = 6$ min which is equivalent to 360 s, whereas the sub-interval in which he is *moving* is only $\Delta t' = 8 - 5 = 3\text{min} = 180$ s. His position at t = 2 min is x = 0 and his position at t = 8 min is $x = v\Delta t' = (2.2)(180) = 396$ m. Therefore,

$$v_{\text{avg}} = \frac{396 \text{ m} - 0}{360 \text{ s}} = 1.10 \text{ m/s}.$$

(b) The man is at rest at t = 2 min and has velocity v = +2.2 m/s at t = 8 min. Thus, keeping the answer to 3 significant figures,

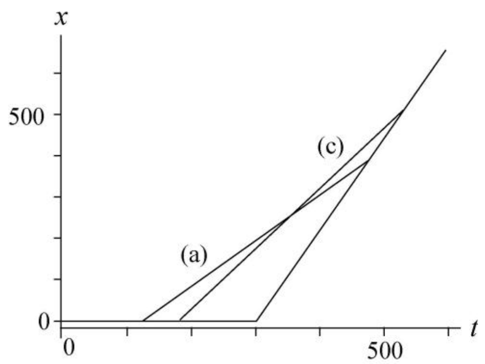
$$a_{\text{avg}} = \frac{2.2 \text{ m/s} - 0}{360 \text{ s}} = 0.00611 \text{ m/s}^2 .$$

(c) Now, the entire interval considered is $\Delta t = 9 - 3 = 6$ min (360 s again), whereas the sub-interval in which he is moving is $\Delta t' = 9 - 5 = 4\text{ min} = 240$ s). His position at $t = 3$ min is x = 0 and his position at t = 9 min is $x = v\Delta t' = (2.2)(240) = 528$ m. Therefore,

$$v_{\text{avg}} = \frac{528 \text{ m} - 0}{360 \text{ s}} = 1.47 \text{ m/s}.$$

(d) The man is at rest at t = 3 min and has velocity v = +2.2 m/s at t = 9 min. Consequently, a_{avg} = 2.2/360 = 0.00611 m/s^2 just as in part (b).

(e) The horizontal line near the bottom of this x-vs-t graph represents the man standing at x = 0 for $0 \le t < 300$ s and the linearly rising line for $300 \le t \le 600$ s represents his constant-velocity motion. The dotted lines represent the answers to part (a) and (c) in the sense that their slopes yield those results.



The graph of v-vs-t is not shown here, but would consist of two horizontal "steps" (one at v = 0 for $0 \le t < 300$ s and the next at v = 2.2 m/s for $300 \le t \le 600$ s). The indications of the average accelerations found in parts (b) and (d) would be dotted lines connecting the "steps" at the appropriate t values (the slopes of the dotted lines representing the values of a_{avg}).

23. We use $v = v_0 + at$, with $t = 0$ as the instant when the velocity equals +9.6 m/s.

(a) Since we wish to calculate the velocity for a time *before* $t = 0$, we set $t = -2.5$ s. Thus, Eq. 2-11 gives

$$v = (9.6\ \text{m/s}) + \left(3.2\ \text{m/s}^2\right)(-2.5\ \text{s}) = 1.6\ \text{m/s}.$$

(b) Now, $t = +2.5$ s and we find

$$v = (9.6\ \text{m/s}) + \left(3.2\ \text{m/s}^2\right)(2.5\ \text{s}) = 18\ \text{m/s}.$$

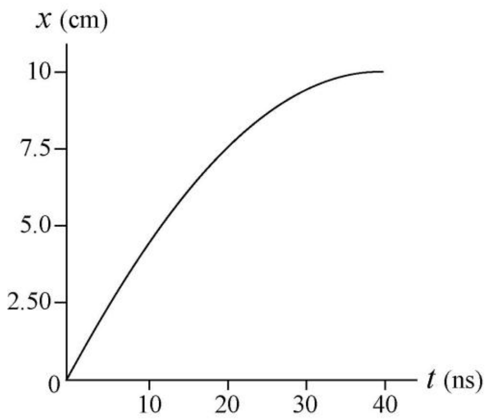
24. The constant-acceleration condition permits the use of Table 2-1.

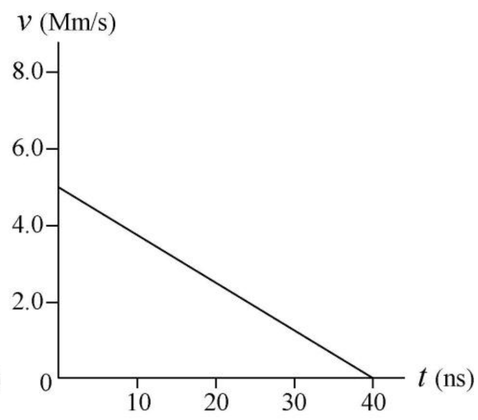
(a) Setting $v = 0$ and $x_0 = 0$ in $v^2 = v_0^2 + 2a(x - x_0)$, we find

$$x = -\frac{1}{2}\frac{v_0^2}{a} = -\frac{1}{2}\frac{(5.00\times 10^6)^2}{-1.25\times 10^{14}} = 0.100\ \text{m}\ .$$

Since the muon is slowing, the initial velocity and the acceleration must have opposite signs.

(b) Below are the time-plots of the position x and velocity v of the muon from the moment it enters the field to the time it stops. The computation in part (a) made no reference to t, so that other equations from Table 2-1 (such as $v = v_0 + at$ and $x = v_0 t + \frac{1}{2}at^2$) are used in making these plots.





25. The constant acceleration stated in the problem permits the use of the equations in Table 2-1.

(a) We solve $v = v_0 + at$ for the time:

$$t = \frac{v - v_0}{a} = \frac{\frac{1}{10}(3.0\times 10^8\ \text{m/s})}{9.8\ \text{m/s}^2} = 3.1\times 10^6\ \text{s}$$

which is equivalent to 1.2 months.

(b) We evaluate $x = x_0 + v_0 t + \frac{1}{2}at^2$, with $x_0 = 0$. The result is

$$x = \frac{1}{2}\left(9.8 \text{ m/s}^2\right)(3.1\times10^6 \text{s})^2 = 4.6\times10^{13} \text{ m} .$$

26. We take $+x$ in the direction of motion, so $v_0 = +24.6$ m/s and $a = -4.92$ m/s^2. We also take $x_0 = 0$.

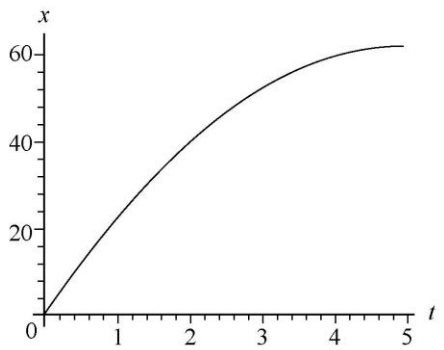
(a) The time to come to a halt is found using Eq. 2-11:

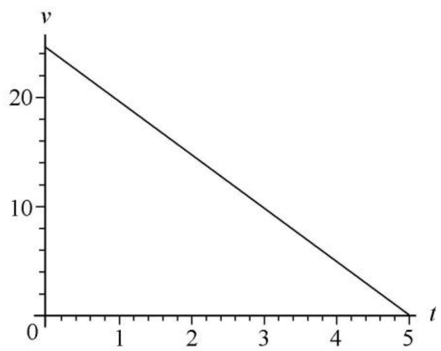
$$0 = v_0 + at \;\Rightarrow\; t = \frac{24.6 \text{ m/s}}{-4.92 \text{ m/s}^2} = 5.00 \text{ s} .$$

(b) Although several of the equations in Table 2-1 will yield the result, we choose Eq. 2-16 (since it does not depend on our answer to part (a)).

$$0 = v_0^2 + 2ax \;\Rightarrow\; x = -\frac{(24.6 \text{ m/s})^2}{2\left(-4.92 \text{ m/s}^2\right)} = 61.5 \text{ m} .$$

(c) Using these results, we plot $v_0 t + \frac{1}{2}at^2$ (the x graph, shown next, on the left) and $v_0 + at$ (the v graph, on the right) over $0 \le t \le 5$ s, with SI units understood.





27. Assuming constant acceleration permits the use of the equations in Table 2-1. We solve $v^2 = v_0^2 + 2a(x - x_0)$ with $x_0 = 0$ and $x = 0.010$ m. Thus,

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(5.7\times10^5 \text{ m/s})^2 - (1.5\times10^5 \text{ m/s})^2}{2(0.010 \text{ m})} = 1.62\times10^{15} \text{ m/s}^2.$$

28. In this problem we are given the initial and final speeds, and the displacement, and asked to find the acceleration. We use the constant-acceleration equation given in Eq. 2-16, $v^2 = v^2_{,0} + 2a(x - x_0)$.

(a) With $v_0 = 0$, $v = 1.6$ m/s and $\Delta x = 5.0\,\mu$m, the acceleration of the spores during the launch is

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(1.6 \text{ m/s})^2}{2(5.0\times10^{-6} \text{ m})} = 2.56\times10^5 \text{ m/s}^2 = 2.6\times10^4 g$$

(b) During the speed-reduction stage, the acceleration is

$$a = \frac{v^2 - v_0^2}{2x} = \frac{0-(1.6 \text{ m/s})^2}{2(1.0\times10^{-3} \text{ m})} = -1.28\times10^3 \text{ m/s}^2 = -1.3\times10^2 g$$

The negative sign means that the spores are decelerating.

29. We separate the motion into two parts, and take the direction of motion to be positive. In part 1, the vehicle accelerates from rest to its highest speed; we are given $v_0 = 0$; $v = 20$ m/s and $a = 2.0$ m/s^2. In part 2, the vehicle decelerates from its highest speed to a halt; we are given $v_0 = 20$ m/s; $v = 0$ and $a = -1.0$ m/s^2 (negative because the acceleration vector points opposite to the direction of motion).

(a) From Table 2-1, we find t_1 (the duration of part 1) from $v = v_0 + at$. In this way, $20 = 0 + 2.0t_1$ yields $t_1 = 10$ s. We obtain the duration t_2 of part 2 from the same equation. Thus, $0 = 20 + (-1.0)t_2$ leads to $t_2 = 20$ s, and the total is $t = t_1 + t_2 = 30$ s.

(b) For part 1, taking $x_0 = 0$, we use the equation $v^2 = v^2{}_{,0} + 2a(x - x_0)$ from Table 2-1 and find

$$x = \frac{v^2 - v_0^2}{2a} = \frac{(20 \text{ m/s})^2 - (0)^2}{2(2.0 \text{ m/s}^2)} = 100 \text{ m}.$$

This position is then the *initial* position for part 2, so that when the same equation is used in part 2 we obtain

$$x - 100 \text{ m} = \frac{v^2 - v_0^2}{2a} = \frac{(0)^2 - (20 \text{ m/s})^2}{2(-1.0 \text{ m/s}^2)}.$$

Thus, the final position is $x = 300$ m. That this is also the total distance traveled should be evident (the vehicle did not "backtrack" or reverse its direction of motion).

30. The acceleration is found from Eq. 2-11 (or, suitably interpreted, Eq. 2-7).

$$a = \frac{\Delta v}{\Delta t} = \frac{(1020 \text{ km/h})\left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}}\right)}{1.4 \text{ s}} = 202.4 \text{ m/s}^2.$$

In terms of the gravitational acceleration g, this is expressed as a multiple of 9.8 m/s^2 as follows:

$$a = \left(\frac{202.4 \text{ m/s}^2}{9.8 \text{ m/s}^2}\right) g = 21g.$$

31. We assume the periods of acceleration (duration t_1) and deceleration (duration t_2) are periods of constant a so that Table 2-1 can be used. Taking the direction of motion to be $+x$ then $a_1 = +1.22$ m/s^2 and $a_2 = -1.22$ m/s^2. We use SI units so the velocity at $t = t_1$ is $v = 305/60 = 5.08$ m/s.

(a) We denote Δx as the distance moved during t_1, and use Eq. 2-16:

$$v^2 = v_0^2 + 2a_1\Delta x \quad \Rightarrow \quad \Delta x = \frac{(5.08\text{ m/s})^2}{2(1.22\text{ m/s}^2)} = 10.59\text{ m} \approx 10.6\text{ m}.$$

(b) Using Eq. 2-11, we have

$$t_1 = \frac{v - v_0}{a_1} = \frac{5.08\text{ m/s}}{1.22\text{ m/s}^2} = 4.17\text{ s}.$$

The deceleration time t_2 turns out to be the same so that $t_1 + t_2 = 8.33$ s. The distances traveled during t_1 and t_2 are the same so that they total to 2(10.59 m) = 21.18 m. This implies that for a distance of 190 m – 21.18 m = 168.82 m, the elevator is traveling at constant velocity. This time of constant velocity motion is

$$t_3 = \frac{168.82\text{ m}}{5.08\text{ m/s}} = 33.21\text{ s}.$$

Therefore, the total time is 8.33 s + 33.21 s ≈ 41.5 s.

32. We choose the positive direction to be that of the initial velocity of the car (implying that $a < 0$ since it is slowing down). We assume the acceleration is constant and use Table 2-1.

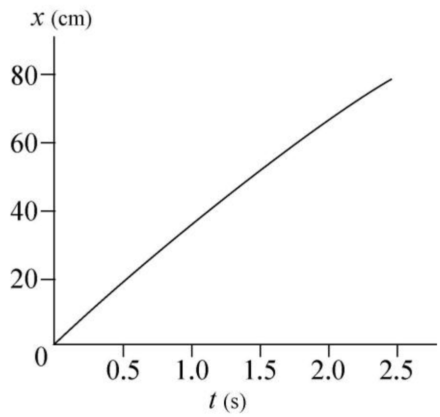
(a) Substituting v_0 = 137 km/h = 38.1 m/s, v = 90 km/h = 25 m/s, and $a = -5.2$ m/s^2 into $v = v_0 + at$, we obtain

$$t = \frac{25\text{ m/s} - 38\text{ m/s}}{-5.2\text{ m/s}^2} = 2.5\text{ s}.$$

(b) We take the car to be at $x = 0$ when the brakes are applied (at time $t = 0$). Thus, the coordinate of the car as a function of time is given by

$$x = (38\text{ m/s})t + \frac{1}{2}(-5.2\text{ m/s}^2)t^2$$

in SI units. This function is plotted from $t = 0$ to $t = 2.5$ s on the graph below. We have not shown the v-vs-t graph here; it is a descending straight line from v_0 to v.



33. The problem statement (see part (a)) indicates that a = constant, which allows us to use Table 2-1.

(a) We take $x_0 = 0$, and solve $x = v_0 t + \frac{1}{2}at^2$ (Eq. 2-15) for the acceleration: $a = 2(x - v_0 t)/t^2$. Substituting $x = 24.0$ m, $v_0 = 56.0$ km/h = 15.55 m/s and $t = 2.00$ s, we find

$$a = \frac{2\left(24.0\text{m} - \left(15.55\text{m/s}\right)\left(2.00\text{s}\right)\right)}{\left(2.00\text{s}\right)^2} = -3.56\text{m/s}^2,$$

or $|a| = 3.56 \text{ m/s}^2$. The negative sign indicates that the acceleration is opposite to the direction of motion of the car. The car is slowing down.

(b) We evaluate $v = v_0 + at$ as follows:

$$v = 15.55 \text{ m/s} - \left(3.56 \text{ m/s}^2\right)\left(2.00 \text{ s}\right) = 8.43 \text{ m/s}$$

which can also be converted to 30.3 km/h.

34. (a) Eq. 2-15 is used for part 1 of the trip and Eq. 2-18 is used for part 2:

$\Delta x_1 = v_{o1}\, t_1 +$ **Erro!**$a_1\, t_1^2$ where $a_1 = 2.25$ m/s^2 and $\Delta x_1 =$ **Erro!**m

$\Delta x_2 = v_2\, t_2 -$ **Erro!**$a_2\, t_2^2$ where $a_2 = -0.75$ m/s^2 and $\Delta x_2 =$ **Erro!**m

In addition, $v_{o1} = v_2 = 0$. Solving these equations for the times and adding the results gives $t = t_1 + t_2 = 56.6$ s.

(b) Eq. 2-16 is used for part 1 of the trip:

$v^2 = (v_{o1})^2 + 2a_1\Delta x_1 = 0 + 2(2.25)($ **Erro!**$) = 1013 \text{ m}^2/\text{s}^2$

which leads to $v = 31.8$ m/s for the maximum speed.

35. (a) From the figure, we see that $x_0 = -2.0$ m. From Table 2-1, we can apply $x - x_0 = v_0 t + \frac{1}{2}at^2$ with $t = 1.0$ s, and then again with $t = 2.0$ s. This yields two equations for the two unknowns, v_0 and a:

$$0.0 - (-2.0 \text{ m}) = v_0\left(1.0 \text{ s}\right) + \frac{1}{2}a\left(1.0 \text{ s}\right)^2$$
$$6.0 \text{ m} - (-2.0 \text{ m}) = v_0\left(2.0 \text{ s}\right) + \frac{1}{2}a\left(2.0 \text{ s}\right)^2.$$

Solving these simultaneous equations yields the results $v_0 = 0$ and $a = 4.0$ m/s^2.

(b) The fact that the answer is positive tells us that the acceleration vector points in the $+x$ direction.

36. We assume the train accelerates from rest ($v_0 = 0$ and $x_0 = 0$) at

$a_1 = +1.34 \text{ m/s}^2$ until the midway point and then decelerates at $a_2 = -1.34 \text{ m/s}^2$ until it comes to a stop $(v_2 = 0)$ at the next station. The velocity at the midpoint is v_1 which occurs at $x_1 = 806/2 = 403$m.

(a) Eq. 2-16 leads to

$$v_1^2 = v_0^2 + 2a_1x_1 \quad \Rightarrow \quad v_1 = \sqrt{2\left(1.34 \text{ m/s}^2\right)\left(403 \text{ m}\right)} = 32.9 \text{ m/s}.$$

(b) The time t_1 for the accelerating stage is (using Eq. 2-15)

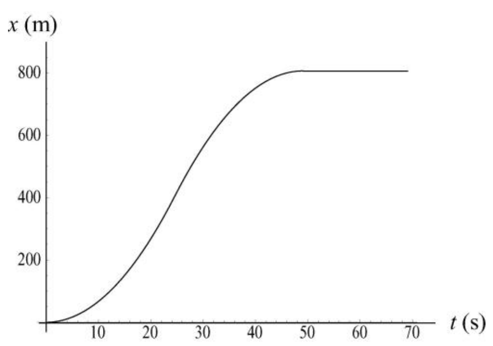
$$x_1 = v_0t_1 + \frac{1}{2}a_1t_1^2 \quad \Rightarrow \quad t_1 = \sqrt{\frac{2\left(403 \text{ m}\right)}{1.34 \text{ m/s}^2}} = 24.53 \text{ s}.$$

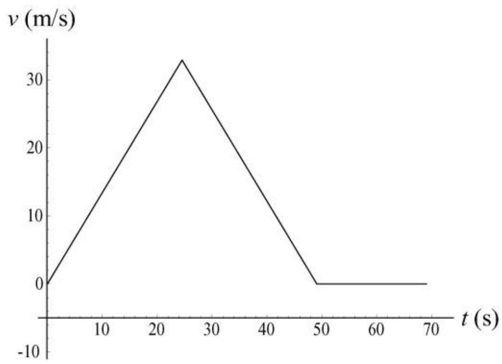
Since the time interval for the decelerating stage turns out to be the same, we double this result and obtain t = 49.1 s for the travel time between stations.

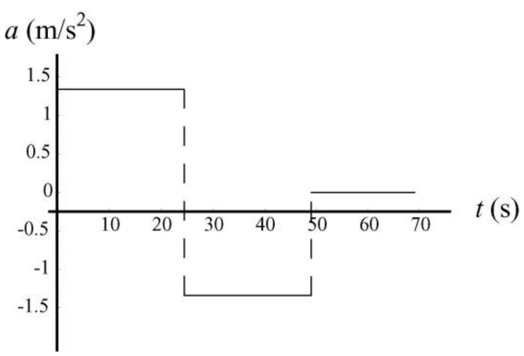
(c) With a "dead time" of 20 s, we have $T = t + 20 = 69.1$ s for the total time between start-ups. Thus, Eq. 2-2 gives

$$v_{\text{avg}} = \frac{806 \text{ m}}{69.1 \text{ s}} = 11.7 \text{ m/s}.$$

(d) The graphs for x, v and a as a function of t are shown below. SI units are understood. The third graph, $a(t)$, consists of three horizontal "steps" — one at 1.34 during $0 < t < 24.53$ and the next at -1.34 during $24.53 < t < 49.1$ and the last at zero during the "dead time" $49.1 < t < 69.1$).







37. (a) We note that $v_A = 12/6 = 2$ m/s (with two significant figures understood). Therefore, with an initial x value of 20 m, car A will be at $x = 28$ m when $t = 4$ s. This must be the value of x for car B at that time; we use Eq. 2-15:

$$28 \text{ m} = (12 \text{ m/s})t + \textbf{Erro!} a_B t^2 \qquad \text{where } t = 4.0 \text{ s} .$$

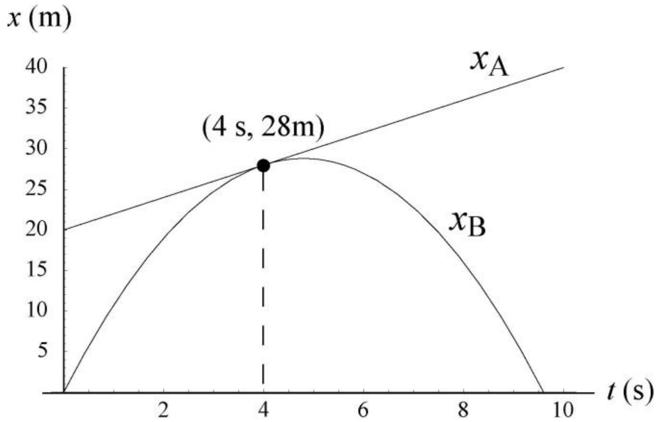
This yields $a_B = -2.5$ m/s^2.

(b) The question is: using the value obtained for a_B in part (a), are there other values of t (besides $t = 4$ s) such that $x_A = x_B$? The requirement is

$$20 + 2t = 12t + \textbf{Erro!} a_B t^2$$

where $a_B = -5/2$. There are two distinct roots unless the discriminant $\sqrt{10^2 - 2(-20)(a_B)}$ is zero. In our case, it <u>is</u> zero – which means there is only one root. The cars are side by side only once at $t = 4$ s.

(c) A sketch is shown below. It consists of a straight line (x_A) tangent to a parabola (x_B) at $t = 4$.



(d) We only care about <u>real</u> roots, which means $10^2 - 2(-20)(a_B) \geq 0$. If $|a_B| > 5/2$ then there are no (real) solutions to the equation; the cars are never side by side.

(e) Here we have $10^2 - 2(-20)(a_B) > 0 \quad \Rightarrow \quad$ two real roots. The cars are side by side at two different times.

38. We take the direction of motion as $+x$, so $a = -5.18$ m/s^2, and we use SI units, so $v_0 = 55(1000/3600) = 15.28$ m/s.

(a) The velocity is constant during the reaction time T, so the distance traveled during it is

$$d_r = v_0T - (15.28 \text{ m/s}) (0.75 \text{ s}) = 11.46 \text{ m}.$$

We use Eq. 2-16 (with $v = 0$) to find the distance d_b traveled during braking:

$$v^2 = v_0^2 + 2ad_b \;\;\Rightarrow\;\; d_b = -\frac{(15.28 \text{ m/s})^2}{2\left(-5.18 \text{ m/s}^2\right)}$$

which yields d_b = 22.53 m. Thus, the total distance is $d_r + d_b$ = 34.0 m, which means that the driver *is* able to stop in time. And if the driver were to continue at v_0, the car would enter the intersection in t = (40 m)/(15.28 m/s) = 2.6 s which is (barely) enough time to enter the intersection before the light turns, which many people would consider an acceptable situation.

(b) In this case, the total distance to stop (found in part (a) to be 34 m) is greater than the distance to the intersection, so the driver cannot stop without the front end of the car being a couple of meters into the intersection. And the time to reach it at constant speed is 32/15.28 = 2.1 s, which is too long (the light turns in 1.8 s). The driver is caught between a rock and a hard place.

39. The displacement (Δx) for each train is the "area" in the graph (since the displacement is the integral of the velocity). Each area is triangular, and the area of a triangle is 1/2(base) × (height). Thus, the (absolute value of the) displacement for one train (1/2)(40 m/s)(5 s) = 100 m, and that of the other train is (1/2)(30 m/s)(4 s) = 60 m. The initial "gap" between the trains was 200 m, and according to our displacement computations, the gap has narrowed by 160 m. Thus, the answer is 200 – 160 = 40 m.

40. Let d be the 220 m distance between the cars at t = 0, and v_1 be the 20 km/h = 50/9 m/s speed (corresponding to a passing point of x_1 = 44.5 m) and v_2 be the 40 km/h =100/9 m/s speed (corresponding to passing point of x_2 = 76.6 m) of the red car. We have two equations (based on Eq. 2-17):

$$d - x_1 = v_o t_1 + \textbf{Erro!} a\, t_1^2 \qquad \text{where } t_1 = x_1 / v_1$$

$$d - x_2 = v_o t_2 + \textbf{Erro!} a\, t_2^2 \qquad \text{where } t_2 = x_2 / v_2$$

We simultaneously solve these equations and obtain the following results:

(a) v_o = – 13.9 m/s. or roughly – 50 km/h (the negative sign means that it's along the $-x$ direction).

(b) a = – 2.0 m/s^2 (the negative sign means that it's along the $-x$ direction).

41. The positions of the cars as a function of time are given by

$$x_r(t) = x_{r0} + \frac{1}{2}a_r t^2 = (-35.0 \text{ m}) + \frac{1}{2}a_r t^2$$
$$x_g(t) = x_{g0} + v_g t = (270 \text{ m}) - (20 \text{ m/s})t$$

where we have substituted the velocity and not the speed for the green car. The two cars pass each other at $t = 12.0$ s when the graphed lines cross. This implies that

$$(270 \text{ m}) - (20 \text{ m/s})(12.0 \text{ s}) = 30 \text{ m} = (-35.0 \text{ m}) + \frac{1}{2}a_r(12.0 \text{ s})^2$$

which can be solved to give $a_r = 0.90 \text{ m/s}^2$.

42. In this solution we elect to wait until the last step to convert to SI units. Constant acceleration is indicated, so use of Table 2-1 is permitted. We start with Eq. 2-17 and denote the train's initial velocity as v_t and the locomotive's velocity as v_ℓ (which is also the final velocity of the train, if the rear-end collision is barely avoided). We note that the distance Δx consists of the original gap between them D as well as the forward distance traveled during this time by the locomotive $v_\ell t$. Therefore,

$$\frac{v_t + v_\ell}{2} = \frac{\Delta x}{t} = \frac{D + v_\ell t}{t} = \frac{D}{t} + v_\ell .$$

We now use Eq. 2-11 to eliminate time from the equation. Thus,

$$\frac{v_t + v_\ell}{2} = \frac{D}{(v_\ell - v_t)/a} + v_\ell$$

which leads to

$$a = \left(\frac{v_t + v_\ell}{2} - v_\ell\right)\left(\frac{v_\ell - v_t}{D}\right) = -\frac{1}{2D}(v_\ell - v_t)^2 .$$

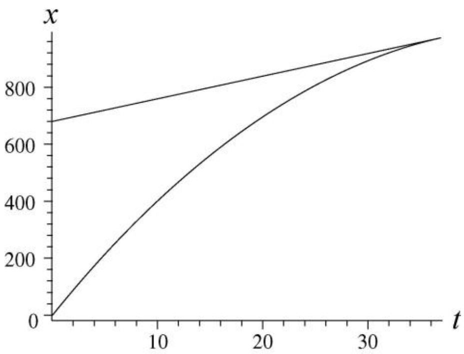
Hence,

$$a = -\frac{1}{2(0.676 \text{ km})}\left(29\frac{\text{km}}{\text{h}} - 161\frac{\text{km}}{\text{h}}\right)^2 = -12888 \text{ km/h}^2$$

which we convert as follows:

$$a = \left(-12888 \text{ km/h}^2\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)^2 = -0.994 \text{ m/s}^2$$

so that its *magnitude* is $|a| = 0.994 \text{ m/s}^2$. A graph is shown below for the case where a collision is just avoided (x along the vertical axis is in meters and t along the horizontal axis is in seconds). The top (straight) line shows the motion of the locomotive and the bottom curve shows the motion of the passenger train.



The other case (where the collision is not quite avoided) would be similar except that the slope of the bottom curve would be greater than that of the top line at the point where they meet.

43. (a) Note that 110 km/h is equivalent to 30.56 m/s. During a two second interval, you travel 61.11 m. The decelerating police car travels (using Eq. 2-15) 51.11 m. In light of the fact that the initial "gap" between cars was 25 m, this means the gap

has narrowed by 10.0 m – that is, to a distance of 15.0 m between cars.

(b) First, we add 0.4 s to the considerations of part (a). During a 2.4 s interval, you travel 73.33 m. The decelerating police car travels (using Eq. 2-15) 58.93 m during that time. The initial distance between cars of 25 m has therefore narrowed by 14.4 m. Thus, at the start of your braking (call it t_o) the gap between the cars is 10.6 m. The speed of the police car at t_o is 30.56 – 5(2.4) = 18.56 m/s. Collision occurs at time t when $x_{you} = x_{police}$ (we choose coordinates such that your position is $x = 0$ and the police car's position is $x = 10.6$ m at t_o). Eq. 2-15 becomes, for each car:

$$x_{police} - 10.6 = 18.56(t - t_o) - \textbf{Erro!}(5)(t - t_o)^2$$
$$x_{you} = 30.56(t - t_o) - \textbf{Erro!}(5)(t - t_o)^2 \quad .$$

Subtracting equations, we find

$$10.6 = (30.56 - 18.56)(t - t_o) \quad \Rightarrow \quad 0.883 \text{ s} = t - t_o.$$

At that time your speed is $30.56 + a(t - t_o) = 30.56 - 5(0.883) \approx 26$ m/s (or 94 km/h).

44. Neglect of air resistance justifies setting $a = -g = -9.8$ m/s^2 (where *down* is our $-y$ direction) for the duration of the fall. This is constant acceleration motion, and we may use Table 2-1 (with Δy replacing Δx).

(a) Using Eq. 2-16 and taking the negative root (since the final velocity is downward), we have

$$v = -\sqrt{v_0^2 - 2g\Delta y} = -\sqrt{0 - 2(9.8 \text{ m/s}^2)(-1700 \text{ m})} = -183 \text{ m/s}.$$

Its magnitude is therefore 183 m/s.

(b) No, but it is hard to make a convincing case without more analysis. We estimate the mass of a raindrop to be about a gram or less, so that its mass and speed (from part (a)) would be less than that of a typical bullet, which is good news. But the fact that one is dealing with *many* raindrops leads us to suspect that this scenario poses an unhealthy situation. If we factor in air resistance, the final speed is smaller, of course, and we return to the relatively healthy situation with which we are familiar.

45. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the fall. This is constant acceleration motion, which justifies the use of Table 2-1 (with Δy replacing Δx).

(a) Starting the clock at the moment the wrench is dropped ($v_0 = 0$), then $v^2 = v_o^2 - 2g\Delta y$ leads to

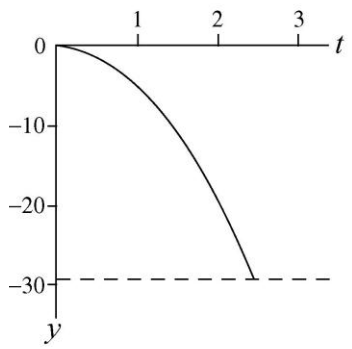
$$\Delta y = -\frac{(-24 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = -29.4 \text{ m}$$

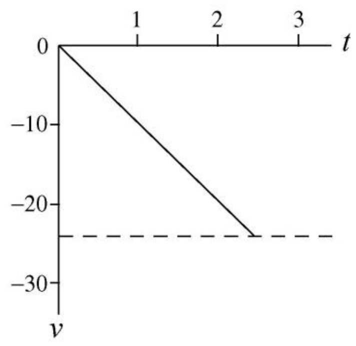
so that it fell through a height of 29.4 m.

(b) Solving $v = v_0 - gt$ for time, we find:

$$t = \frac{v_0 - v}{g} = \frac{0-(-24\text{ m/s})}{9.8\text{ m/s}^2} = 2.45\text{ s.}$$

(c) SI units are used in the graphs, and the initial position is taken as the coordinate origin. In the interest of saving space, we do not show the acceleration graph, which is a horizontal line at –9.8 m/s^2.





46. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the fall. This is constant acceleration motion, which justifies the use of Table 2-1 (with Δy replacing Δx).

(a) Noting that $\Delta y = y - y_0 = -30$ m, we apply Eq. 2-15 and the quadratic formula (Appendix E) to compute t:

$$\Delta y = v_0 t - \frac{1}{2} g t^2 \quad \Rightarrow \quad t = \frac{v_0 \pm \sqrt{v_0^2 - 2g\Delta y}}{g}$$

which (with $v_0 = -12$ m/s since it is downward) leads, upon choosing the positive root (so that $t > 0$), to the result:

$$t = \frac{-12\text{ m/s} + \sqrt{(-12\text{ m/s})^2 - 2(9.8\text{ m/s}^2)(-30\text{ m})}}{9.8\text{ m/s}^2} = 1.54\text{ s.}$$

(b) Enough information is now known that any of the equations in Table 2-1 can be used to obtain v; however, the one equation that does *not* use our result from part (a) is Eq. 2-16:

$$v = \sqrt{v_0^2 - 2g\Delta y} = 27.1\text{ m/s}$$

where the positive root has been chosen in order to give *speed* (which is the magnitude of the velocity vector).

47. We neglect air resistance for the duration of the motion (between "launching" and "landing"), so $a = -g = -9.8$ m/s^2 (we take downward to be the $-y$ direction). We use the equations in Table 2-1 (with Δy replacing Δx) because this is a = constant motion.

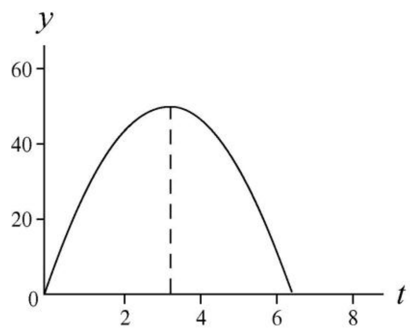
(a) At the highest point the velocity of the ball vanishes. Taking $y_0 = 0$, we set $v = 0$ in $v^2 = v_0^2 - 2gy$, and solve for the initial velocity: $v_0 = \sqrt{2gy}$. Since $y = 50$ m we find $v_0 = 31$ m/s.

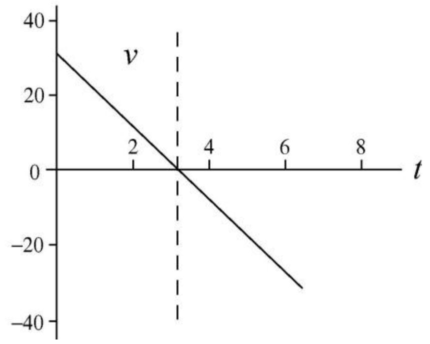
(b) It will be in the air from the time it leaves the ground until the time it returns to the ground ($y = 0$). Applying Eq. 2-15 to the entire motion (the rise and the fall, of total time $t > 0$) we have

$$y = v_0 t - \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \frac{2v_0}{g}$$

which (using our result from part (a)) produces $t = 6.4$ s. It is possible to obtain this without using part (a)'s result; one can find the time just for the rise (from ground to highest point) from Eq. 2-16 and then double it.

(c) SI units are understood in the x and v graphs shown. In the interest of saving space, we do not show the graph of a, which is a horizontal line at –9.8 m/s^2.





48. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y axis.

(a) Using $y = v_0 t - \frac{1}{2}gt^2$, with $y = 0.544$ m and $t = 0.200$ s, we find

$$v_0 = \frac{y + gt^2/2}{t} = \frac{0.544 \text{ m} + (9.8 \text{ m/s}^2)(0.200 \text{ s})^2/2}{0.200 \text{ s}} = 3.70 \text{ m/s} .$$

(b) The velocity at $y = 0.544$ m is

$$v = v_0 - gt = 3.70 \text{ m/s} - (9.8 \text{ m/s}^2)(0.200 \text{ s}) = 1.74 \text{ m/s} .$$

(c) Using $v^2 = v_0^2 - 2gy$ (with different values for y and v than before), we solve for the value of y corresponding to maximum height (where $v = 0$).

$$y = \frac{v_0^2}{2g} = \frac{(3.7 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 0.698 \text{ m}.$$

Thus, the armadillo goes 0.698 – 0.544 = 0.154 m higher.

49. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. We are placing the coordinate origin on the ground. We note that the initial velocity of the package is the same as the velocity of the balloon, $v_0 = +12$ m/s and that its initial coordinate is $y_0 = +80$ m.

(a) We solve $y = y_0 + v_0 t - \frac{1}{2}gt^2$ for time, with $y = 0$, using the quadratic formula (choosing the positive root to yield a positive value for t).

$$t = \frac{v_0 + \sqrt{v_0^2 + 2gy_0}}{g} = \frac{12 + \sqrt{12^2 + 2(9.8)(80)}}{9.8} = 5.4 \text{ s}$$

(b) If we wish to avoid using the result from part (a), we could use Eq. 2-16, but if that is not a concern, then a variety of formulas from Table 2-1 can be used. For instance, Eq. 2-11 leads to

$$v = v_0 - gt = 12 - (9.8)(5.4) = -41 \text{ m/s}.$$

Its final *speed* is 41 m/s.

50. The full extent of the bolt's fall is given by $y - y_o = -$ **Erro!**$g\,t^2$ where $y - y_o = -90$ m (if upwards is chosen as the positive y direction). Thus the time for the full fall is found to be $t = 4.29$ s. The first 80% of its free fall distance is given by $-72 = -g\,\tau^2/2$, which requires time $\tau = 3.83$ s.

(a) Thus, the final 20% of its fall takes $t - \tau = 0.45$ s.

(b) We can find that speed using $v = -g\tau$. Therefore, $|v| = 38$ m/s, approximately.

(c) Similarly, $v_{final} = -g\,t \quad \Rightarrow \quad |v_{final}| = 42$ m/s.

51. The speed of the boat is constant, given by $v_b = d/t$. Here, d is the distance of the boat from the bridge when the key is dropped (12 m) and t is the time the key takes in falling. To calculate t, we put the origin of the coordinate system at the point where the key is dropped and take the y axis to be positive in the *downward* direction. Taking the time to be zero at the instant the key is dropped, we compute the time t when $y = 45$ m. Since the initial velocity of the key is zero, the coordinate of the key is given by $y = \frac{1}{2}gt^2$. Thus,

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(45\text{ m})}{9.8\text{ m/s}^2}} = 3.03\text{ s}.$$

Therefore, the speed of the boat is

$$v_b = \frac{12\ \text{m}}{3.03\ \text{s}} = 4.0\ \text{m/s}\ .$$

52. The y coordinate of Apple 1 obeys $y - y_{o1} = -$ **Erro!**$g\ t^2$ where $y = 0$ when t = 2.0 s. This allows us to solve for y_{o1}, and we find y_{o1} = 19.6 m.

The graph for the coordinate of Apple 2 (which is thrown apparently at t = 1.0 s with velocity v_2) is

$$y - y_{o2} = v_2(t - 1.0) - \textbf{Erro!} g\ (t - 1.0)^2$$

where $y_{o2} = y_{o1}$ = 19.6 m and where y = 0 when t = 2.25 s. Thus we obtain $|v_2|$ = 9.6 m/s, approximately.

53. (a) With upward chosen as the $+y$ direction, we use Eq. 2-11 to find the initial velocity of the package:

$$v = v_o + at \quad \Rightarrow \quad 0 = v_o - (9.8\ \text{m/s}^2)(2.0\ \text{s})$$

which leads to v_o = 19.6 m/s. Now we use Eq. 2-15:

$$\Delta y = (19.6\ \text{m/s})(2.0\ \text{s}) + \textbf{Erro!}(-9.8\ \text{m/s}^2)(2.0\ \text{s})^2 \approx 20\ \text{m}\ .$$

We note that the "2.0 s" in this second computation refers to the time interval $2 < t < 4$ in the graph (whereas the "2.0 s" in the first computation referred to the $0 < t < 2$ time interval shown in the graph).

(b) In our computation for part (b), the time interval ("6.0 s") refers to the $2 < t < 8$ portion of the graph:

$$\Delta y = (19.6\ \text{m/s})(6.0\ \text{s}) + \textbf{Erro!}(-9.8\ \text{m/s}^2)(6.0\ \text{s})^2 \approx -59\ \text{m}\ ,$$

or $|\Delta y| = 59\ \text{m}$.

54. We use Eq. 2-16, $v_B^2 = v_A^2 + 2a(y_B - y_A)$, with a = –9.8 m/s^2, $y_B - y_A$ = 0.40 m, and v_B = **Erro!**v_A. It is then straightforward to solve: v_A = 3.0 m/s, approximately.

55. (a) We first find the velocity of the ball just before it hits the ground. During contact with the ground its average acceleration is given by

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}$$

where Δv is the change in its velocity during contact with the ground and $\Delta t = 20.0 \times 10^{-3}$ s is the duration of contact. Now, to find the velocity just *before* contact, we put the origin at the point where the ball is dropped (and take $+y$ upward) and take t = 0 to be when it is dropped. The ball strikes the ground at y = –15.0 m. Its velocity there is found from Eq. 2-16: $v^2 = -2gy$. Therefore,

$$v = -\sqrt{-2gy} = -\sqrt{-2(9.8\ \text{m/s}^2)(-15.0\ \text{m})} = -17.1\ \text{m/s}$$

where the negative sign is chosen since the ball is traveling downward at the moment of contact. Consequently, the average acceleration during contact with the ground is

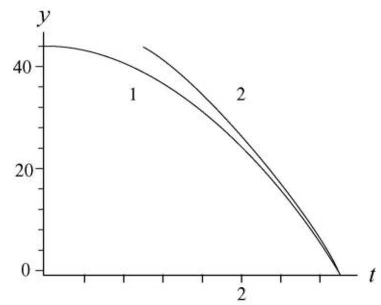
$$a_{\text{avg}} = \frac{0-(-17.1\ \text{m/s})}{20.0\times 10^{-3}\ \text{s}} = 857\ \text{m/s}^2.$$

(b) The fact that the result is positive indicates that this acceleration vector points upward. In a later chapter, this will be directly related to the magnitude and direction of the force exerted by the ground on the ball during the collision.

56. (a) We neglect air resistance, which justifies setting $a = -g = -9.8\ \text{m/s}^2$ (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Eq. 2-15 (with Δy replacing Δx) because this is constant acceleration motion. We use primed variables (except t) with the first stone, which has zero initial velocity, and unprimed variables with the second stone (with initial downward velocity $-v_0$, so that v_0 is being used for the initial *speed*). SI units are used throughout.

$$\Delta y' = 0(t) - \frac{1}{2}gt^2$$
$$\Delta y = (-v_0)(t-1) - \frac{1}{2}g(t-1)^2$$

Since the problem indicates $\Delta y' = \Delta y = -43.9$ m, we solve the first equation for t (finding $t = 2.99$ s) and use this result to solve the second equation for the initial speed of the second stone:



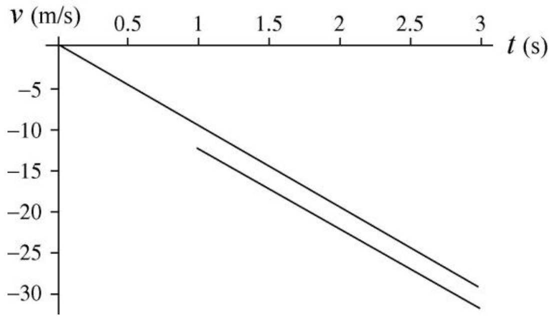
$$-43.9\ \text{m} = (-v_0)(1.99\ \text{s}) - \frac{1}{2}(9.8\ \text{m/s}^2)(1.99\ \text{s})^2$$

which leads to $v_0 = 12.3$ m/s.

(b) The velocity of the stones are given by

$$v'_y = \frac{d(\Delta y')}{dt} = -gt, \qquad v_y = \frac{d(\Delta y)}{dt} = -v_0 - g(t-1)$$

The plot is shown below:



57. The average acceleration during contact with the floor is $a_{avg} = (v_2 - v_1) / \Delta t$, where v_1 is its velocity just before striking the floor, v_2 is its velocity just as it leaves the floor, and Δt is the duration of contact with the floor (12×10^{-3} s).

(a) Taking the y axis to be positively upward and placing the origin at the point where the ball is dropped, we first find the velocity just before striking the floor, using $v_1^2 = v_0^2 - 2gy$. With $v_0 = 0$ and $y = -4.00$ m, the result is

$$v_1 = -\sqrt{-2gy} = -\sqrt{-2(9.8\ \text{m/s}^2)(-4.00\ \text{m})} = -8.85\ \text{m/s}$$

where the negative root is chosen because the ball is traveling downward. To find the velocity just after hitting the floor (as it ascends without air friction to a height of 2.00 m), we use $v^2 = v_2^2 - 2g(y - y_0)$ with $v = 0$, $y = -2.00$ m (it ends up two meters *below* its initial drop height), and $y_0 = -4.00$ m. Therefore,

$$v_2 = \sqrt{2g(y - y_0)} = \sqrt{2(9.8\ \text{m/s}^2)(-2.00\ \text{m} + 4.00\ \text{m})} = 6.26\ \text{m/s}\ .$$

Consequently, the average acceleration is

$$a_{avg} = \frac{v_2 - v_1}{\Delta t} = \frac{6.26\ \text{m/s} - (-8.85\ \text{m/s})}{12.0 \times 10^{-3}\ \text{s}} = 1.26 \times 10^3\ \text{m/s}^2.$$

(b) The positive nature of the result indicates that the acceleration vector points upward. In a later chapter, this will be directly related to the magnitude and direction of the force exerted by the ground on the ball during the collision.

58. To find the "launch" velocity of the rock, we apply Eq. 2-11 to the maximum height (where the speed is momentarily zero)

$$v = v_0 - gt \quad \Rightarrow \quad 0 = v_0 - \left(9.8\ \text{m/s}^2\right)\left(2.5\ \text{s}\right)$$

so that $v_0 = 24.5$ m/s (with $+y$ up). Now we use Eq. 2-15 to find the height of the tower (taking $y_0 = 0$ at the ground level)

$$y - y_0 = v_0 t + \frac{1}{2}at^2 \quad \Rightarrow \quad y - 0 = (24.5\text{ m/s})(1.5\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(1.5\text{ s})^2 .$$

Thus, we obtain y = 26 m.

59. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y axis.

(a) The time drop 1 leaves the nozzle is taken as $t = 0$ and its time of landing on the floor t_1 can be computed from Eq. 2-15, with $v_0 = 0$ and $y_1 = -2.00$ m.

$$y_1 = -\frac{1}{2}gt_1^2 \quad \Rightarrow \quad t_1 = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-2.00\text{ m})}{9.8\text{ m/s}^2}} = 0.639\text{ s} .$$

At that moment, the fourth drop begins to fall, and from the regularity of the dripping we conclude that drop 2 leaves the nozzle at t = 0.639/3 = 0.213 s and drop 3 leaves the nozzle at t = 2(0.213 s) = 0.426 s. Therefore, the time in free fall (up to the moment drop 1 lands) for drop 2 is $t_2 = t_1 - 0.213$ s = 0.426 s. Its position at the moment drop 1 strikes the floor is

$$y_2 = -\frac{1}{2}gt_2^2 = -\frac{1}{2}(9.8\text{ m/s}^2)(0.426\text{ s})^2 = -0.889\text{ m},$$

or 89 cm below the nozzle.

(b) The time in free fall (up to the moment drop 1 lands) for drop 3 is $t_3 = t_1 - 0.426$ s = 0.213 s. Its position at the moment drop 1 strikes the floor is

$$y_3 = -\frac{1}{2}gt_3^2 = -\frac{1}{2}(9.8\text{ m/s}^2)(0.213\text{ s})^2 = -0.222\text{ m},$$

or 22 cm below the nozzle.

60. We choose *down* as the $+y$ direction and set the coordinate origin at the point where it was dropped (which is when we start the clock). We denote the 1.00 s duration mentioned in the problem as $t - t'$ where t is the value of time when it lands and t' is one second prior to that. The corresponding distance is $y - y' = 0.50h$, where y denotes the location of the ground. In these terms, y is the same as h, so we have $h - y' = 0.50h$ or $0.50h = y'$.

(a) We find t' and t from Eq. 2-15 (with $v_0 = 0$):

$$y' = \frac{1}{2}gt'^2 \;\Rightarrow\; t' = \sqrt{\frac{2y'}{g}}$$
$$y = \frac{1}{2}gt^2 \;\Rightarrow\; t = \sqrt{\frac{2y}{g}}\ .$$

Plugging in $y = h$ and $y' = 0.50h$, and dividing these two equations, we obtain

$$\frac{t'}{t} = \sqrt{\frac{2\left(0.50h\right)/g}{2h/g}} = \sqrt{0.50}\ .$$

Letting $t' = t - 1.00$ (SI units understood) and cross-multiplying, we find

$$t - 1.00 = t\sqrt{0.50} \;\Rightarrow\; t = \frac{1.00}{1-\sqrt{0.50}}$$

which yields $t = 3.41$ s.

(b) Plugging this result into $y = \frac{1}{2}gt^2$ we find $h = 57$ m.

(c) In our approach, we did not use the quadratic formula, but we did "choose a root" when we assumed (in the last calculation in part (a)) that $\sqrt{0.50} = +0.707$ instead of -0.707. If we had instead let $\sqrt{0.50} = -0.707$ then our answer for t would have been roughly 0.6 s which would imply that $t' = t - 1$ would equal a negative number (indicating a time *before* it was dropped) which certainly does not fit with the physical situation described in the problem.

61. The time t the pot spends passing in front of the window of length $L = 2.0$ m is 0.25 s each way. We use v for its velocity as it passes the top of the window (going up). Then, with $a = -g = -9.8$ m/s^2 (taking *down* to be the $-y$ direction), Eq. 2-18 yields

$$L = vt - \frac{1}{2}gt^2 \quad\Rightarrow\quad v = \frac{L}{t} - \frac{1}{2}gt.$$

The distance H the pot goes above the top of the window is therefore (using Eq. 2-16 with the *final velocity* being zero to indicate the highest point)

$$H = \frac{v^2}{2g} = \frac{\left(L/t - gt/2\right)^2}{2g} = \frac{\left(2.00\text{ m}/0.25\text{ s} - (9.80\text{ m/s}^2)(0.25\text{ s})/2\right)^2}{2(9.80\text{ m/s}^2)} = 2.34\text{ m}.$$

62. The graph shows $y = 25$ m to be the highest point (where the speed momentarily vanishes). The neglect of "air friction" (or whatever passes for that on the distant planet) is certainly reasonable due to the symmetry of the graph.

(a) To find the acceleration due to gravity g_p on that planet, we use Eq. 2-15 (with $+y$ up)

$$y - y_0 = vt + \frac{1}{2}g_p t^2 \quad \Rightarrow \quad 25\text{ m} - 0 = (0)(2.5\text{ s}) + \frac{1}{2}g_p(2.5\text{ s})^2$$

so that $g_p = 8.0\text{ m/s}^2$.

(b) That same (max) point on the graph can be used to find the initial velocity.

$$y - y_0 = \frac{1}{2}(v_0 + v)t \quad \Rightarrow \quad 25\text{ m} - 0 = \frac{1}{2}(v_0 + 0)(2.5\text{ s})$$

Therefore, $v_0 = 20$ m/s.

63. We choose *down* as the $+y$ direction and place the coordinate origin at the top of the building (which has height H). During its fall, the ball passes (with velocity v_1) the top of the window (which is at y_1) at time t_1, and passes the bottom (which is at y_2) at time t_2. We are told $y_2 - y_1 = 1.20$ m and $t_2 - t_1 = 0.125$ s. Using Eq. 2-15 we have

$$y_2 - y_1 = v_1(t_2 - t_1) + \frac{1}{2}g(t_2 - t_1)^2$$

which immediately yields

$$v_1 = \frac{1.20\text{ m} - \frac{1}{2}(9.8\text{ m/s}^2)(0.125\text{ s})^2}{0.125\text{ s}} = 8.99\text{ m/s}.$$

From this, Eq. 2-16 (with $v_0 = 0$) reveals the value of y_1:

$$v_1^2 = 2gy_1 \quad \Rightarrow \quad y_1 = \frac{(8.99\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 4.12\text{ m}.$$

It reaches the ground ($y_3 = H$) at t_3. Because of the symmetry expressed in the problem ("upward flight is a reverse of the fall'') we know that $t_3 - t_2 = 2.00/2 = 1.00$ s. And this means $t_3 - t_1 = 1.00\text{ s} + 0.125\text{ s} = 1.125$ s. Now Eq. 2-15 produces

$$y_3 - y_1 = v_1(t_3 - t_1) + \frac{1}{2}g(t_3 - t_1)^2$$

$$y_3 - 4.12\text{ m} = (8.99\text{ m/s})(1.125\text{ s}) + \frac{1}{2}(9.8\text{ m/s}^2)(1.125\text{ s})^2$$

which yields $y_3 = H = 20.4$ m.

64. The height reached by the player is $y = 0.76$ m (where we have taken the origin of the y axis at the floor and $+y$ to be upward).

(a) The initial velocity v_0 of the player is

$$v_0 = \sqrt{2gy} = \sqrt{2(9.8\text{ m/s}^2)(0.76\text{ m})} = 3.86\text{ m/s}.$$

This is a consequence of Eq. 2-16 where velocity v vanishes. As the player reaches y_1 = 0.76 m – 0.15 m = 0.61 m, his speed v_1 satisfies $v_0^2 - v_1^2 = 2gy_1$, which yields

$$v_1 = \sqrt{v_0^2 - 2gy_1} = \sqrt{(3.86\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(0.61\text{ m})} = 1.71\text{ m/s}\ .$$

The time t_1 that the player spends *ascending* in the top Δy_1 = 0.15 m of the jump can now be found from Eq. 2-17:

$$\Delta y_1 = \frac{1}{2}\left(v_1 + v\right)t_1 \ \Rightarrow\ t_1 = \frac{2\left(0.15\text{ m}\right)}{1.71\text{ m/s} + 0} = 0.175\text{ s}$$

which means that the total time spent in that top 15 cm (both ascending and descending) is 2(0.175 s) = 0.35 s = 350 ms.

(b) The time t_2 when the player reaches a height of 0.15 m is found from Eq. 2-15:

$$0.15\text{ m} = v_0 t_2 - \frac{1}{2}gt_2^2 = (3.86\text{ m/s})t_2 - \frac{1}{2}(9.8\text{ m/s}^2)t_2^2\ ,$$

which yields (using the quadratic formula, taking the smaller of the two positive roots) t_2 = 0.041 s = 41 ms, which implies that the total time spent in that bottom 15 cm (both ascending and descending) is 2(41 ms) = 82 ms.

65. The key idea here is that the speed of the head (and the torso as well) at any given time can be calculated by finding the area on the graph of the head's acceleration versus time, as shown in Eq. 2-26:

$$v_1 - v_0 = \begin{pmatrix}\text{area between the acceleration curve}\\ \text{and the time axis, from } t_0 \text{ to } t_1\end{pmatrix}$$

(a) From Fig. 2.14a, we see that the head begins to accelerate from rest (v_0 = 0) at t_0 = 110 ms and reaches a maximum value of 90 m/s^2 at t_1 = 160 ms. The area of this region is

$$\text{area} = \frac{1}{2}(160 - 110)\times 10^{-3}\text{s}\cdot\left(90\text{ m/s}^2\right) = 2.25\text{ m/s}$$

which is equal to v_1, the speed at t_1.

(b) To compute the speed of the torso at t_1=160 ms, we divide the area into 4 regions: From 0 to 40 ms, region A has zero area. From 40 ms to 100 ms, region *B* has the shape of a triangle with area

$$\text{area}_B = \frac{1}{2}(0.0600\text{ s})(50.0\text{ m/s}^2) = 1.50\text{ m/s}\ .$$

From 100 to 120 ms, region *C* has the shape of a rectangle with area

$$\text{area}_C = (0.0200\text{ s})\,(50.0\text{ m/s}^2) = 1.00\text{ m/s}.$$

From 110 to 160 ms, region D has the shape of a trapezoid with area

$$\text{area}_D = \frac{1}{2}(0.0400\text{ s})\,(50.0 + 20.0)\text{ m/s}^2 = 1.40\text{ m/s}.$$

Substituting these values into Eq. 2-26, with v_0=0 then gives

$$v_1 - 0 = 0 + 1.50\text{ m/s} + 1.00\text{ m/s} + 1.40\text{ m/s} = 3.90\text{ m/s},$$

or $v_1 = 3.90$ m/s.

66. This problem can be solved by noting that velocity can be determined by the graphical integration of acceleration versus time. The speed of the tongue of the salamander is simply equal to the area under the acceleration curve:

$$v = \text{area} = \frac{1}{2}(10^{-2}\text{ s})(100\text{ m/s}^2) + \frac{1}{2}(10^{-2}\text{ s})(100\text{ m/s}^2 + 400\text{ m/s}^2) + \frac{1}{2}(10^{-2}\text{ s})(400\text{ m/s}^2)$$
$$= 5.0\text{ m/s}.$$

67. Since $v = dx/dt$ (Eq. 2-4), then $\Delta x = \int v\,dt$, which corresponds to the area under the v vs t graph. Dividing the total area A into rectangular (base × height) and triangular $\left(\frac{1}{2}\text{base} \times \text{height}\right)$ areas, we have

$$\begin{aligned} A &= A_{0<t<2} + A_{2<t<10} + A_{10<t<12} + A_{12<t<16} \\ &= \frac{1}{2}(2)(8) + (8)(8) + \left[(2)(4) + \frac{1}{2}(2)(4)\right] + (4)(4) \end{aligned}$$

with SI units understood. In this way, we obtain Δx = 100 m.

68. The key idea here is that the position of an object at any given time can be calculated by finding the area on the graph of the object's velocity versus time, as shown in Eq. 2-25:

$$x_1 - x_0 = \left(\begin{array}{c}\text{area between the velocity curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1\end{array}\right).$$

(a) To compute the position of the fist at t = 50 ms, we divide the area in Fig. 2-37 into two regions. From 0 to 10 ms, region A has the shape of a triangle with area

$$\text{area}_A = \frac{1}{2}(0.010\text{ s})\,(2\text{ m/s}) = 0.01\text{ m}.$$

From 10 to 50 ms, region B has the shape of a trapezoid with area

$$\text{area}_B = \frac{1}{2}(0.040\text{ s})\,(2 + 4)\text{ m/s} = 0.12\text{ m}.$$

Substituting these values into Eq. 2-25, with x_0=0 then gives

$$x_1 - 0 = 0 + 0.01 \text{ m} + 0.12 \text{ m} = 0.13 \text{ m},$$

or $x_1 = 0.13$ m.

(b) The speed of the fist reaches a maximum at t_1 = 120 ms. From 50 to 90 ms, region *C* has the shape of a trapezoid with area

$$\text{area}_C = \frac{1}{2}(0.040 \text{ s})\,(4 + 5) \text{ m/s} = 0.18 \text{ m}.$$

From 90 to 120 ms, region *D* has the shape of a trapezoid with area

$$\text{area}_D = \frac{1}{2}(0.030 \text{ s})\,(5 + 7.5) \text{ m/s} = 0.19 \text{ m}.$$

Substituting these values into Eq. 2-25, with x_0=0 then gives

$$x_1 - 0 = 0 + 0.01 \text{ m} + 0.12 \text{ m} + 0.18 \text{ m} + 0.19 \text{ m} = 0.50 \text{ m},$$

or $x_1 = 0.50$ m.

69. The problem is solved using Eq. 2-26:

$$v_1 - v_0 = \begin{pmatrix} \text{area between the acceleration curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{pmatrix}$$

To compute the speed of the unhelmeted, bare head at t_1=7.0 ms, we divide the area under the *a* vs. *t* graph into 4 regions: From 0 to 2 ms, region A has the shape of a triangle with area

$$\text{area}_A = \frac{1}{2}(0.0020 \text{ s})\,(120 \text{ m/s}^2) = 0.12 \text{ m/s}.$$

From 2 ms to 4 ms, region *B* has the shape of a trapezoid with area

$$\text{area}_B = \frac{1}{2}(0.0020 \text{ s})\,(120 + 140) \text{ m/s}^2 = 0.26 \text{ m/s}.$$

From 4 to 6 ms, region *C* has the shape of a trapezoid with area

$$\text{area}_C = \frac{1}{2}(0.0020 \text{ s})\,(140 + 200) \text{ m/s}^2 = 0.34 \text{ m/s}.$$

From 6 to 7 ms, region *D* has the shape of a triangle with area

$$\text{area}_{\text{D}} = \frac{1}{2}(0.0010 \text{ s}) (200 \text{ m/s}^2) = 0.10 \text{ m/s}.$$

Substituting these values into Eq. 2-26, with v_0=0 then gives

$$v_{unhelmeted} = 0.12 \text{ m/s} + 0.26 \text{ m/s} + 0.34 \text{ m/s} + 0.10 \text{ m/s} = 0.82 \text{ m/s}.$$

Carrying out similar calculations for the helmeted head, we have the following results: From 0 to 3 ms, region A has the shape of a triangle with area

$$\text{area}_{\text{A}} = \frac{1}{2}(0.0030 \text{ s}) (40 \text{ m/s}^2) = 0.060 \text{ m/s}.$$

From 3 ms to 4 ms, region *B* has the shape of a rectangle with area

$$\text{area}_{\text{B}} = (0.0010 \text{ s}) (40 \text{ m/s}^2) = 0.040 \text{ m/s}.$$

From 4 to 6 ms, region *C* has the shape of a trapezoid with area

$$\text{area}_{\text{C}} = \frac{1}{2}(0.0020 \text{ s}) (40 + 80) \text{ m/s}^2 = 0.12 \text{ m/s}.$$

From 6 to 7 ms, region *D* has the shape of a triangle with area

$$\text{area}_{\text{D}} = \frac{1}{2}(0.0010 \text{ s}) (80 \text{ m/s}^2) = 0.040 \text{ m/s}.$$

Substituting these values into Eq. 2-26, with v_0=0 then gives

$$v_{helmeted} = 0.060 \text{ m/s} + 0.040 \text{ m/s} + 0.12 \text{ m/s} + 0.040 \text{ m/s} = 0.26 \text{ m/s}.$$

Thus, the difference in the speed is

$$\Delta v = v_{unhelmeted} - v_{helmeted} = 0.82 \text{ m/s} - 0.26 \text{ m/s} = 0.56 \text{ m/s}.$$

70. To solve this problem, we note that velocity is equal to the time derivative of a position function, as well as the time integral of an acceleration function, with the integration constant being the initial velocity. Thus, the velocity of particle 1 can be written as

$$v_1 = \frac{dx_1}{dt} = \frac{d}{dt}\left(6.00t^2 + 3.00t + 2.00\right) = 12.0t + 3.00.$$

Similarly, the velocity of particle 2 is

$$v_2 = v_{20} + \int a_2 dt = 20.0 + \int (-8.00t) dt = 20.0 - 4.00t^2.$$

The condition that $v_1 = v_2$ implies

$$12.0t + 3.00 = 20.0 - 4.00t^2 \quad \Rightarrow \quad 4.00t^2 + 12.0t - 17.0 = 0$$

which can be solved to give (taking positive root) $t = (-3+\sqrt{26})/2 = 1.05\ \text{s}$. Thus, the velocity at this time is $v_1 = v_2 = 12.0(1.05) + 3.00 = 15.6\ \text{m/s}$.

71. We denote the required time as *t*, assuming the light turns green when the clock reads zero. By this time, the distances traveled by the two vehicles must be the same.

(a) Denoting the acceleration of the automobile as *a* and the (constant) speed of the truck as *v* then

$$\Delta x = \left(\frac{1}{2}at^2\right)_{\text{car}} = \left(vt\right)_{\text{truck}}$$

which leads to

$$t = \frac{2v}{a} = \frac{2\left(9.5\ \text{m/s}\right)}{2.2\ \text{m/s}^2} = 8.6\ \text{s}\ .$$

Therefore,

$$\Delta x = vt = \left(9.5\ \text{m/s}\right)\left(8.6\ \text{s}\right) = 82\ \text{m}\ .$$

(b) The speed of the car at that moment is

$$v_{\text{car}} = at = \left(2.2\ \text{m/s}^2\right)\left(8.6\ \text{s}\right) = 19\ \text{m/s}\ .$$

72. (a) A constant velocity is equal to the ratio of displacement to elapsed time. Thus, for the vehicle to be traveling at a constant speed v_p over a distance D_{23}, the time delay should be $t = D_{23}/v_p$.

(b) The time required for the car to accelerate from rest to a cruising speed v_p is $t_0 = v_p/a$. During this time interval, the distance traveled is $\Delta x_0 = at_0^2/2 = v_p^2/2a$. The car then moves at a constant speed v_p over a distance $D_{12} - \Delta x_0 - d$ to reach intersection 2, and the time elapsed is $t_1 = (D_{12} - \Delta x_0 - d)/v_p$. Thus, the time delay at intersection 2 should be set to

$$t_{\text{total}} = t_r + t_0 + t_1 = t_r + \frac{v_p}{a} + \frac{D_{12} - \Delta x_0 - d}{v_p} = t_r + \frac{v_p}{a} + \frac{D_{12} - (v_p^2/2a) - d}{v_p}$$

$$= t_r + \frac{1}{2}\frac{v_p}{a} + \frac{D_{12} - d}{v_p}$$

73. (a) The derivative (with respect to time) of the given expression for *x* yields the

"velocity" of the spot:

$$v(t) = 9 - \textbf{Erro!}t^2$$

with 3 significant figures understood. It is easy to see that $v = 0$ when $t = 2.00$ s.

(b) At $t = 2$ s, $x = 9(2) - ¾(2)^3 = 12$. Thus, the location of the spot when $v = 0$ is 12.0 cm from left edge of screen.

(c) The derivative of the velocity is $a = -$ **Erro!**t which gives an acceleration (leftward) of magnitude 9.00 m/s^2 when the spot is 12 cm from left edge of screen.

(d) Since $v > 0$ for times less than $t = 2$ s, then the spot had been moving rightwards.

(e) As implied by our answer to part (c), it moves leftward for times immediately after $t = 2$ s. In fact, the expression found in part (a) guarantees that for all $t > 2$, $v < 0$ (that is, until the clock is "reset" by reaching an edge).

(f) As the discussion in part (e) shows, the edge that it reaches at some $t > 2$ s cannot be the right edge; it is the left edge ($x = 0$). Solving the expression given in the problem statement (with $x = 0$) for positive t yields the answer: the spot reaches the left edge at $t = \sqrt{12}$ s ≈ 3.46 s.

74. (a) Let the height of the diving board be h. We choose *down* as the $+y$ direction and set the coordinate origin at the point where it was dropped (which is when we start the clock). Thus, $y = h$ designates the location where the ball strikes the water. Let the depth of the lake be D, and the total time for the ball to descend be T. The speed of the ball as it reaches the surface of the lake is then $v = \sqrt{2gh}$ (from Eq. 2-16), and the time for the ball to fall from the board to the lake surface is $t_1 = \sqrt{2h/g}$ (from Eq. 2-15). Now, the time it spends descending in the lake (at constant velocity v) is

$$t_2 = \frac{D}{v} = \frac{D}{\sqrt{2gh}} \ .$$

Thus, $T = t_1 + t_2 = \sqrt{\dfrac{2h}{g}} + \dfrac{D}{\sqrt{2gh}}$, which gives

$$D = T\sqrt{2gh} - 2h = (4.80\ \text{s})\sqrt{(2)(9.80\ \text{m/s}^2)(5.20\ \text{m})} - 2(5.20\ \text{m}) = 38.1\ \text{m}\ .$$

(b) Using Eq. 2-2, the magnitude of the average velocity is

$$v_{\text{avg}} = \frac{D + h}{T} = \frac{38.1\ \text{m} + 5.20\ \text{m}}{4.80\ \text{s}} = 9.02\ \text{m/s}$$

(c) In our coordinate choices, a positive sign for v_{avg} means that the ball is going downward. If, however, upwards had been chosen as the positive direction, then this answer in (b) would turn out negative-valued.

(d) We find v_0 from $\Delta y = v_0 t + \frac{1}{2} g t^2$ with $t = T$ and $\Delta y = h + D$. Thus,

$$v_0 = \frac{h + D}{T} - \frac{gT}{2} = \frac{5.20 \text{ m} + 38.1 \text{ m}}{4.80 \text{ s}} - \frac{\left(9.8 \text{ m/s}^2\right)\left(4.80 \text{ s}\right)}{2} = 14.5 \text{ m/s}$$

(e) Here in our coordinate choices the negative sign means that the ball is being thrown upward.

75. We choose *down* as the $+y$ direction and use the equations of Table 2-1 (replacing x with y) with $a = +g$, $v_0 = 0$ and $y_0 = 0$. We use subscript 2 for the elevator reaching the ground and 1 for the halfway point.

(a) Eq. 2-16, $v_2^2 = v_0^2 + 2a(y_2 - y_0)$, leads to

$$v_2 = \sqrt{2gy_2} = \sqrt{2\left(9.8 \text{ m/s}^2\right)\left(120 \text{ m}\right)} = 48.5 \text{ m/s} .$$

(b) The time at which it strikes the ground is (using Eq. 2-15)

$$t_2 = \sqrt{\frac{2y_2}{g}} = \sqrt{\frac{2\left(120 \text{ m}\right)}{9.8 \text{ m/s}^2}} = 4.95 \text{ s} .$$

(c) Now Eq. 2-16, in the form $v_1^2 = v_0^2 + 2a(y_1 - y_0)$, leads to

$$v_1 = \sqrt{2gy_1} = \sqrt{2(9.8 \text{ m/s}^2)(60 \text{ m})} = 34.3 \text{m/s}.$$

(d) The time at which it reaches the halfway point is (using Eq. 2-15)

$$t_1 = \sqrt{\frac{2y_1}{g}} = \sqrt{\frac{2(60 \text{ m})}{9.8 \text{ m/s}^2}} = 3.50 \text{ s} .$$

76. Taking $+y$ to be upward and placing the origin at the point from which the objects are dropped, then the location of diamond 1 is given by $y_1 = -\frac{1}{2} g t^2$ and the location of diamond 2 is given by $y_2 = -\frac{1}{2} g (t-1)^2$. We are starting the clock when the first object is dropped. We want the time for which $y_2 - y_1 = 10$ m. Therefore,

$$-\frac{1}{2} g (t-1)^2 + \frac{1}{2} g t^2 = 10 \quad \Rightarrow \quad t = (10/g) + 0.5 = 1.5 \text{ s}.$$

77. Assuming the horizontal velocity of the ball is constant, the horizontal displacement is

$$\Delta x = v \Delta t$$

where Δx is the horizontal distance traveled, Δt is the time, and v is the (horizontal) velocity. Converting v to meters per second, we have 160 km/h = 44.4 m/s. Thus

$$\Delta t = \frac{\Delta x}{v} = \frac{18.4 \text{ m}}{44.4 \text{ m/s}} = 0.414 \text{ s}.$$

The velocity-unit conversion implemented above can be figured "from basics" (1000 m = 1 km, 3600 s = 1 h) or found in Appendix D.

78. In this solution, we make use of the notation $x(t)$ for the value of x at a particular t. Thus, $x(t) = 50t + 10t^2$ with SI units (meters and seconds) understood.

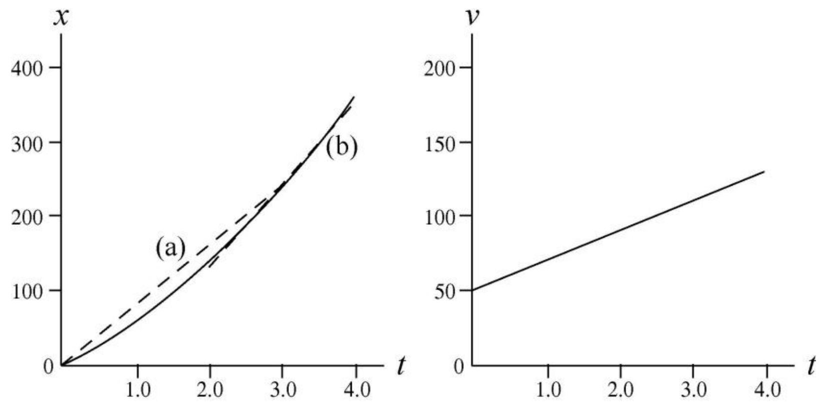
(a) The average velocity during the first 3 s is given by

$$v_{\text{avg}} = \frac{x(3) - x(0)}{\Delta t} = \frac{(50)(3) + (10)(3)^2 - 0}{3} = 80 \text{ m/s}.$$

(b) The instantaneous velocity at time t is given by $v = dx/dt = 50 + 20t$, in SI units. At $t = 3.0$ s, $v = 50 + (20)(3.0) = 110$ m/s.

(c) The instantaneous acceleration at time t is given by $a = dv/dt = 20 \text{ m/s}^2$. It is constant, so the acceleration at any time is 20 m/s^2.

(d) and (e) The graphs that follow show the coordinate x and velocity v as functions of time, with SI units understood. The dashed line marked (a) in the first graph runs from $t = 0$, $x = 0$ to $t = 3.0$s, $x = 240$ m. Its slope is the average velocity during the first 3s of motion. The dashed line marked (b) is tangent to the x curve at $t = 3.0$ s. Its slope is the instantaneous velocity at $t = 3.0$ s.



79. We take $+x$ in the direction of motion, so $v_0 = +30$ m/s, $v_1 = +15$ m/s and $a < 0$. The acceleration is found from Eq. 2-11: $a = (v_1 - v_0)/t_1$ where $t_1 = 3.0$ s. This gives $a = -5.0 \text{ m/s}^2$. The displacement (which in this situation is the same as the distance traveled) to the point it stops ($v_2 = 0$) is, using Eq. 2-16,

$$v_2^2 = v_0^2 + 2a\Delta x \quad \Rightarrow \quad \Delta x = -\frac{(30 \text{ m/s})^2}{2(-5 \text{ m/s}^2)} = 90 \text{ m}.$$

80. If the plane (with velocity v) maintains its present course, and if the terrain

continues its upward slope of 4.3°, then the plane will strike the ground after traveling

$$\Delta x = \frac{h}{\tan\theta} = \frac{35\text{ m}}{\tan 4.3°} = 465.5\text{ m} \approx 0.465\ \text{ km}.$$

This corresponds to a time of flight found from Eq. 2-2 (with $v = v_{\text{avg}}$ since it is constant)

$$t = \frac{\Delta x}{v} = \frac{0.465\text{ km}}{1300\text{ km/h}} = 0.000358\text{ h} \approx 1.3\text{ s}.$$

This, then, estimates the time available to the pilot to make his correction.

81. The problem consists of two constant-acceleration parts: part 1 with $v_0 = 0$, $v = 6.0$ m/s, $x = 1.8$ m, and $x_0 = 0$ (if we take its original position to be the coordinate origin); and, part 2 with $v_0 = 6.0$ m/s, $v = 0$, and $a_2 = -2.5$ m/s^2 (negative because we are taking the positive direction to be the direction of motion).

(a) We can use Eq. 2-17 to find the time for the first part

$x - x_0 =$ **Erro!** $(v_0 + v)\, t_1 \quad \Rightarrow \quad 1.8\text{ m} - 0 =$ **Erro!**$(0 + 6.0\text{ m/s})\, t_1$

so that $t_1 = 0.6$ s. And Eq. 2-11 is used to obtain the time for the second part

$$v = v_0 + a_2 t_2 \quad \Rightarrow \quad 0 = 6.0\text{ m/s} + (-2.5\text{ m/s}^2)t_2$$

from which $t_2 = 2.4$ s is computed. Thus, the total time is $t_1 + t_2 = 3.0$ s.

(b) We already know the distance for part 1. We could find the distance for part 2 from several of the equations, but the one that makes no use of our part (a) results is Eq. 2-16

$$v^2 = v_0^2 + 2a_2\Delta x_2 \quad \Rightarrow \quad 0 = (6.0\text{ m/s})^2 + 2(-2.5\text{ m/s}^2)\Delta x_2$$

which leads to $\Delta x_2 = 7.2$ m. Therefore, the total distance traveled by the shuffleboard disk is (1.8 + 7.2) m = 9.0 m.

82. The time required is found from Eq. 2-11 (or, suitably interpreted, Eq. 2-7). First, we convert the velocity change to SI units:

$$\Delta v = (100\text{ km/h})\left(\frac{1000\text{ m/km}}{3600\text{ s/h}}\right) = 27.8\text{ m/s}.$$

Thus, $\Delta t = \Delta v/a = 27.8/50 = 0.556$ s.

83. From Table 2-1, $v^2 - v_0^2 = 2a\Delta x$ is used to solve for a. Its minimum value is

$$a_{\min} = \frac{v_2 - v_0^2}{2\Delta x_{\max}} = \frac{(360 \text{ km/h})^2}{2(1.80 \text{ km})} = 36000 \text{ km/h}^2$$

which converts to 2.78 m/s^2.

84. (a) For the automobile Δv = 55 – 25 = 30 km/h, which we convert to SI units:

$$a = \frac{\Delta v}{\Delta t} = \frac{(30 \text{ km/h})\left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}}\right)}{(0.50 \text{ min})(60 \text{ s/min})} = 0.28 \text{ m/s}^2 \ .$$

(b) The change of velocity for the bicycle, for the same time, is identical to that of the car, so its acceleration is also 0.28 m/s^2.

85. We denote t_r as the reaction time and t_b as the braking time. The motion during t_r is of the constant-velocity (call it v_0) type. Then the position of the car is given by

$$x = v_0 t_r + v_0 t_b + \frac{1}{2} a t_b^2$$

where v_0 is the initial velocity and a is the acceleration (which we expect to be negative-valued since we are taking the velocity in the positive direction and we know the car is decelerating). *After* the brakes are applied the velocity of the car is given by $v = v_0 + at_b$. Using this equation, with $v = 0$, we eliminate t_b from the first equation and obtain

$$x = v_0 t_r - \frac{v_0^2}{a} + \frac{1}{2}\frac{v_0^2}{a} = v_0 t_r - \frac{1}{2}\frac{v_0^2}{a} \ .$$

We write this equation for each of the initial velocities:

$$x_1 = v_{01} t_r - \frac{1}{2}\frac{v_{01}^2}{a}$$

and

$$x_2 = v_{02} t_r - \frac{1}{2}\frac{v_{02}^2}{a} .$$

Solving these equations simultaneously for t_r and a we get

$$t_r = \frac{v_{02}^2 x_1 - v_{01}^2 x_2}{v_{01} v_{02}\left(v_{02} - v_{01}\right)}$$

and

$$a = -\frac{1}{2}\frac{v_{02} v_{01}^2 - v_{01} v_{02}^2}{v_{02} x_1 - v_{01} x_2} .$$

(a) Substituting x_1 = 56.7 m, v_{01} = 80.5 km/h = 22.4 m/s, x_2 = 24.4 m and v_{02} = 48.3 km/h = 13.4 m/s, we find

$$t_r = \frac{v_{02}^2 x_1 - v_{01}^2 x_2}{v_{01}v_{02}(v_{02}-v_{01})} = \frac{(13.4\ \text{m/s})^2(56.7\ \text{m}) - (22.4\ \text{m/s})^2(24.4\ \text{m})}{(22.4\ \text{m/s})(13.4\ \text{m/s})(13.4\ \text{m/s} - 22.4\ \text{m/s})} = 0.74\ \text{s}.$$

(b) In a similar manner, substituting x_1 = 56.7 m, v_{01} = 80.5 km/h = 22.4 m/s, x_2 = 24.4 m and v_{02} = 48.3 km/h = 13.4 m/s gives

$$a = -\frac{1}{2}\frac{v_{02}v_{01}^2 - v_{01}v_{02}^2}{v_{02}x_1 - v_{01}x_2} = -\frac{1}{2}\frac{(13.4\ \text{m/s})(22.4\ \text{m/s})^2 - (22.4\ \text{m/s})(13.4\ \text{m/s})^2}{(13.4\ \text{m/s})(56.7\ \text{m}) - (22.4\ \text{m/s})(24.4\ \text{m})} = -6.2\ \text{m/s}^2.$$

The *magnitude* of the deceleration is therefore 6.2 m/s^2. Although rounded off values are displayed in the above substitutions, what we have input into our calculators are the “exact” values (such as $v_{02} = \frac{161}{12}$ m/s).

86. We take the moment of applying brakes to be t = 0. The deceleration is constant so that Table 2-1 can be used. Our primed variables (such as $v_0' = 72\ \text{km/h} = 20\ \text{m/s}$) refer to one train (moving in the $+x$ direction and located at the origin when t = 0) and unprimed variables refer to the other (moving in the $-x$ direction and located at x_0 = +950 m when t = 0). We note that the acceleration vector of the unprimed train points in the *positive* direction, even though the train is slowing down; its initial velocity is v_o = –144 km/h = –40 m/s. Since the primed train has the lower initial speed, it should stop sooner than the other train would (were it not for the collision). Using Eq 2-16, it should stop (meaning $v' = 0$) at

$$x' = \frac{(v')^2 - (v_0')^2}{2a'} = \frac{0 - (20\ \text{m/s})^2}{-2\ \text{m/s}^2} = 200\ \text{m}.$$

The speed of the other train, when it reaches that location, is

$$v = \sqrt{v_o^2 + 2a\Delta x} = \sqrt{(-40\ \text{m/s})^2 + 2(1.0\ \text{m/s}^2)(200\ \text{m} - 950\ \text{m})} = 10\ \text{m/s}$$

using Eq 2-16 again. Specifically, its velocity at that moment would be –10 m/s since it is still traveling in the $-x$ direction when it crashes. If the computation of v had failed (meaning that a negative number would have been inside the square root) then we would have looked at the possibility that there was no collision and examined how far apart they finally were. A concern that can be brought up is whether the primed train collides before it comes to rest; this can be studied by computing the time it stops (Eq. 2-11 yields t = 20 s) and seeing where the unprimed train is at that moment (Eq. 2-18 yields x = 350 m, still a good distance away from contact).

87. The y coordinate of Piton 1 obeys $y - y_{o1}$ = – **Erro!**$g\ t^2$ where y = 0 when t = 3.0 s. This allows us to solve for y_{o1}, and we find y_{o1} = 44.1 m. The graph for the coordinate of Piton 2 (which is thrown apparently at t = 1.0 s with velocity v_1) is

$y - y_{o2} = v_1(t–1.0)$ – **Erro!**$g\ (t–1.0)^2$

where $y_{o2} = y_{o1} + 10 = 54.1$ m and where (again) $y = 0$ when $t = 3.0$ s. Thus we obtain $|v_1| = 17$ m/s, approximately.

88. We adopt the convention frequently used in the text: that "up" is the positive y direction.

(a) At the highest point in the trajectory $v = 0$. Thus, with $t = 1.60$ s, the equation $v = v_0 - gt$ yields $v_0 = 15.7$ m/s.

(b) One equation that is not dependent on our result from part (a) is $y - y_0 = vt +$ **Erro!**gt^2; this readily gives $y_{max} - y_0 = 12.5$ m for the highest ("max") point measured relative to where it started (the top of the building).

(c) Now we use our result from part (a) and plug into $y - y_0 = v_0t +$ **Erro!**gt^2 with $t =$ 6.00 s and $y = 0$ (the ground level). Thus, we have

$$0 - y_0 = (15.68 \text{ m/s})(6.00 \text{ s}) - \textbf{Erro!}\ (9.8 \text{ m/s}^2)(6.00 \text{ s})^2 \quad .$$

Therefore, y_0 (the height of the building) is equal to 82.3 m.

89. Integrating (from $t = 2$ s to variable $t = 4$ s) the acceleration to get the velocity and using the values given in the problem, leads to

$$v = v_0 + \int_{t_0}^{t} a\,dt = v_0 + \int_{t_0}^{t}(5.0t)\,dt = v_0 + \frac{1}{2}(5.0)(t^2 - t_0^2) = 17 + \textbf{Erro!}(5.0)(4^2 - 2^2) = 47$$

m/s.

90. We take $+x$ in the direction of motion. We use subscripts 1 and 2 for the data. Thus, $v_1 = +30$ m/s, $v_2 = +50$ m/s and $x_2 - x_1 = +160$ m.

(a) Using these subscripts, Eq. 2-16 leads to

$$a = \frac{v_2^2 - v_1^2}{2(x_2 - x_1)} = \frac{(50 \text{ m/s})^2 - (30 \text{ m/s})^2}{2(160 \text{ m})} = 5.0 \text{ m/s}^2 \ .$$

(b) We find the time interval corresponding to the displacement $x_2 - x_1$ using Eq. 2-17:

$$t_2 - t_1 = \frac{2(x_2 - x_1)}{v_1 + v_2} = \frac{2(160 \text{ m})}{30 \text{ m/s} + 50 \text{ m/s}} = 4.0 \text{ s} \ .$$

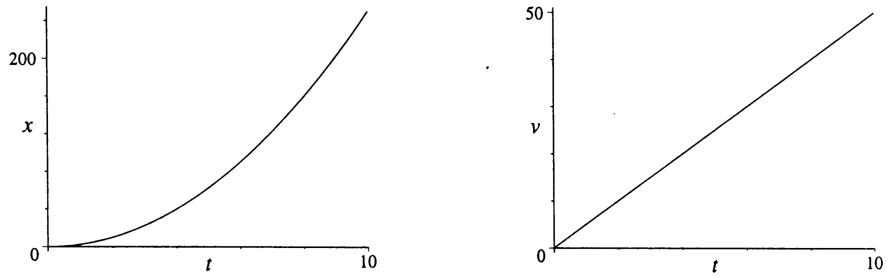
(c) Since the train is at rest ($v_0 = 0$) when the clock starts, we find the value of t_1 from Eq. 2-11:

$$v_1 = v_0 + at_1 \quad \Rightarrow \quad t_1 = \frac{30 \text{ m/s}}{5.0 \text{ m/s}^2} = 6.0 \text{ s} \ .$$

(d) The coordinate origin is taken to be the location at which the train was initially at rest (so $x_0 = 0$). Thus, we are asked to find the value of x_1. Although any of several equations could be used, we choose Eq. 2-17:

$$x_1 = \frac{1}{2}(v_0 + v_1)t_1 = \frac{1}{2}(30\ \text{m/s})(6.0\ \text{s}) = 90\ \text{m}\ .$$

(e) The graphs are shown below, with SI units assumed.



91. We take +x in the direction of motion, so

$$v = (60\ \text{km/h})\left(\frac{1000\ \text{m/km}}{3600\ \text{s/h}}\right) = +16.7\ \text{m/s}$$

and $a > 0$. The location where it starts from rest ($v_0 = 0$) is taken to be $x_0 = 0$.

(a) Eq. 2-7 gives $a_{avg} = (v - v_0)/t$ where $t = 5.4$ s and the velocities are given above. Thus, $a_{avg} = 3.1\ \text{m/s}^2$.

(b) The assumption that a = constant permits the use of Table 2-1. From that list, we choose Eq. 2-17:

$$x = \frac{1}{2}(v_0 + v)\ t = \frac{1}{2}(16.7\ \text{m/s})(5.4\ \text{s}) = 45\ \text{m}.$$

(c) We use Eq. 2-15, now with $x = 250$ m:

$$x = \frac{1}{2}at^2 \;\Rightarrow\; t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(250\ \text{m})}{3.1\ \text{m/s}^2}}$$

which yields $t = 13$ s.

92. We take the direction of motion as +x, take $x_0 = 0$ and use SI units, so v = 1600(1000/3600) = 444 m/s.

(a) Eq. 2-11 gives 444 = a(1.8) or $a = 247\ \text{m/s}^2$. We express this as a multiple of g by setting up a ratio:

$$a = \left(\frac{247\ \text{m/s}^2}{9.8\ \text{m/s}^2}\right) g = 25g.$$

(b) Eq. 2-17 readily yields

$$x = \frac{1}{2}\left(v_0 + v\right) t = \frac{1}{2}\left(444 \text{ m/s}\right)\left(1.8 \text{ s}\right) = 400 \text{ m}.$$

93. The object, once it is dropped (v_0 = 0) is in free-fall ($a = -g = -9.8$ m/s^2 if we take *down* as the $-y$ direction), and we use Eq. 2-15 repeatedly.

(a) The (positive) distance D from the lower dot to the mark corresponding to a certain reaction time t is given by $\Delta y = -D = -\frac{1}{2} g t^2$, or $D = gt^2/2$. Thus, for $t_1 = 50.0$ ms,

$$D_1 = \frac{\left(9.8 \text{ m/s}^2\right)\left(50.0 \times 10^{-3} \text{ s}\right)^2}{2} = 0.0123 \text{ m} = 1.23 \text{ cm}.$$

(b) For t_2 = 100 ms, $D_2 = \dfrac{\left(9.8 \text{m/s}^2\right)\left(100 \times 10^{-3} \text{ s}\right)^2}{2} = 0.049 \text{m} = 4D_1.$

(c) For t_3 = 150 ms, $D_3 = \dfrac{\left(9.8 \text{m/s}^2\right)\left(150 \times 10^{-3} \text{ s}\right)^2}{2} = 0.11 \text{m} = 9D_1.$

(d) For t_4 = 200 ms, $D_4 = \dfrac{\left(9.8 \text{m/s}^2\right)\left(200 \times 10^{-3} \text{ s}\right)^2}{2} = 0.196 \text{m} = 16D_1.$

(e) For t_4 = 250 ms, $D_5 = \dfrac{\left(9.8 \text{ m/s}^2\right)\left(250 \times 10^{-3} \text{ s}\right)^2}{2} = 0.306 \text{ m} = 25D_1.$

The velocity v at t = 6 (SI units and two significant figures understood) is $v_{\text{given}} + \int_{-2}^{6} a\,dt$. A quick way to implement this is to recall the area of a triangle (**Erro!** base × height). The result is v = 7 m/s + 32 m/s = 39 m/s.

95. Let D be the distance up the hill. Then

average speed = **Erro!** = **Erro!** ≈ 25 km/h .

96. Converting to SI units, we have v = 3400(1000/3600) = 944 m/s (presumed constant) and Δt = 0.10 s. Thus, $\Delta x = v\Delta t$ = 94 m.

97. The (ideal) driving time before the change was $t = \Delta x/v$, and after the change it is $t' = \Delta x/v'$. The time saved by the change is therefore

$$t - t' = \Delta x\left(\frac{1}{v} - \frac{1}{v'}\right) = \Delta x\left(\frac{1}{55} - \frac{1}{65}\right) = \Delta x\,(0.0028 \text{ h/mi})$$

which becomes, converting Δx = 700/1.61 = 435 mi (using a conversion found on the inside front cover of the textbook), $t - t'$ = (435)(0.0028) = 1.2 h. This is equivalent to 1 h and 13 min.

98. We obtain the velocity by integration of the acceleration:

$$v - v_0 = \int_0^t (6.1 - 1.2t')dt'.$$

Lengths are in meters and times are in seconds. The student is encouraged to look at the discussion in the textbook in §2-7 to better understand the manipulations here.

(a) The result of the above calculation is

$$v = v_0 + 6.1t - 0.6t^2 ,$$

where the problem states that v_0 = 2.7 m/s. The maximum of this function is found by knowing when its derivative (the acceleration) is zero (a = 0 when t = 6.1/1.2 = 5.1 s) and plugging that value of t into the velocity equation above. Thus, we find $v = 18$ m/s.

(b) We integrate again to find x as a function of t:

$$x - x_0 = \int_0^t v\,dt' = \int_0^t (v_0 + 6.1t' - 0.6t'^2)\,dt' = v_0 t + 3.05t^2 - 0.2t^3 .$$

With x_0 = 7.3 m, we obtain x = 83 m for t = 6. This is the correct answer, but one has the right to worry that it might not be; after all, the problem asks for the total distance traveled (and x - x_0 is just the *displacement*). If the cyclist backtracked, then his total distance would be greater than his displacement. Thus, we might ask, "did he backtrack?" To do so would require that his velocity be (momentarily) zero at some point (as he reversed his direction of motion). We could solve the above quadratic equation for velocity, for a positive value of t where v = 0; if we did, we would find that at t = 10.6 s, a reversal does indeed happen. However, in the time interval concerned with in our problem ($0 \le t \le 6$ s), there is no reversal and the displacement is the same as the total distance traveled.

99. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. When something is thrown straight up and is caught at the level it was thrown from (with a trajectory similar to that shown in Fig. 2-31), the time of flight t is half of its time of ascent t_a, which is given by Eq. 2-18 with $\Delta y = H$ and v = 0 (indicating the maximum point).

$$H = vt_a + \frac{1}{2}gt_a^2 \quad \Rightarrow \quad t_a = \sqrt{\frac{2H}{g}}$$

Writing these in terms of the total time in the air $t = 2t_a$ we have

$$H = \frac{1}{8}gt^2 \quad \Rightarrow \quad t = 2\sqrt{\frac{2H}{g}}.$$

We consider two throws, one to height H_1 for total time t_1 and another to height H_2 for total time t_2, and we set up a ratio:

$$\frac{H_2}{H_1} = \frac{\frac{1}{8}gt_2^2}{\frac{1}{8}gt_1^2} = \left(\frac{t_2}{t_1}\right)^2$$

from which we conclude that if $t_2 = 2t_1$ (as is required by the problem) then $H_2 = 2^2 H_1 = 4H_1$.

100. The acceleration is constant and we may use the equations in Table 2-1.

(a) Taking the first point as coordinate origin and time to be zero when the car is there, we apply Eq. 2-17:

$$x = \frac{1}{2}\left(v + v_0\right)t = \frac{1}{2}\left(15.0\text{ m/s} + v_0\right)\left(6.00\text{ s}\right).$$

With $x = 60.0$ m (which takes the direction of motion as the $+x$ direction) we solve for the initial velocity: $v_0 = 5.00$ m/s.

(b) Substituting $v = 15.0$ m/s, $v_0 = 5.00$ m/s and $t = 6.00$ s into $a = (v - v_0)/t$ (Eq. 2-11), we find $a = 1.67$ m/s^2.

(c) Substituting $v = 0$ in $v^2 = v_0^2 + 2ax$ and solving for x, we obtain

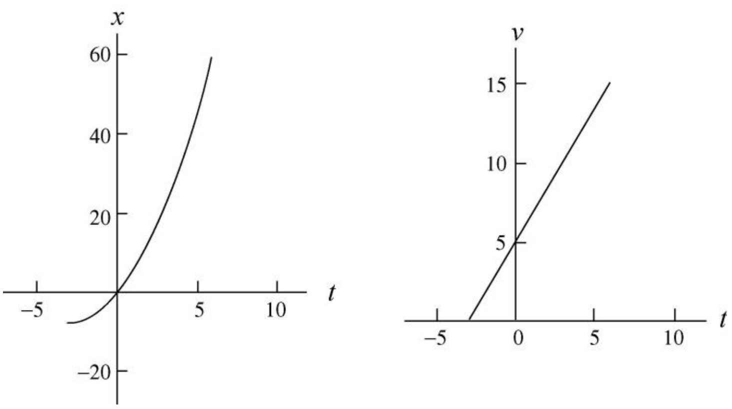
$$x = -\frac{v_0^2}{2a} = -\frac{(5.00\text{ m/s})^2}{2\left(1.67\text{ m/s}^2\right)} = -7.50\text{m},$$

or $|x| = 7.50$ m.

(d) The graphs require computing the time when $v = 0$, in which case, we use $v = v_0 + at' = 0$. Thus,

$$t' = \frac{-v_0}{a} = \frac{-5.00\text{ m/s}}{1.67\text{ m/s}^2} = -3.0\text{s}$$

indicates the moment the car was at rest. SI units are assumed.



101. Taking the +*y* direction *downward* and y_0 = 0, we have $y = v_0 t + \frac{1}{2} g t^2$ which (with v_0 = 0) yields $t = \sqrt{2y/g}$.

(a) For this part of the motion, *y* = 50 m so that

$$t = \sqrt{\frac{2(50 \text{ m})}{9.8 \text{ m/s}^2}} = 3.2 \text{ s} .$$

(b) For this next part of the motion, we note that the total displacement is *y* = 100 m. Therefore, the total time is

$$t = \sqrt{\frac{2(100 \text{ m})}{9.8 \text{ m/s}^2}} = 4.5 \text{ s} .$$

The different between this and the answer to part (a) is the time required to fall through that second 50 m distance: 4.5 – 3.2 = 1.3 s.

102. Direction of +*x* is implicit in the problem statement. The initial position (when the clock starts) is x_0 = 0 (where v_0 = 0), the end of the speeding-up motion occurs at x_1 = 1100/2 = 550 m, and the subway comes to a halt (v_2 = 0) at x_2 = 1100 m.

(a) Using Eq. 2-15, the subway reaches x_1 at

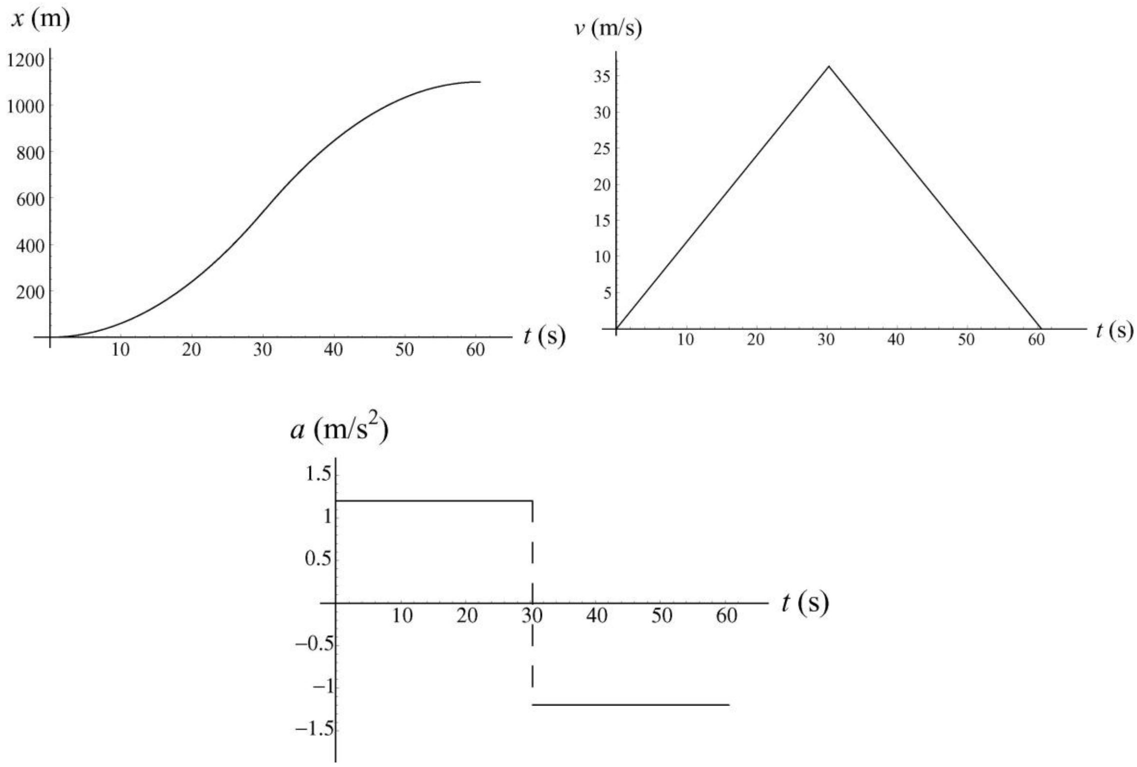
$$t_1 = \sqrt{\frac{2x_1}{a_1}} = \sqrt{\frac{2(550 \text{ m})}{1.2 \text{ m/s}^2}} = 30.3 \text{ s} .$$

The time interval $t_2 - t_1$ turns out to be the same value (most easily seen using Eq. 2-18 so the total time is t_2 = 2(30.3) = 60.6 s.

(b) Its maximum speed occurs at t_1 and equals

$$v_1 = v_0 + a_1 t_1 = 36.3 \text{ m/s} .$$

(c) The graphs are shown below:



103. This problem consists of two parts: part 1 with constant acceleration (so that the equations in Table 2-1 apply), $v_0 = 0$, $v = 11.0$ m/s, $x = 12.0$ m, and $x_0 = 0$ (adopting the starting line as the coordinate origin); and, part 2 with constant velocity (so that $x - x_0 = vt$ applies) with $v = 11.0$ m/s, $x_0 = 12.0$, and $x = 100.0$ m.

(a) We obtain the time for part 1 from Eq. 2-17

$$x - x_0 = \frac{1}{2}\left(v_0 + v\right)t_1 \;\Rightarrow\; 12.0 - 0 = \frac{1}{2}\left(0 + 11.0\right)t_1$$

so that $t_1 = 2.2$ s, and we find the time for part 2 simply from $88.0 = (11.0)t_2 \rightarrow t_2 = 8.0$ s. Therefore, the total time is $t_1 + t_2 = 10.2$ s.

(b) Here, the total time is required to be 10.0 s, and we are to locate the point x_p where the runner switches from accelerating to proceeding at constant speed. The equations for parts 1 and 2, used above, therefore become

$$x_p - 0 = \frac{1}{2}\left(0 + 11.0 \text{ m/s}\right)t_1$$
$$100.0 \text{ m} - x_p = \left(11.0 \text{ m/s}\right)\left(10.0 \text{ s} - t_1\right)$$

where in the latter equation, we use the fact that $t_2 = 10.0 - t_1$. Solving the equations for the two unknowns, we find that $t_1 = 1.8$ s and $x_p = 10.0$ m.

104. (a) Using the fact that the area of a triangle is $\frac{1}{2}$ (base) (height) (and the fact that the integral corresponds to the area under the curve) we find, from $t = 0$ through $t = 5$ s, the integral of v with respect to t is 15 m. Since we are told that $x_0 = 0$ then we

conclude that $x = 15$ m when $t = 5.0$ s.

(b) We see directly from the graph that $v = 2.0$ m/s when $t = 5.0$ s.

(c) Since $a = dv/dt$ = slope of the graph, we find that the acceleration during the interval $4 < t < 6$ is uniformly equal to -2.0 m/s^2.

(d) Thinking of $x(t)$ in terms of accumulated area (on the graph), we note that $x(1) = 1$ m; using this and the value found in part (a), Eq. 2-2 produces

$$v_{\text{avg}} = \frac{x(5) - x(1)}{5-1} = \frac{15 \text{ m} - 1 \text{ m}}{4 \text{ s}} = 3.5 \text{ m/s}.$$

(e) From Eq. 2-7 and the values $v(t)$ we read directly from the graph, we find

$$a_{\text{avg}} = \frac{v(5) - v(1)}{5-1} = \frac{2 \text{ m/s} - 2 \text{ m/s}}{4 \text{ s}} = 0.$$

105. We neglect air resistance, which justifies setting $a = -g = -9.8 \text{ m/s}^2$ (taking *down* as the $-y$ direction) for the duration of the stone's motion. We are allowed to use Table 2-1 (with Δx replaced by y) because the ball has constant acceleration motion (and we choose $y_o = 0$).

(a) We apply Eq. 2-16 to both measurements, with SI units understood.

$$v_B^2 = v_0^2 - 2gy_B \quad \Rightarrow \quad \left(\frac{1}{2}v\right)^2 + 2g\left(y_A + 3\right) = v_0^2$$
$$v_A^2 = v_0^2 - 2gy_A \quad \Rightarrow \quad v^2 + 2gy_A = v_0^2$$

We equate the two expressions that each equal v_0^2 and obtain

$$\frac{1}{4}v^2 + 2gy_A + 2g(3) = v^2 + 2gy_A \quad \Rightarrow \quad 2g(3) = \frac{3}{4}v^2$$

which yields $v = \sqrt{2g(4)} = 8.85 \text{ m/s}.$

(b) An object moving upward at A with speed $v = 8.85$ m/s will reach a maximum height $y - y_A = v^2/2g = 4.00$ m above point A (this is again a consequence of Eq. 2-16, now with the "final" velocity set to zero to indicate the highest point). Thus, the top of its motion is 1.00 m above point B.

106. We neglect air resistance, which justifies setting $a = -g = -9.8 \text{ m/s}^2$ (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y-axis. The total time of fall can be computed from Eq. 2-15 (using the quadratic formula).

$$\Delta y = v_0 t - \frac{1}{2} g t^2 \quad \Rightarrow \quad t = \frac{v_0 + \sqrt{v_0^2 - 2g\Delta y}}{g}$$

with the positive root chosen. With $y = 0$, $v_0 = 0$ and $y_0 = h = 60$ m, we obtain

$$t = \frac{\sqrt{2gh}}{g} = \sqrt{\frac{2h}{g}} = 3.5 \text{ s} .$$

Thus, "1.2 s earlier" means we are examining where the rock is at $t = 2.3$ s:

$$y - h = v_0(2.3\,\text{s}) - \frac{1}{2} g (2.3\,\text{s})^2 \quad \Rightarrow \quad y = 34 \text{ m}$$

where we again use the fact that $h = 60$ m and $v_0 = 0$.

107. (a) The wording of the problem makes it clear that the equations of Table 2-1 apply, the challenge being that v_0, v, and a are not explicitly given. We can, however, apply $x - x_0 = v_0 t +$ **Erro!**at^2 to a variety of points on the graph and solve for the unknowns from the simultaneous equations. For instance,

$16 \text{ m} - 0 = v_0(2.0 \text{ s}) +$ **Erro!** $a(2.0 \text{ s})^2$
$27 \text{ m} - 0 = v_0(3.0 \text{ s}) +$ **Erro!** $a(3.0 \text{ s})^2$

lead to the values $v_0 = 6.0$ m/s and $a = 2.0 \text{ m/s}^2$.

(b) From Table 2-1,
$x - x_0 = vt -$ **Erro!**at^2 $\Rightarrow$ $27 \text{ m} - 0 = v(3.0 \text{ s}) -$ **Erro!** $(2.0 \text{ m/s}^2)(3.0 \text{ s})^2$

which leads to $v = 12$ m/s.

(c) Assuming the wind continues during $3.0 \le t \le 6.0$, we apply $x - x_0 = v_0 t +$ **Erro!** at^2 to this interval (where $v_0 = 12.0$ m/s from part (b)) to obtain

$\Delta x = (12.0 \text{ m/s})(3.0 \text{ s}) +$ **Erro!** $(2.0 \text{ m/s}^2)(3.0 \text{ s})^2 = 45 \text{ m}$.

108. With $+y$ upward, we have $y_0 = 36.6$ m and $y = 12.2$ m. Therefore, using Eq. 2-18 (the last equation in Table 2-1), we find

$$y - y_0 = vt + \frac{1}{2} g t^2 \quad \Rightarrow \quad v = -22.0 \text{ m/s}$$

at $t = 2.00$ s. The term *speed* refers to the magnitude of the velocity vector, so the answer is $|v| = 22.0$ m/s.

109. The bullet starts at rest ($v_0 = 0$) and after traveling the length of the barrel ($\Delta x = 1.2 \text{ m}$) emerges with the given velocity ($v = 640$ m/s), where the direction of motion is the positive direction. Turning to the constant acceleration equations in Table 2-1, we use

$$\Delta x = \frac{1}{2}(v_0 + v)\ t\ .$$

Thus, we find t = 0.00375 s (or about 3.8 ms).

110. During free fall, we ignore the air resistance and set $a = -g = -9.8$ m/s^2 where we are choosing *down* to be the $-y$ direction. The initial velocity is zero so that Eq. 2-15 becomes $\Delta y = -\frac{1}{2}gt^2$ where Δy represents the *negative* of the distance d she has fallen. Thus, we can write the equation as $d = \frac{1}{2}gt^2$ for simplicity.

(a) The time t_1 during which the parachutist is in free fall is (using Eq. 2-15) given by

$$d_1 = 50\ \text{m} = \frac{1}{2}gt_1^2 = \frac{1}{2}\left(9.80\ \text{m/s}^2\right)t_1^2$$

which yields t_1 = 3.2 s. The *speed* of the parachutist just before he opens the parachute is given by the positive root $v_1^2 = 2gd_1$, or

$$v_1 = \sqrt{2gh_1} = \sqrt{(2)\left(9.80\ \text{m/s}^2\right)(50\ \text{m})} = 31\ \text{m/s}.$$

If the final speed is v_2, then the time interval t_2 between the opening of the parachute and the arrival of the parachutist at the ground level is

$$t_2 = \frac{v_1 - v_2}{a} = \frac{31\ \text{m/s} - 3.0\ \text{m/s}}{2\ \text{m/s}^2} = 14\ \text{s}.$$

This is a result of Eq. 2-11 where *speeds* are used instead of the (negative-valued) velocities (so that final-velocity minus initial-velocity turns out to equal initial-speed minus final-speed); we also note that the acceleration vector for this part of the motion is positive since it points upward (opposite to the direction of motion — which makes it a deceleration). The total time of flight is therefore $t_1 + t_2$ = 17 s.

(b) The distance through which the parachutist falls after the parachute is opened is given by

$$d = \frac{v_1^2 - v_2^2}{2a} = \frac{(31\ \text{m/s})^2 - (3.0\ \text{m/s})^2}{(2)\left(2.0\ \text{m/s}^2\right)} \approx 240\ \text{m}.$$

In the computation, we have used Eq. 2-16 with both sides multiplied by –1 (which changes the negative-valued Δy into the positive d on the left-hand side, and switches the order of v_1 and v_2 on the right-hand side). Thus the fall begins at a height of h = 50 + $d \approx$ 290 m.

111. There is no air resistance, which makes it quite accurate to set $a = -g = -9.8$ m/s^2 (where downward is the $-y$ direction) for the duration of the fall. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion; in fact, when the acceleration changes (during the process of catching the ball) we will

again assume constant acceleration conditions; in this case, we have $a_2 = +25g = 245$ m/s^2.

(a) The time of fall is given by Eq. 2-15 with $v_0 = 0$ and $y = 0$. Thus,

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(145\text{ m})}{9.8\text{ m/s}^2}} = 5.44\text{ s.}$$

(b) The final velocity for its free-fall (which becomes the initial velocity during the catching process) is found from Eq. 2-16 (other equations can be used but they would use the result from part (a)).

$$v = -\sqrt{v_0^2 - 2g(y - y_0)} = -\sqrt{2gy_0} = -53.3\text{ m/s}$$

where the negative root is chosen since this is a downward velocity. Thus, the speed is $|v| = 53.3$ m/s.

(c) For the catching process, the answer to part (b) plays the role of an *initial* velocity ($v_0 = -53.3$ m/s) and the final velocity must become zero. Using Eq. 2-16, we find

$$\Delta y_2 = \frac{v^2 - v_0^2}{2a_2} = \frac{-(-53.3\text{ m/s})^2}{2(245\text{ m/s}^2)} = -5.80\text{ m},$$

or $|\Delta y_2| = 5.80$ m. The negative value of Δy_2 signifies that the distance traveled while arresting its motion is downward.

112. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking down as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to $y = 0$.

(a) With $y_0 = h$ and v_0 replaced with $-v_0$, Eq. 2-16 leads to

$$v = \sqrt{(-v_0)^2 - 2g(y - y_0)} = \sqrt{v_0^2 + 2gh}\,.$$

The positive root is taken because the problem asks for the speed (the *magnitude* of the velocity).

(b) We use the quadratic formula to solve Eq. 2-15 for t, with v_0 replaced with $-v_0$,

$$\Delta y = -v_0 t - \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \frac{-v_0 + \sqrt{(-v_0)^2 - 2g\Delta y}}{g}$$

where the positive root is chosen to yield $t > 0$. With $y = 0$ and $y_0 = h$, this becomes

$$t = \frac{\sqrt{v_0^2 + 2gh} - v_0}{g}.$$

(c) If it were thrown upward with that speed from height h then (in the absence of air friction) it would return to height h with that same downward speed and would therefore yield the same final speed (before hitting the ground) as in part (a). An important perspective related to this is treated later in the book (in the context of energy conservation).

(d) Having to travel up before it starts its descent certainly requires more time than in part (b). The calculation is quite similar, however, except for now having $+v_0$ in the equation where we had put in $-v_0$ in part (b). The details follow:

$$\Delta y = v_0 t - \frac{1}{2} g t^2 \quad \Rightarrow \quad t = \frac{v_0 + \sqrt{v_0^2 - 2g\Delta y}}{g}$$

with the positive root again chosen to yield $t > 0$. With $y = 0$ and $y_0 = h$, we obtain

$$t = \frac{\sqrt{v_0^2 + 2gh} + v_0}{g}.$$

113. During T_r the velocity v_0 is constant (in the direction we choose as $+x$) and obeys $v_0 = D_r/T_r$ where we note that in SI units the velocity is $v_0 = 200(1000/3600) = 55.6$ m/s. During T_b the acceleration is opposite to the direction of v_0 (hence, for us, $a < 0$) until the car is stopped ($v = 0$).

(a) Using Eq. 2-16 (with $\Delta x_b = 170$ m) we find

$$v^2 = v_0^2 + 2a\Delta x_b \quad \Rightarrow \quad a = -\frac{v_0^2}{2\Delta x_b}$$

which yields $|a| = 9.08$ m/s^2.

(b) We express this as a multiple of g by setting up a ratio:

$$a = \left(\frac{9.08\ \text{m/s}^2}{9.8\ \text{m/s}^2}\right) g = 0.926g\ .$$

(c) We use Eq. 2-17 to obtain the braking time:

$$\Delta x_b = \frac{1}{2}(v_0 + v)T_b \quad \Rightarrow \quad T_b = \frac{2(170\ \text{m})}{55.6\ \text{m/s}} = 6.12\ \text{s}\ .$$

(d) We express our result for T_b as a multiple of the reaction time T_r by setting up a ratio:

$$T_b = \left(\frac{6.12 \text{ s}}{400 \times 10^{-3} \text{ s}} \right) T_r = 15.3 T_r \ .$$

(e) Since $T_b > T_r$, most of the full time required to stop is spent in braking.

(f) We are only asked what the *increase* in distance D is, due to ΔT_r = 0.100 s, so we simply have

$$\Delta D = v_0 \Delta T_r = (55.6 \text{ m/s})(0.100 \text{ s}) = 5.56 \text{ m} \ .$$

114. We assume constant velocity motion and use Eq. 2-2 (with $v_{avg} = v > 0$). Therefore,

$$\Delta x = v\Delta t = \left(303 \frac{\text{km}}{\text{h}}\right)\left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}}\right)\left(100 \times 10^{-3} \text{ s}\right) = 8.4 \text{ m}.$$

Chapter 3

1. A vector $\vec{a}$ can be represented in the *magnitude-angle* notation (a, θ), where

$$a = \sqrt{a_x^2 + a_y^2}$$

is the magnitude and

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$

is the angle $\vec{a}$ makes with the positive x axis.

(a) Given $A_x = -25.0$ m and $A_y = 40.0$ m, $A = \sqrt{(-25.0\text{ m})^2 + (40.0\text{ m})^2} = 47.2\text{ m}$

(b) Recalling that tan θ = tan (θ + 180°), $\tan^{-1}$ [(40.0 m)/ (– 25.0 m)] = – 58° or 122°. Noting that the vector is in the third quadrant (by the signs of its x and y components) we see that 122° is the correct answer. The graphical calculator "shortcuts" mentioned above are designed to correctly choose the right possibility.

2. The angle described by a full circle is 360° = 2π rad, which is the basis of our conversion factor.

(a)

$$20.0° = (20.0°)\frac{2\pi \text{ rad}}{360°} = 0.349 \text{ rad}.$$

(b)

$$50.0° = (50.0°)\frac{2\pi \text{ rad}}{360°} = 0.873 \text{ rad}.$$

(c)

$$100° = (100°)\frac{2\pi \text{ rad}}{360°} = 1.75 \text{ rad}.$$

(d)

$$0.330 \text{ rad} = (0.330 \text{ rad})\frac{360°}{2\pi \text{ rad}} = 18.9°.$$

(e)

$$2.10 \text{ rad} = (2.10 \text{ rad})\frac{360°}{2\pi \text{ rad}} = 120°.$$

(f)

$$7.70 \text{ rad} = (7.70 \text{ rad})\frac{360°}{2\pi \text{ rad}} = 441°.$$

3. The x and the y components of a vector $\vec{a}$ lying on the xy plane are given by

$$a_x = a\cos\theta, \qquad a_y = a\sin\theta$$

where $a = |\vec{a}\,|$ is the magnitude and θ is the angle between $\vec{a}$ and the positive x axis.

(a) The x component of $\vec{a}$ is given by $a_x = 7.3 \cos 250° = -2.5$ m.

(b) and the y component is given by $a_y = 7.3 \sin 250° = -6.9$ m.

In considering the variety of ways to compute these, we note that the vector is 70° below the $-x$ axis, so the components could also have been found from $a_x = -7.3 \cos 70°$ and $a_y = -7.3 \sin 70°$. In a similar vein, we note that the vector is 20° to the left from the $-y$ axis, so one could use $a_x = -7.3 \sin 20°$ and $a_y = -7.3 \cos 20°$ to achieve the same results.

4. (a) The height is $h = d \sin\theta$, where $d = 12.5$ m and $\theta = 20.0°$. Therefore, $h = 4.28$ m.

(b) The horizontal distance is $d \cos\theta = 11.7$ m.

5. The vector sum of the displacements $\vec{d}_{\text{storm}}$ and $\vec{d}_{\text{new}}$ must give the same result as its originally intended displacement $\vec{d}_{\text{o}} = (120 \text{ km})\hat{\text{j}}$ where east is $\hat{\text{i}}$, north is $\hat{\text{j}}$. Thus, we write

$$\vec{d}_{\text{storm}} = (100 \text{ km})\,\hat{\text{i}}, \;\; \vec{d}_{\text{new}} = A\,\hat{\text{i}} + B\,\hat{\text{j}}.$$

(a) The equation $\vec{d}_{\text{storm}} + \vec{d}_{\text{new}} = \vec{d}_{\text{o}}$ readily yields A = –100 km and B = 120 km. The magnitude of $\vec{d}_{\text{new}}$ is therefore equal to $|\,\vec{d}_{\text{new}}\,| = \sqrt{A^2 + B^2} = 156 \text{ km}$.

(b) The direction is $\tan^{-1}$ (B/A) = –50.2° or 180° + (–50.2°) = 129.8°. We choose the latter value since it indicates a vector pointing in the second quadrant, which is what we expect here. The answer can be phrased several equivalent ways: 129.8° counterclockwise from east, or 39.8° west from north, or 50.2° north from west.

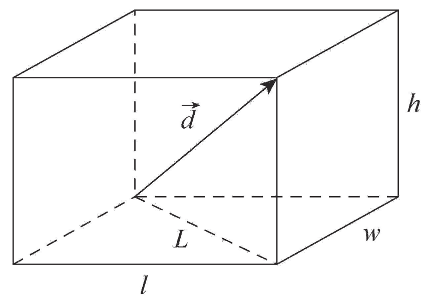
6. (a) With $r = 15$ m and $\theta = 30°$, the x component of $\vec{r}$ is given by

$$r_x = r\cos\theta = (15\text{ m})\cos 30° = 13\text{ m}.$$

(b) Similarly, the y component is given by $r_y = r\sin\theta = (15\text{ m})\sin 30° = 7.5$ m.

7. The length unit meter is understood throughout the calculation.

(a) We compute the distance from one corner to the diametrically opposite corner: $\sqrt{(3.00\text{ m})^2 + (3.70\text{ m})^2 + (4.30\text{ m})^2}$.



(b) The displacement vector is along the straight line from the beginning to the end point of the trip. Since a straight line is the shortest distance between two points, the length of the path cannot be less than the magnitude of the displacement.

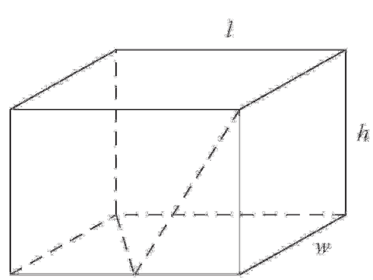
(c) It can be greater, however. The fly might, for example, crawl along the edges of the room. Its displacement would be the same but the path length would be $\ell + w + h = 11.0$ m.

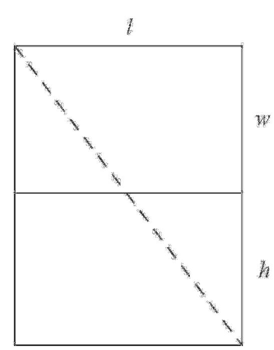
(d) The path length is the same as the magnitude of the displacement if the fly flies along the displacement vector.

(e) We take the x axis to be out of the page, the y axis to be to the right, and the z axis to be upward. Then the x component of the displacement is $w = 3.70$ m, the y component of the displacement is 4.30 m, and the z component is 3.00 m. Thus,

$$\vec{d} = (3.70\text{ m})\hat{\text{i}} + (4.30\text{ m})\hat{\text{j}} + (3.00\text{ m})\hat{\text{k}} .$$

An equally correct answer is gotten by interchanging the length, width, and height.

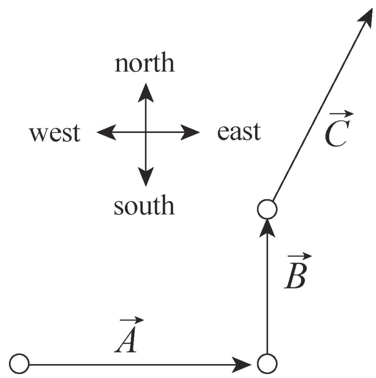




(f) Suppose the path of the fly is as shown by the dotted lines on the upper diagram. Pretend there is a hinge where the front wall of the room joins the floor and lay the wall down as shown on the lower diagram. The shortest walking distance between the lower left back of the room and the upper right front corner is the dotted straight line shown on the diagram. Its length is

$$L_{\min} = \sqrt{(w+h)^2 + \ell^2} = \sqrt{(3.70\text{ m} + 3.00\text{ m})^2 + (4.30\text{ m})^2} = 7.96\text{ m}.$$

8. We label the displacement vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ (and denote the result of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\mathrm{i}}$ direction (+x direction) and *north* as the $\hat{\mathrm{j}}$ direction (+y direction). We note that the angle between $\vec{C}$ and the x axis is 60°. Thus,



$$\vec{A} = (50\text{ km})\,\hat{\mathrm{i}}$$
$$\vec{B} = (30\text{ km})\,\hat{\mathrm{j}}$$
$$\vec{C} = (25\text{ km})\cos(60°)\,\hat{\mathrm{i}} + (25\text{ km})\sin(60°)\,\hat{\mathrm{j}}$$

(a) The total displacement of the car from its initial position is represented by

$$\vec{r} = \vec{A} + \vec{B} + \vec{C} = (62.5\text{ km})\,\hat{\mathrm{i}} + (51.7\text{ km})\,\hat{\mathrm{j}}$$

which means that its magnitude is

$$|\vec{r}| = \sqrt{(62.5\,\text{km})^2 + (51.7\,\text{km})^2} = 81\text{ km}.$$

(b) The angle (counterclockwise from +x axis) is $\tan^{-1}$ (51.7 km/62.5 km) = 40°, which is to say that it points 40° *north of east*. Although the resultant $\vec{r}$ is shown in our sketch, it would be a direct line from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

9. We write $\vec{r} = \vec{a} + \vec{b}$. When not explicitly displayed, the units here are assumed to be meters.

(a) The x and the y components of $\vec{r}$ are $r_x = a_x + b_x$ = (4.0 m) – (13 m) = –9.0 m and $r_y = a_y + b_y$ = (3.0 m) + (7.0 m) = 10 m, respectively. Thus $\vec{r} = (-9.0\,\text{m})\,\hat{\text{i}} + (10\,\text{m})\,\hat{\text{j}}$.

(b) The magnitude of $\vec{r}$ is

$$r = |\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{(-9.0\text{ m})^2 + (10\text{ m})^2} = 13\text{ m}.$$

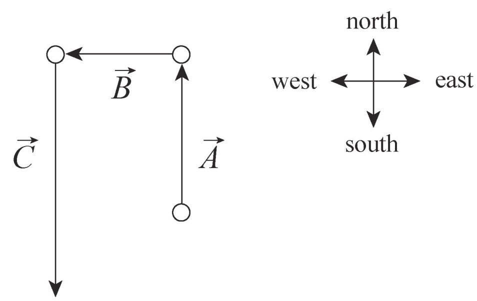
(c) The angle between the resultant and the $+x$ axis is given by

$$\theta = \tan^{-1}(r_y/r_x) = \tan^{-1}[(10\text{ m})/(-9.0\text{ m})] = -48° \text{ or } 132°.$$

Since the x component of the resultant is negative and the y component is positive, characteristic of the second quadrant, we find the angle is 132° (measured counterclockwise from $+x$ axis).

10. We label the displacement vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ (and denote the result of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\mathrm{i}}$ direction (+x direction) and *north* as the $\hat{\mathrm{j}}$ direction (+y direction) All distances are understood to be in kilometers.

(a) The vector diagram representing the motion is shown below:



$$\vec{A} = (3.1 \text{ km})\,\hat{\mathrm{j}}$$
$$\vec{B} = (-2.4 \text{ km})\,\hat{\mathrm{i}}$$
$$\vec{C} = (-5.2 \text{ km})\,\hat{\mathrm{j}}$$

(b) The final point is represented by

$$\vec{r} = \vec{A} + \vec{B} + \vec{C} = (-2.4 \text{ km})\hat{\mathrm{i}} + (-2.1 \text{ km})\hat{\mathrm{j}}$$

whose magnitude is

$$|\vec{r}| = \sqrt{(-2.4 \text{ km})^2 + (-2.1 \text{ km})^2} \approx 3.2 \text{ km} .$$

(c) There are two possibilities for the angle:

$$\theta = \tan^{-1}\left(\frac{-2.1 \text{ km}}{-2.4 \text{ km}}\right) = 41°, \text{or } 221° .$$

We choose the latter possibility since $\vec{r}$ is in the third quadrant. It should be noted that many graphical calculators have polar ↔ rectangular "shortcuts" that automatically produce the correct answer for angle (measured counterclockwise from the +x axis). We may phrase the angle, then, as 221° counterclockwise from East (a phrasing that sounds peculiar, at best) or as 41° south from west or 49° west from south. The resultant $\vec{r}$ is not shown in our sketch; it would be an arrow directed from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

11. We find the components and then add them (as scalars, not vectors). With d = 3.40 km and θ= 35.0° we find d cos θ+ d sin θ= 4.74 km.

12. (a) $\vec{a}+\vec{b}=(3.0\hat{\mathrm{i}}+4.0\hat{\mathrm{j}})\text{ m}+(5.0\hat{\mathrm{i}}-2.0\hat{\mathrm{j}})\text{ m}=(8.0\text{ m})\hat{\mathrm{i}}+(2.0\text{ m})\hat{\mathrm{j}}$.

(b) The magnitude of $\vec{a}+\vec{b}$ is

$$|\vec{a}+\vec{b}|=\sqrt{(8.0\text{ m})^2+(2.0\text{ m})^2}=8.2\text{ m}.$$

(c) The angle between this vector and the $+x$ axis is $\tan^{-1}[(2.0\text{ m})/(8.0\text{ m})] = 14°$.

(d) $\vec{b}-\vec{a}=(5.0\hat{\mathrm{i}}-2.0\hat{\mathrm{j}})\text{ m}-(3.0\hat{\mathrm{i}}+4.0\hat{\mathrm{j}})\text{ m}=(2.0\text{ m})\hat{\mathrm{i}}-(6.0\text{ m})\hat{\mathrm{j}}$.

(e) The magnitude of the difference vector $\vec{b}-\vec{a}$ is

$$|\vec{b}-\vec{a}|=\sqrt{(2.0\text{ m})^2+(-6.0\text{ m})^2}=6.3\text{ m}.$$

(f) The angle between this vector and the $+x$ axis is $\tan^{-1}[(-6.0\text{ m})/(2.0\text{ m})] = -72°$. The vector is 72° *clockwise* from the axis defined by $\hat{\mathrm{i}}$.

13. All distances in this solution are understood to be in meters.

(a) $\vec{a}+\vec{b}=[4.0+(-1.0)]\,\hat{\mathrm{i}}+[(-3.0)+1.0]\,\hat{\mathrm{j}}+(1.0+4.0)\hat{\mathrm{k}}=(3.0\hat{\mathrm{i}}-2.0\hat{\mathrm{j}}+5.0\,\hat{\mathrm{k}})$ m.

(b) $\vec{a}-\vec{b}=[4.0-(-1.0)]\hat{\mathrm{i}}+[(-3.0)-1.0]\hat{\mathrm{j}}+(1.0-4.0)\hat{\mathrm{k}}=(5.0\,\hat{\mathrm{i}}-4.0\hat{\mathrm{j}}-3.0\,\hat{\mathrm{k}})$ m.

(c) The requirement $\vec{a}-\vec{b}+\vec{c}=0$ leads to $\vec{c}=\vec{b}-\vec{a}$, which we note is the opposite of what we found in part (b). Thus, $\vec{c}=(-5.0\hat{\mathrm{i}}\ +\ 4.0\,\hat{\mathrm{j}}\ +\ 3.0\hat{\mathrm{k}})$ m.

14. The *x, y* and *z* components of $\vec{r} = \vec{c} + \vec{d}$ are, respectively,

(a) $r_x = c_x + d_x = 7.4 \text{ m} + 4.4 \text{ m} = 12 \text{ m}$,

(b) $r_y = c_y + d_y = -3.8 \text{ m} - 2.0 \text{ m} = -5.8 \text{ m}$, and

(c) $r_z = c_z + d_z = -6.1 \text{ m} + 3.3 \text{ m} = -2.8 \text{ m}$.

15. Reading carefully, we see that the (x, y) specifications for each "dart" are to be interpreted as $(\Delta x, \Delta y)$ descriptions of the corresponding displacement vectors. We combine the different parts of this problem into a single exposition.

(a) Along the x axis, we have (with the centimeter unit understood)

$$30.0 + b_x - 20.0 - 80.0 = -140,$$

which gives $b_x = -70.0$ cm.

(b) Along the y axis we have

$$40.0 - 70.0 + c_y - 70.0 = -20.0$$

which yields $c_y = 80.0$ cm.

(c) The magnitude of the final location $(-140, -20.0)$ is $\sqrt{(-140)^2 + (-20.0)^2} = 141$ cm.

(d) Since the displacement is in the third quadrant, the angle of the overall displacement is given by $\pi + \tan^{-1}[(-20.0)/(-140)]$ or 188° counterclockwise from the $+x$ axis (172° clockwise from the $+x$ axis).

16. If we wish to use Eq. 3-5 in an unmodified fashion, we should note that the angle between $\vec{C}$ and the +x axis is 180° + 20.0° = 200°.

(a) The x and y components of $\vec{B}$ are given by

$$B_x = C_x - A_x = (15.0 \text{ m}) \cos 200° - (12.0 \text{ m}) \cos 40° = -23.3 \text{ m},$$
$$B_y = C_y - A_y = (15.0 \text{ m}) \sin 200° - (12.0 \text{ m}) \sin 40° = -12.8 \text{ m}.$$

Consequently, its magnitude is $|\vec{B}| = \sqrt{(-23.3 \text{ m})^2 + (-12.8 \text{ m})^2} = 26.6 \text{ m}$.

(b) The two possibilities presented by a simple calculation for the angle between $\vec{B}$ and the +x axis are $\tan^{-1}$[(–12.8 m)/(–23.3 m)] = 28.9°, and 180° + 28.9° = 209°. We choose the latter possibility as the correct one since it indicates that $\vec{B}$ is in the third quadrant (indicated by the signs of its components). We note, too, that the answer can be equivalently stated as $-151°$.

17. It should be mentioned that an efficient way to work this vector addition problem is with the cosine law for general triangles (and since $\vec{a}, \vec{b}$ and $\vec{r}$ form an isosceles triangle, the angles are easy to figure). However, in the interest of reinforcing the usual systematic approach to vector addition, we note that the angle $\vec{b}$ makes with the +*x* axis is 30° +105° = 135° and apply Eq. 3-5 and Eq. 3-6 where appropriate.

(a) The *x* component of $\vec{r}$ is r_x = (10.0 m) cos 30° + (10.0 m) cos 135° = 1.59 m.

(b) The *y* component of $\vec{r}$ is r_y = (10.0 m) sin 30° + (10.0 m) sin 135° = 12.1 m.

(c) The magnitude of $\vec{r}$ is $r = |\vec{r}| = \sqrt{(1.59\text{ m})^2 + (12.1\text{ m})^2} = 12.2\text{ m}.$

(d) The angle between $\vec{r}$ and the +*x* direction is $\tan^{-1}$[(12.1 m)/(1.59 m)] = 82.5°.

18. (a) Summing the x components, we have

$$20 \text{ m} + b_x - 20 \text{ m} - 60 \text{ m} = -140 \text{ m},$$

which gives $b_x = -80 \text{ m}$.

(b) Summing the y components, we have

$$60 \text{ m} - 70 \text{ m} + c_y - 70 \text{ m} = 30 \text{ m},$$

which implies $c_y = 110$ m.

(c) Using the Pythagorean theorem, the magnitude of the overall displacement is given by $\sqrt{(-140 \text{ m})^2 + (30 \text{ m})^2} \approx 143 \text{ m}$.

(d) The angle is given by $\tan^{-1}(30/(-140)) = -12°$, (which would be 12° measured clockwise from the $-x$ axis, or 168° measured counterclockwise from the $+x$ axis)

19. Many of the operations are done efficiently on most modern graphical calculators using their built-in vector manipulation and rectangular $\leftrightarrow$ polar "shortcuts." In this solution, we employ the "traditional" methods (such as Eq. 3-6). Where the length unit is not displayed, the unit meter should be understood.

(a) Using unit-vector notation,

$$\begin{aligned}
\vec{a} &= (50\text{ m})\cos(30°)\hat{\text{i}} + (50\text{ m})\sin(30°)\hat{\text{j}} \\
\vec{b} &= (50\text{ m})\cos(195°)\,\hat{\text{i}} + (50\text{ m})\sin(195°)\,\hat{\text{j}} \\
\vec{c} &= (50\text{ m})\cos(315°)\,\hat{\text{i}} + (50\text{ m})\sin(315°)\,\hat{\text{j}} \\
\vec{a}+\vec{b}+\vec{c} &= (30.4\text{ m})\,\hat{\text{i}} - (23.3\text{ m})\,\hat{\text{j}}.
\end{aligned}$$

The magnitude of this result is $\sqrt{(30.4\text{ m})^2 + (-23.3\text{ m})^2} = 38\text{ m}$.

(b) The two possibilities presented by a simple calculation for the angle between the vector described in part (a) and the +*x* direction are $\tan^{-1}$[(–23.2 m)/(30.4 m)] = –37.5°, and 180° + (–37.5°) = 142.5°. The former possibility is the correct answer since the vector is in the fourth quadrant (indicated by the signs of its components). Thus, the angle is –37.5°, which is to say that it is 37.5° *clockwise* from the +*x* axis. This is equivalent to 322.5° counterclockwise from +*x*.

(c) We find

$$\vec{a}-\vec{b}+\vec{c} = [43.3-(-48.3)+35.4]\,\hat{\text{i}} - [25-(-12.9)+(-35.4)]\,\hat{\text{j}} = (127\,\hat{\text{i}} + 2.60\,\hat{\text{j}})\text{ m}$$

in unit-vector notation. The magnitude of this result is

$$|\vec{a}-\vec{b}+\vec{c}| = \sqrt{(127\text{ m})^2 + (2.6\text{ m})^2} \approx 1.30\times 10^2\text{ m}.$$

(d) The angle between the vector described in part (c) and the +*x* axis is $\tan^{-1}(2.6\text{ m}/127\text{ m}) \approx 1.2°$.

(e) Using unit-vector notation, $\vec{d}$ is given by $\vec{d} = \vec{a}+\vec{b}-\vec{c} = (-40.4\,\hat{\text{i}} + 47.4\,\hat{\text{j}})\text{ m}$, which has a magnitude of $\sqrt{(-40.4\text{ m})^2 + (47.4\text{ m})^2} = 62\text{ m}$.

(f) The two possibilities presented by a simple calculation for the angle between the vector described in part (e) and the +*x* axis are $\tan^{-1}(47.4/(-40.4)) = -50.0°$, and $180° + (-50.0°) = 130°$. We choose the latter possibility as the correct one since it indicates that $\vec{d}$ is in the second quadrant (indicated by the signs of its components).

20. Angles are given in ‘standard’ fashion, so Eq. 3-5 applies directly. We use this to write the vectors in unit-vector notation before adding them. However, a very different-looking approach using the special capabilities of most graphical calculators can be imagined. Wherever the length unit is not displayed in the solution below, the unit meter should be understood.

(a) Allowing for the different angle units used in the problem statement, we arrive at

$$\vec{E} = 3.73\,\hat{\text{i}} + 4.70\,\hat{\text{j}}$$
$$\vec{F} = 1.29\,\hat{\text{i}} - 4.83\,\hat{\text{j}}$$
$$\vec{G} = 1.45\,\hat{\text{i}} + 3.73\,\hat{\text{j}}$$
$$\vec{H} = -5.20\,\hat{\text{i}} + 3.00\,\hat{\text{j}}$$
$$\vec{E} + \vec{F} + \vec{G} + \vec{H} = 1.28\,\hat{\text{i}} \; + 6.60\,\hat{\text{j}}.$$

(b) The magnitude of the vector sum found in part (a) is $\sqrt{(1.28\text{ m})^2 + (6.60\text{ m})^2} = 6.72\text{ m}$.

(c) Its angle measured counterclockwise from the +x axis is $\tan^{-1}(6.60/1.28) = 79.0°$.

(d) Using the conversion factor $\pi\ \text{rad} = 180°$, 79.0° = 1.38 rad.

21. (a) With $\hat{\text{i}}$ directed forward and $\hat{\text{j}}$ directed leftward, then the resultant is $(5.00\,\hat{\text{i}} + 2.00\,\hat{\text{j}})$ m . The magnitude is given by the Pythagorean theorem: $\sqrt{(5.00\text{ m})^2 + (2.00\text{ m})^2}$ = 5.385 m ≈ 5.39 m.

(b) The angle is $\tan^{-1}(2.00/5.00) \approx 21.8°$ (left of forward).

22. The desired result is the displacement vector, in units of km, $\vec{A}$ = (5.6 km), 90º (measured counterclockwise from the $+x$ axis), or $\vec{A} = (5.6 \text{ km})\hat{\text{j}}$, where $\hat{\text{j}}$ is the unit vector along the positive y axis (north). This consists of the sum of two displacements: during the whiteout, $\vec{B} = (7.8 \text{ km}),\ 50°$, or

$$\vec{B} = (7.8 \text{ km})(\cos 50°\hat{\text{i}} + \sin 50°\ \hat{\text{j}}) = (5.01 \text{ km})\hat{\text{i}} + (5.98 \text{ km})\hat{\text{j}}$$

and the unknown $\vec{C}$. Thus, $\vec{A} = \vec{B} + \vec{C}$.

(a) The desired displacement is given by $\vec{C} = \vec{A} - \vec{B} = (-5.01 \text{ km})\ \hat{\text{i}} - (0.38 \text{ km})\ \hat{\text{j}}$. The magnitude is $\sqrt{(-5.01 \text{ km})^2 + (-0.38 \text{ km})^2} = 5.0 \text{ km}$.

(b) The angle is $\tan^{-1}[(-0.38 \text{ km})/(-5.01 \text{ km})] = 4.3°$, south of due west.

23. The strategy is to find where the camel is ($\vec{C}$) by adding the two consecutive displacements described in the problem, and then finding the difference between that location and the oasis ($\vec{B}$). Using the magnitude-angle notation

$$\vec{C} = (24 \angle -15°) + (8.0 \angle 90°) = (23.25 \angle 4.41°)$$

so

$$\vec{B} - \vec{C} = (25 \angle 0°) - (23.25 \angle 4.41°) = (2.5 \angle -45°)$$

which is efficiently implemented using a vector capable calculator in polar mode. The distance is therefore 2.6 km.

24. Let $\vec{A}$ represent the first part of Beetle 1's trip (0.50 m east or $0.5\ \hat{\text{i}}$) and $\vec{C}$ represent the first part of Beetle 2's trip intended voyage (1.6 m at 50º north of east). For their respective second parts: $\vec{B}$ is 0.80 m at 30º north of east and $\vec{D}$ is the unknown. The final position of Beetle 1 is

$$\vec{A}+\vec{B}=(0.5\text{ m})\hat{\text{i}}+(0.8\text{ m})(\cos 30°\ \hat{\text{i}}+\sin 30°\ \hat{\text{j}})=(1.19\text{ m})\ \hat{\text{i}}+(0.40\text{ m})\ \hat{\text{j}}.$$

The equation relating these is $\vec{A}+\vec{B}=\vec{C}+\vec{D}$, where

$$\vec{C}=(1.60\text{ m})(\cos 50.0°\hat{\text{i}}+\sin 50.0°\hat{\text{j}})=(1.03\text{ m})\hat{\text{i}}+(1.23\text{ m})\hat{\text{j}}$$

(a) We find $\vec{D}=\vec{A}+\vec{B}-\vec{C}=(0.16\text{ m})\hat{\text{i}}+(-0.83\text{ m})\hat{\text{j}}$, and the magnitude is D = 0.84 m.

(b) The angle is $\tan^{-1}(-0.83/0.16)=-79°$ which is interpreted to mean 79º south of east (or 11º east of south).

25. The resultant (along the y axis, with the same magnitude as $\vec{C}$) forms (along with $\vec{C}$) a side of an isosceles triangle (with $\vec{B}$ forming the base). If the angle between $\vec{C}$ and the y axis is $\theta = \tan^{-1}(3/4) = 36.87°$, then it should be clear that (referring to the magnitudes of the vectors) $B = 2C\sin(\theta/2)$. Thus (since $C = 5.0$) we find $B = 3.2$.

26. As a vector addition problem, we express the situation (described in the problem statement) as $\vec{A} + \vec{B} = (3A)\hat{\text{j}}$, where $\vec{A} = A\,\hat{\text{i}}$ and $B = 7.0$ m. Since $\hat{\text{i}} \perp \hat{\text{j}}$ we may use the Pythagorean theorem to express B in terms of the magnitudes of the other two vectors:

$$B = \sqrt{(3A)^2 + A^2} \quad \Rightarrow \quad A = \frac{1}{\sqrt{10}}B = 2.2 \text{ m}.$$

27. Let $l_0 = 2.0\text{ cm}$ be the length of each segment. The nest is located at the endpoint of segment *w*.

(a) Using unit-vector notation, the displacement vector for point A is

$$\begin{aligned}\vec{d}_A &= \vec{w}+\vec{v}+\vec{i}+\vec{h} = l_0(\cos 60°\hat{\text{i}}+\sin 60°\,\hat{\text{j}})+\left(l_0\,\hat{\text{j}}\right)+l_0(\cos 120°\hat{\text{i}}+\sin 120°\,\hat{\text{j}})+\left(l_0\,\hat{\text{j}}\right)\\ &= (2+\sqrt{3})l_0\,\hat{\text{j}}.\end{aligned}$$

Therefore, the magnitude of $\vec{d}_A$ is $|\vec{d}_A| = (2+\sqrt{3})(2.0\text{ cm}) = 7.5\text{ cm}$.

(b) The angle of $\vec{d}_A$ is $\theta = \tan^{-1}(d_{A,y}/d_{A,x}) = \tan^{-1}(\infty) = 90°$.

(c) Similarly, the displacement for point B is

$$\begin{aligned}\vec{d}_B &= \vec{w}+\vec{v}+\vec{j}+\vec{p}+\vec{o}\\ &= l_0(\cos 60°\hat{\text{i}}+\sin 60°\,\hat{\text{j}})+\left(l_0\,\hat{\text{j}}\right)+l_0(\cos 60°\hat{\text{i}}+\sin 60°\,\hat{\text{j}})+l_0(\cos 30°\hat{\text{i}}+\sin 30°\,\hat{\text{j}})+\left(l_0\,\hat{\text{i}}\right)\\ &= (2+\sqrt{3}/2)l_0\,\hat{\text{i}}+(3/2+\sqrt{3})l_0\,\hat{\text{j}}.\end{aligned}$$

Therefore, the magnitude of $\vec{d}_B$ is

$$|\vec{d}_B| = l_0\sqrt{(2+\sqrt{3}/2)^2+(3/2+\sqrt{3})^2} = (2.0\text{ cm})(4.3) = 8.6\text{ cm}.$$

(d) The direction of $\vec{d}_B$ is

$$\theta_B = \tan^{-1}\left(\frac{d_{B,y}}{d_{B,x}}\right) = \tan^{-1}\left(\frac{3/2+\sqrt{3}}{2+\sqrt{3}/2}\right) = \tan^{-1}(1.13) = 48°.$$

28. Many of the operations are done efficiently on most modern graphical calculators using their built-in vector manipulation and rectangular $\leftrightarrow$ polar "shortcuts." In this solution, we employ the "traditional" methods (such as Eq. 3-6).

(a) The magnitude of $\vec{a}$ is $a = \sqrt{(4.0 \text{ m})^2 + (-3.0 \text{ m})^2} = 5.0 \text{ m}$.

(b) The angle between $\vec{a}$ and the +x axis is $\tan^{-1}$ [(–3.0 m)/(4.0 m)] = –37°. The vector is 37° *clockwise* from the axis defined by $\hat{\text{i}}$.

(c) The magnitude of $\vec{b}$ is $b = \sqrt{(6.0 \text{ m})^2 + (8.0 \text{ m})^2} = 10 \text{ m}$.

(d) The angle between $\vec{b}$ and the +x axis is $\tan^{-1}$[(8.0 m)/(6.0 m)] = 53°.

(e) $\vec{a}+\vec{b} = (4.0 \text{ m} + 6.0 \text{ m})\,\hat{\text{i}} + [(-3.0 \text{ m}) + 8.0 \text{ m}]\hat{\text{j}} = (10 \text{ m})\hat{\text{i}} + (5.0 \text{ m})\hat{\text{j}}$. The magnitude of this vector is $|\vec{a}+\vec{b}| = \sqrt{(10 \text{ m})^2 + (5.0 \text{ m})^2} = 11 \text{ m}$; we round to two significant figures in our results.

(f) The angle between the vector described in part (e) and the +x axis is $\tan^{-1}$[(5.0 m)/(10 m)] = 27°.

(g) $\vec{b}-\vec{a} = (6.0 \text{ m} - 4.0 \text{ m})\,\hat{\text{i}} + [8.0 \text{ m} - (-3.0 \text{ m})]\,\hat{\text{j}} = (2.0 \text{ m})\,\hat{\text{i}} + (11 \text{ m})\,\hat{\text{j}}$. The magnitude of this vector is $|\vec{b}-\vec{a}| = \sqrt{(2.0 \text{ m})^2 + (11 \text{ m})^2} = 11 \text{ m}$, which is, interestingly, the same result as in part (e) (exactly, not just to 2 significant figures) (this curious coincidence is made possible by the fact that $\vec{a} \perp \vec{b}$).

(h) The angle between the vector described in part (g) and the +x axis is $\tan^{-1}$[(11 m)/(2.0 m)] = 80°.

(i) $\vec{a}-\vec{b} = (4.0 \text{ m} - 6.0 \text{ m})\,\hat{\text{i}} + [(-3.0 \text{ m}) - 8.0 \text{ m}]\,\hat{\text{j}} = (-2.0 \text{ m})\,\hat{\text{i}} + (-11 \text{ m})\,\hat{\text{j}}$. The magnitude of this vector is $|\vec{a}-\vec{b}| = \sqrt{(-2.0 \text{ m})^2 + (-11 \text{ m})^2} = 11 \text{ m}$.

(j) The two possibilities presented by a simple calculation for the angle between the vector described in part (i) and the +x direction are $\tan^{-1}$ [(–11 m)/(–2.0 m)] = 80°, and 180° + 80° = 260°. The latter possibility is the correct answer (see part (k) for a further observation related to this result).

(k) Since $\vec{a}-\vec{b} = (-1)(\vec{b}-\vec{a})$, they point in opposite (anti-parallel) directions; the angle between them is 180°.

29. Solving the simultaneous equations yields the answers:

(a) $\vec{d}_1 = 4\,\vec{d}_3 = 8\,\hat{\mathrm{i}} + 16\,\hat{\mathrm{j}}$, and

(b) $\vec{d}_2 = \vec{d}_3 = 2\,\hat{\mathrm{i}} + 4\,\hat{\mathrm{j}}$.

30. The vector equation is $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$. Expressing $\vec{B}$ and $\vec{D}$ in unit-vector notation, we have $(1.69\hat{\mathrm{i}} + 3.63\hat{\mathrm{j}})$ m and $(-2.87\hat{\mathrm{i}} + 4.10\hat{\mathrm{j}})$ m, respectively. Where the length unit is not displayed in the solution below, the unit meter should be understood.

(a) Adding corresponding components, we obtain $\vec{R} = (-3.18 \text{ m})\hat{\mathrm{i}} + (4.72 \text{ m})\hat{\mathrm{j}}$.

(b) Using Eq. 3-6, the magnitude is

$$|\vec{R}| = \sqrt{(-3.18 \text{ m})^2 + (4.72 \text{ m})^2} = 5.69 \text{ m}.$$

(c) The angle is

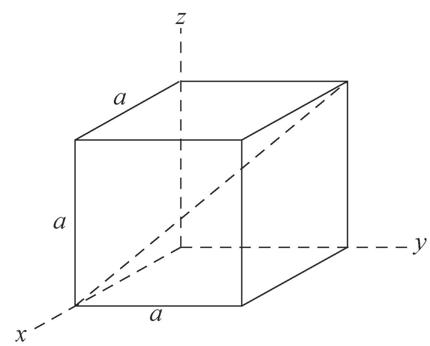
$$\theta = \tan^{-1}\left(\frac{4.72 \text{ m}}{-3.18 \text{ m}}\right) = -56.0° \;\; (\text{with } -x \text{ axis}).$$

If measured counterclockwise from +*x*-axis, the angle is then $180° - 56.0° = 124°$. Thus, converting the result to polar coordinates, we obtain

$$(-3.18, 4.72) \;\rightarrow\; (5.69 \angle 124°)$$

31. (a) As can be seen from Figure 3-32, the point diametrically opposite the origin (0,0,0) has position vector $a\,\hat{\mathrm{i}} + a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$ and this is the vector along the "body diagonal."

(b) From the point (a, 0, 0) which corresponds to the position vector $a\,\hat{\mathrm{i}}$, the diametrically opposite point is (0, a, a) with the position vector $a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$. Thus, the vector along the line is the difference $-a\hat{\mathrm{i}} + a\hat{\mathrm{j}} + a\hat{\mathrm{k}}$.



(c) If the starting point is (0, a, 0) with the corresponding position vector $a\,\hat{\mathrm{j}}$, the diametrically opposite point is (a, 0, a) with the position vector $a\,\hat{\mathrm{i}} + a\,\hat{\mathrm{k}}$. Thus, the vector along the line is the difference $a\,\hat{\mathrm{i}} - a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$.

(d) If the starting point is (a, a, 0) with the corresponding position vector $a\,\hat{\mathrm{i}} + a\,\hat{\mathrm{j}}$, the diametrically opposite point is (0, 0, a) with the position vector $a\,\hat{\mathrm{k}}$. Thus, the vector along the line is the difference $-a\,\hat{\mathrm{i}} - a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$.

(e) Consider the vector from the back lower left corner to the front upper right corner. It is $a\,\hat{\mathrm{i}} + a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$. We may think of it as the sum of the vector $a\,\hat{\mathrm{i}}$ parallel to the x axis and the vector $a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$ perpendicular to the x axis. The tangent of the angle between the vector and the x axis is the perpendicular component divided by the parallel component. Since the magnitude of the perpendicular component is $\sqrt{a^2 + a^2} = a\sqrt{2}$ and the magnitude of the parallel component is a, $\tan\theta = \left(a\sqrt{2}\right)/a = \sqrt{2}$. Thus $\theta = 54.7°$. The angle between the vector and each of the other two adjacent sides (the y and z axes) is the same as is the angle between any of the other diagonal vectors and any of the cube sides adjacent to them.

(f) The length of any of the diagonals is given by $\sqrt{a^2 + a^2 + a^2} = a\sqrt{3}$.

32. (a) With a = 17.0 m and θ = 56.0° we find $a_x = a \cos \theta$ = 9.51 m.

(b) Similarly, $a_y = a \sin \theta$ = 14.1 m.

(c) The angle relative to the new coordinate system is θ' = (56.0° – 18.0°) = 38.0°. Thus, $a_x' = a \cos \theta' = 13.4$ m.

(d) Similarly, $a_y' = a \sin \theta'$ = 10.5 m.

33. (a) The scalar (dot) product is $(4.50)(7.30)\cos(320° - 85.0°) = -18.8$.

(b) The vector (cross) product is in the $\hat{k}$ direction (by the right-hand rule) with magnitude $|(4.50)(7.30)\sin(320° - 85.0°)| = 26.9$.

34. First, we rewrite the given expression as $4(\vec{d}_{\text{plane}} \cdot \vec{d}_{\text{cross}})$ where $\vec{d}_{\text{plane}} = \vec{d}_1 + \vec{d}_2$ and in the plane of $\vec{d}_1$ and $\vec{d}_2$, and $\vec{d}_{\text{cross}} = \vec{d}_1 \times \vec{d}_2$. Noting that $\vec{d}_{\text{cross}}$ is perpendicular to the plane of $\vec{d}_1$ and $\vec{d}_2$, we see that the answer must be 0 (the scalar [dot] product of perpendicular vectors is zero).

35. We apply Eq. 3-30 and Eq.3-23. If a vector-capable calculator is used, this makes a good exercise for getting familiar with those features. Here we briefly sketch the method.

(a) We note that $\vec{b}\times\vec{c} = -8.0\,\hat{\text{i}} + 5.0\,\hat{\text{j}} + 6.0\,\hat{\text{k}}$. Thus,

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = (3.0)\,(-8.0) + (3.0)(5.0) + (-2.0)\,(6.0) = -21.$$

(b) We note that $\vec{b}+\vec{c} = 1.0\,\hat{\text{i}} - 2.0\,\hat{\text{j}} + 3.0\,\hat{\text{k}}$. Thus,

$$\vec{a}\cdot(\vec{b}+\vec{c}) = (3.0)\,(1.0) + (3.0)\,(-2.0) + (-2.0)\,(3.0) = -9.0.$$

(c) Finally,

$$\begin{aligned}\vec{a}\times(\vec{b}+\vec{c}) &= [(3.0)(3.0)-(-2.0)(-2.0)]\,\hat{\text{i}} + [(-2.0)(1.0)-(3.0)(3.0)]\,\hat{\text{j}} \\ &\quad +[(3.0)(-2.0)-(3.0)(1.0)]\,\hat{\text{k}} \\ &= 5\hat{\text{i}} - 11\hat{\text{j}} - 9\hat{\text{k}}\end{aligned}.$$

36. We apply Eq. 3-30 and Eq. 3-23.

(a) $\vec{a}\times\vec{b} = (a_x b_y - a_y b_x)\,\hat{\mathrm{k}}$ since all other terms vanish, due to the fact that neither $\vec{a}$ nor $\vec{b}$ have any z components. Consequently, we obtain $[(3.0)(4.0)-(5.0)(2.0)]\hat{\mathrm{k}} = 2.0\,\hat{\mathrm{k}}$.

(b) $\vec{a}\cdot b = a_x b_x + a_y b_y$ yields (3.0)(2.0) + (5.0)(4.0) = 26.

(c) $\vec{a}+\vec{b} = (3.0 + 2.0)\,\hat{\mathrm{i}} + (5.0 + 4.0)\,\hat{\mathrm{j}} \Rightarrow \quad (\vec{a}+\vec{b})\cdot\vec{b} = (5.0)\,(2.0) + (9.0)\,(4.0) = 46$.

(d) Several approaches are available. In this solution, we will construct a $\hat{b}$ unit-vector and "dot" it (take the scalar product of it) with $\vec{a}$. In this case, we make the desired unit-vector by

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2.0\,\hat{\mathrm{i}} + 4.0\,\hat{\mathrm{j}}}{\sqrt{(2.0)^2 + (4.0)^2}}\,.$$

We therefore obtain

$$a_b = \vec{a}\cdot\hat{b} = \frac{(3.0)(2.0) + (5.0)(4.0)}{\sqrt{(2.0)^2 + (4.0)^2}} = 5.8.$$

37. Examining the figure, we see that $\vec{a} + \vec{b} + \vec{c} = 0$, where $\vec{a} \perp \vec{b}$.

(a) $|\vec{a} \times \vec{b}| = (3.0)(4.0) = 12$ since the angle between them is 90º.

(b) Using the Right Hand Rule, the vector $\vec{a}\times\vec{b}$ points in the $\hat{\mathrm{i}}\times\hat{\mathrm{j}}=\hat{\mathrm{k}}$, or the $+z$ direction.

(c) $|\vec{a} \times \vec{c}| = |\vec{a} \times (-\vec{a} - \vec{b})| = |-(\vec{a} \times \vec{b})| = 12.$

(d) The vector $-\vec{a}\times\vec{b}$ points in the $-\hat{\mathrm{i}}\times\hat{\mathrm{j}}=-\hat{\mathrm{k}}$, or the $-z$ direction.

(e) $|\vec{b} \times \vec{c}| = |\vec{b} \times (-\vec{a} - \vec{b})| = |-(\vec{b} \times \vec{a})| = |(\vec{a} \times \vec{b})| = 12.$

(f) The vector points in the $+z$ direction, as in part (a).

38. The displacement vectors can be written as (in meters)

$$\vec{d}_1 = (4.50\text{ m})(\cos 63°\,\hat{\text{j}} + \sin 63°\,\hat{\text{k}}) = (2.04\text{ m})\,\hat{\text{j}} + (4.01\text{ m})\,\hat{\text{k}}$$
$$\vec{d}_2 = (1.40\text{ m})(\cos 30°\,\hat{\text{i}} + \sin 30°\,\hat{\text{k}}) = (1.21\text{ m})\,\hat{\text{i}} + (0.70\text{ m})\,\hat{\text{k}}\,.$$

(a) The dot product of $\vec{d}_1$ and $\vec{d}_2$ is

$$\vec{d}_1 \cdot \vec{d}_2 = (2.04\,\hat{\text{j}} + 4.01\,\hat{\text{k}}) \cdot (1.21\,\hat{\text{i}} + 0.70\,\hat{\text{k}}) = (4.01\,\hat{\text{k}}) \cdot (0.70\,\hat{\text{k}}) = 2.81\,\text{m}^2.$$

(b) The cross product of $\vec{d}_1$ and $\vec{d}_2$ is

$$\begin{aligned}\vec{d}_1 \times \vec{d}_2 &= (2.04\,\hat{\text{j}} + 4.01\,\hat{\text{k}}) \times (1.21\,\hat{\text{i}} + 0.70\,\hat{\text{k}}) \\ &= (2.04)(1.21)(-\hat{\text{k}}) + (2.04)(0.70)\hat{\text{i}} + (4.01)(1.21)\hat{\text{j}} \\ &= (1.43\,\hat{\text{i}} + 4.86\,\hat{\text{j}} - 2.48\,\hat{\text{k}})\,\text{m}^2.\end{aligned}$$

(c) The magnitudes of $\vec{d}_1$ and $\vec{d}_2$ are

$$d_1 = \sqrt{(2.04\text{ m})^2 + (4.01\text{ m})^2} = 4.50\text{ m}$$
$$d_2 = \sqrt{(1.21\text{ m})^2 + (0.70\text{ m})^2} = 1.40\text{ m.}$$

Thus, the angle between the two vectors is

$$\theta = \cos^{-1}\left(\frac{\vec{d}_1 \cdot \vec{d}_2}{d_1 d_2}\right) = \cos^{-1}\left(\frac{2.81\text{ m}^2}{(4.50\text{ m})(1.40\text{ m})}\right) = 63.5°.$$

39. Since $ab \cos \phi = a_x b_x + a_y b_y + a_z b_z$,

$$\cos\phi = \frac{a_x b_x + a_y b_y + a_z b_z}{ab}.$$

The magnitudes of the vectors given in the problem are

$$a = |\vec{a}| = \sqrt{(3.00)^2 + (3.00)^2 + (3.00)^2} = 5.20$$

$$b = |\vec{b}| = \sqrt{(2.00)^2 + (1.00)^2 + (3.00)^2} = 3.74.$$

The angle between them is found from

$$\cos\phi = \frac{(3.00)(2.00) + (3.00)(1.00) + (3.00)(3.00)}{(5.20)(3.74)} = 0.926.$$

The angle is $\phi = 22°$.

40. Using the fact that

$$\hat{\mathrm{i}}\times\hat{\mathrm{j}}=\hat{\mathrm{k}},\quad \hat{\mathrm{j}}\times\hat{\mathrm{k}}=\hat{\mathrm{i}},\quad \hat{\mathrm{k}}\times\hat{\mathrm{i}}=\hat{\mathrm{j}}$$

we obtain

$$2\vec{A}\times\vec{B}=2\left(2.00\hat{\mathrm{i}}+3.00\hat{\mathrm{j}}-4.00\hat{\mathrm{k}}\right)\times\left(-3.00\hat{\mathrm{i}}+4.00\hat{\mathrm{j}}+2.00\hat{\mathrm{k}}\right)=44.0\hat{\mathrm{i}}+16.0\hat{\mathrm{j}}+34.0\hat{\mathrm{k}}.$$

Next, making use of

$$\begin{aligned}\hat{\mathrm{i}}\cdot\hat{\mathrm{i}} &= \hat{\mathrm{j}}\cdot\hat{\mathrm{j}} = \hat{\mathrm{k}}\cdot\hat{\mathrm{k}} = 1\\ \hat{\mathrm{i}}\cdot\hat{\mathrm{j}} &= \hat{\mathrm{j}}\cdot\hat{\mathrm{k}} = \hat{\mathrm{k}}\cdot\hat{\mathrm{i}} = 0\end{aligned}$$

we have

$$\begin{aligned}3\vec{C}\cdot\left(2\vec{A}\times\vec{B}\right)&=3\left(7.00\hat{\mathrm{i}}-8.00\hat{\mathrm{j}}\right)\cdot\left(44.0\hat{\mathrm{i}}+16.0\hat{\mathrm{j}}+34.0\hat{\mathrm{k}}\right)\\&=3[(7.00)(44.0)+(-8.00)(16.0)+(0)(34.0)]=540.\end{aligned}$$

41. From the definition of the dot product between $\vec{A}$ and $\vec{B}$, $\vec{A}\cdot\vec{B} = AB\cos\theta$, we have

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

With $A = 6.00$, $B = 7.00$ and $\vec{A}\cdot\vec{B} = 14.0$, $\cos\theta = 0.333$, or $\theta = 70.5°$.

42. Applying Eq. 3-23, $\vec{F} = q\vec{v} \times \vec{B}$ (where q is a scalar) becomes

$$F_x\,\hat{\mathrm{i}} + F_y\,\hat{\mathrm{j}} + F_z\,\hat{\mathrm{k}} = q\left(v_y B_z - v_z B_y\right)\hat{\mathrm{i}} + q\left(v_z B_x - v_x B_z\right)\hat{\mathrm{j}} + q\left(v_x B_y - v_y B_x\right)\hat{\mathrm{k}}$$

which — plugging in values — leads to three equalities:

$$\begin{aligned} 4.0 &= 2\,(4.0B_z - 6.0B_y) \\ -20 &= 2\,(6.0B_x - 2.0B_z) \\ 12 &= 2\,(2.0B_y - 4.0B_x) \end{aligned}$$

Since we are told that $B_x = B_y$, the third equation leads to $B_y = -3.0$. Inserting this value into the first equation, we find $B_z = -4.0$. Thus, our answer is

$$\vec{B} = -3.0\,\hat{\mathrm{i}} - 3.0\,\hat{\mathrm{j}} - 4.0\,\hat{\mathrm{k}}.$$

43. From the figure, we note that $\vec{c} \perp \vec{b}$, which implies that the angle between $\vec{c}$ and the $+x$ axis is 120°. Direct application of Eq. 3-5 yields the answers for this and the next few parts.

(a) $a_x = a \cos 0° = a = 3.00$ m.

(b) $a_y = a \sin 0° = 0$.

(c) $b_x = b \cos 30° = (4.00 \text{ m}) \cos 30° = 3.46$ m.

(d) $b_y = b \sin 30° = (4.00 \text{ m}) \sin 30° = 2.00$ m.

(e) $c_x = c \cos 120° = (10.0 \text{ m}) \cos 120° = -5.00$ m.

(f) $c_y = c \sin 30° = (10.0 \text{ m}) \sin 120° = 8.66$ m.

(g) In terms of components (first x and then y), we must have

$$\begin{aligned} -5.00 \text{ m} &= p\,(3.00 \text{ m}) + q\,(3.46 \text{ m}) \\ 8.66 \text{ m} &= p\,(0) + q\,(2.00 \text{ m}). \end{aligned}$$

Solving these equations, we find $p = -6.67$.

(h) Similarly, $q = 4.33$ (note that it's easiest to solve for q first). The numbers p and q have no units.

44. The two vectors are written as, in unit of meters,

$$\vec{d}_1 = 4.0\hat{\text{i}}+5.0\hat{\text{j}} = d_{1x}\hat{\text{i}} + d_{1y}\hat{\text{j}}, \quad \vec{d}_2 = -3.0\hat{\text{i}}+4.0\hat{\text{j}} = d_{2x}\hat{\text{i}} + d_{2y}\hat{\text{j}}$$

(a) The vector (cross) product gives

$$\vec{d}_1 \times \vec{d}_2 = (d_{1x}d_{2y} - d_{1y}d_{2x})\hat{\text{k}} = [(4.0)(4.0) - (5.0)(-3.0)]\hat{\text{k}} = 31\ \hat{\text{k}}$$

(b) The scalar (dot) product gives

$$\vec{d}_1 \cdot \vec{d}_2 = d_{1x}d_{2x} + d_{1y}d_{2y} = (4.0)(-3.0) + (5.0)(4.0) = 8.0.$$

(c)

$$(\vec{d}_1 + \vec{d}_2) \cdot \vec{d}_2 = \vec{d}_1 \cdot \vec{d}_2 + d_2^2 = 8.0 + (-3.0)^2 + (4.0)^2 = 33.$$

(d) Note that the magnitude of the d_1 vector is $\sqrt{16+25}$ = 6.4. Now, the dot product is (6.4)(5.0)cosθ = 8. Dividing both sides by 32 and taking the inverse cosine yields θ = 75.5°. Therefore the component of the d_1 vector along the direction of the d_2 vector is 6.4cos$\theta \approx$ 1.6.

45. Although we think of this as a three-dimensional movement, it is rendered effectively two-dimensional by referring measurements to its well-defined plane of the fault.

(a) The magnitude of the net displacement is

$$|\vec{AB}| = \sqrt{|AD|^2 + |AC|^2} = \sqrt{(17.0\text{ m})^2 + (22.0\text{ m})^2} = 27.8\text{m}.$$

(b) The magnitude of the vertical component of $\vec{AB}$ is $|AD| \sin 52.0° = 13.4$ m.

46. Where the length unit is not displayed, the unit meter is understood.

(a) We first note that $a = |\vec{a}| = \sqrt{(3.2)^2 + (1.6)^2} = 3.58$ and $b = |\vec{b}| = \sqrt{(0.50)^2 + (4.5)^2} = 4.53$. Now,

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y = ab \cos \phi$$
$$(3.2)(0.50) + (1.6)(4.5) = (3.58)(4.53) \cos \phi$$

which leads to $\phi = 57°$ (the inverse cosine is double-valued as is the inverse tangent, but we know this is the right solution since both vectors are in the same quadrant).

(b) Since the angle (measured from $+x$) for $\vec{a}$ is $\tan^{-1}(1.6/3.2) = 26.6°$, we know the angle for $\vec{c}$ is 26.6° –90° = –63.4° (the other possibility, 26.6° + 90° would lead to a $c_x < 0$). Therefore,

$$c_x = c \cos(-63.4°) = (5.0)(0.45) = 2.2 \text{ m}.$$

(c) Also, $c_y = c \sin(-63.4°) = (5.0)(-0.89) = -4.5$ m.

(d) And we know the angle for $\vec{d}$ to be 26.6° + 90° = 116.6°, which leads to

$$d_x = d \cos(116.6°) = (5.0)(-0.45) = -2.2 \text{ m}.$$

(e) Finally, $d_y = d \sin 116.6° = (5.0)(0.89) = 4.5$ m.

47. We apply Eq. 3-20 and Eq. 3-27.

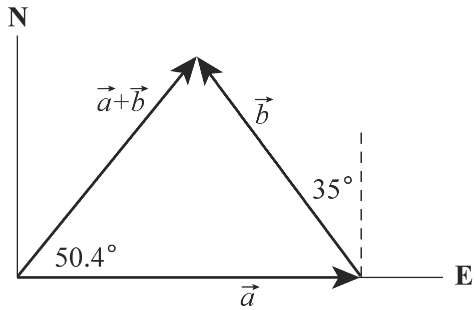
(a) The scalar (dot) product of the two vectors is

$$\vec{a}\cdot\vec{b} = ab\cos\phi = (10)\,(6.0)\cos 60° = 30.$$

(b) The magnitude of the vector (cross) product of the two vectors is

$$|\,\vec{a}\times\vec{b}\,| = ab\sin\phi = (10)\,(6.0)\sin 60° = 52.$$

48. The vectors are shown on the diagram. The x axis runs from west to east and the y axis runs from south to north. Then a_x = 5.0 m, a_y = 0, b_x = –(4.0 m) sin 35° = –2.29 m, and b_y = (4.0 m) cos 35° = 3.28 m.



(a) Let $\vec{c} = \vec{a} + \vec{b}$. Then $c_x = a_x + b_x = 5.00\text{ m} - 2.29\text{ m} = 2.71\text{ m}$ and

$c_y = a_y + b_y = 0 + 3.28\text{ m} = 3.28\text{ m}$. The magnitude of c is

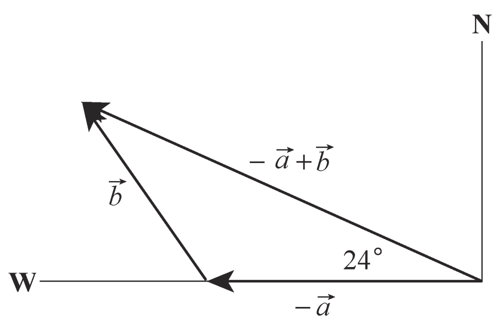
$$c = \sqrt{c_x^2 + c_y^2} = \sqrt{(2.71\text{m})^2 + (3.28\text{m})^2} = 4.2\text{ m}.$$

(b) The angle θ that $\vec{c} = \vec{a} + \vec{b}$ makes with the $+x$ axis is

$$\theta = \tan^{-1}\left(\frac{c_y}{c_x}\right) = \tan^{-1}\left(\frac{3.28}{2.71}\right) = 50°.$$

The second possibility (θ = 50.4° + 180° = 230.4°) is rejected because it would point in a direction opposite to $\vec{c}$.

(c) The vector $\vec{b} - \vec{a}$ is found by adding $-\vec{a}$ to $\vec{b}$. The result is shown on the diagram to the right. Let $\vec{c} = \vec{b} - \vec{a}$. The components are $c_x = b_x - a_x = -2.29\text{ m} - 5.00\text{ m} = -7.29\text{ m}$, and $c_y = b_y - a_y = 3.28\text{ m}$. The magnitude of $\vec{c}$ is $c = \sqrt{c_x^2 + c_y^2} = 8.0\,\text{m}$.



(d) The tangent of the angle θ that $\vec{c}$ makes with the $+x$ axis (east) is

$$\tan\theta = \frac{c_y}{c_x} = \frac{3.28\text{ m}}{-7.29\text{ m}} = -4.50.$$

There are two solutions: –24.2° and 155.8°. As the diagram shows, the second solution is correct. The vector $\vec{c} = -\vec{a} + \vec{b}$ is 24° north of west.

49. We choose $+x$ east and $+y$ north and measure all angles in the "standard" way (positive ones are counterclockwise from $+x$). Thus, vector $\vec{d}_1$ has magnitude $d_1 = 4.00$ m (with the unit meter) and direction $\theta_1 = 225°$. Also, $\vec{d}_2$ has magnitude $d_2 = 5.00$ m and direction $\theta_2 = 0°$, and vector $\vec{d}_3$ has magnitude $d_3 = 6.00$ m and direction $\theta_3 = 60°$.

(a) The x-component of $\vec{d}_1$ is $d_{1x} = d_1 \cos \theta_1 = -2.83$ m.

(b) The y-component of $\vec{d}_1$ is $d_{1y} = d_1 \sin \theta_1 = -2.83$ m.

(c) The x-component of $\vec{d}_2$ is $d_{2x} = d_2 \cos \theta_2 = 5.00$ m.

(d) The y-component of $\vec{d}_2$ is $d_{2y} = d_2 \sin \theta_2 = 0$.

(e) The x-component of $\vec{d}_3$ is $d_{3x} = d_3 \cos \theta_3 = 3.00$ m.

(f) The y-component of $\vec{d}_3$ is $d_{3y} = d_3 \sin \theta_3 = 5.20$ m.

(g) The sum of x-components is

$$d_x = d_{1x} + d_{2x} + d_{3x} = -2.83 \text{ m} + 5.00 \text{ m} + 3.00 \text{ m} = 5.17 \text{ m}.$$

(h) The sum of y-components is

$$d_y = d_{1y} + d_{2y} + d_{3y} = -2.83 \text{ m} + 0 + 5.20 \text{ m} = 2.37 \text{ m}.$$

(i) The magnitude of the resultant displacement is

$$d = \sqrt{d_x^2 + d_y^2} = \sqrt{(5.17 \text{ m})^2 + (2.37 \text{ m})^2} = 5.69 \text{ m}.$$

(j) And its angle is $\theta = \tan^{-1}(2.37/5.17) = 24.6°$ which (recalling our coordinate choices) means it points at about 25° north of east.

(k) and (l) This new displacement (the direct line home) when vectorially added to the previous (net) displacement must give zero. Thus, the new displacement is the negative, or opposite, of the previous (net) displacement. That is, it has the same magnitude (5.69 m) but points in the opposite direction (25° south of west).

50. From the figure, it is clear that $\vec{a} + \vec{b} + \vec{c} = 0$, where $\vec{a} \perp \vec{b}$.

(a) $\vec{a} \cdot \vec{b} = 0$ since the angle between them is 90º.

(b) $\vec{a} \cdot \vec{c} = \vec{a} \cdot (-\vec{a} - \vec{b}) = -|\vec{a}|^2 = -16$.

(c) Similarly, $\vec{b} \cdot \vec{c} = -9.0$.

51. Let $\vec{A}$ represent the first part of his actual voyage (50.0 km east) and $\vec{C}$ represent the intended voyage (90.0 km north). We are looking for a vector $\vec{B}$ such that $\vec{A} + \vec{B} = \vec{C}$.

(a) The Pythagorean theorem yields $B = \sqrt{(50.0 \text{ km})^2 + (90.0 \text{ km})^2} = 103 \text{ km}$.

(b) The direction is $\tan^{-1}(50.0 \text{ km}/90.0 \text{ km}) = 29.1°$ west of north (which is equivalent to 60.9° north of due west).

52. If we wish to use Eq. 3-5 directly, we should note that the angles for $\vec{Q}, \vec{R}$ and $\vec{S}$ are 100°, 250° and 310°, respectively, if they are measured counterclockwise from the +*x* axis.

(a) Using unit-vector notation, with the unit meter understood, we have

$$\begin{aligned}
\vec{P} &= 10.0 \cos(25.0°)\,\hat{\mathrm{i}} + 10.0 \sin(25.0°)\,\hat{\mathrm{j}} \\
\vec{Q} &= 12.0 \cos(100°)\,\hat{\mathrm{i}} + 12.0 \sin(100°)\,\hat{\mathrm{j}} \\
\vec{R} &= 8.00 \cos(250°)\,\hat{\mathrm{i}} + 8.00 \sin(250°)\,\hat{\mathrm{j}} \\
\vec{S} &= 9.00 \cos(310°)\,\hat{\mathrm{i}} + 9.00 \sin(310°)\,\hat{\mathrm{j}} \\
\vec{P} + \vec{Q} + \vec{R} + \vec{S} &= (10.0 \text{ m})\hat{\mathrm{i}} + (1.63 \text{ m})\hat{\mathrm{j}}
\end{aligned}$$

(b) The magnitude of the vector sum is $\sqrt{(10.0 \text{ m})^2 + (1.63 \text{ m})^2} = 10.2 \text{ m}$.

(c) The angle is $\tan^{-1}$ (1.63 m/10.0 m) ≈ 9.24° measured counterclockwise from the +*x* axis.

53. Noting that the given 130° is measured counterclockwise from the $+x$ axis, the two vectors can be written as

$$\vec{A} = 8.00(\cos 130°\,\hat{\text{i}} + \sin 130°\,\hat{\text{j}}) = -5.14\,\hat{\text{i}} + 6.13\,\hat{\text{j}}$$
$$\vec{B} = B_x\hat{\text{i}} + B_y\hat{\text{j}} = -7.72\,\hat{\text{i}} - 9.20\,\hat{\text{j}}.$$

(a) The angle between the negative direction of the y axis ($-\hat{\text{j}}$) and the direction of $\vec{A}$ is

$$\theta = \cos^{-1}\left(\frac{\vec{A}\cdot(-\hat{\text{j}})}{A}\right) = \cos^{-1}\left(\frac{-6.13}{\sqrt{(-5.14)^2 + (6.13)^2}}\right) = \cos^{-1}\left(\frac{-6.13}{8.00}\right) = 140°.$$

Alternatively, one may say that the $-y$ direction corresponds to an angle of 270°, and the answer is simply given by 270°–130° = 140°.

(b) Since the y axis is in the xy plane, and $\vec{A}\times\vec{B}$ is perpendicular to that plane, then the answer is 90.0°.

(c) The vector can be simplified as

$$\begin{aligned}\vec{A}\times(\vec{B}+3.00\,\hat{\text{k}}) &= (-5.14\,\hat{\text{i}} + 6.13\,\hat{\text{j}})\times(-7.72\,\hat{\text{i}} - 9.20\,\hat{\text{j}} + 3.00\,\hat{\text{k}}) \\ &= 18.39\,\hat{\text{i}} + 15.42\,\hat{\text{j}} + 94.61\,\hat{\text{k}}\end{aligned}$$

Its magnitude is $|\vec{A}\times(\vec{B}+3.00\hat{\text{k}})| = 97.6.$ The angle between the negative direction of the y axis ($-\hat{\text{j}}$) and the direction of the above vector is

$$\theta = \cos^{-1}\left(\frac{-15.42}{97.6}\right) = 99.1°.$$

54. The three vectors are

$$\vec{d}_1 = 4.0\hat{\text{i}} + 5.0\hat{\text{j}} - 6.0\hat{\text{k}}$$
$$\vec{d}_2 = -1.0\hat{\text{i}} + 2.0\hat{\text{j}} + 3.0\hat{\text{k}}$$
$$\vec{d}_3 = 4.0\hat{\text{i}} + 3.0\hat{\text{j}} + 2.0\hat{\text{k}}$$

(a) $\vec{r} = \vec{d}_1 - \vec{d}_2 + \vec{d}_3 = (9.0\text{ m})\hat{\text{i}} + (6.0\text{ m})\hat{\text{j}} + (-7.0\text{ m})\hat{\text{k}}$.

(b) The magnitude of $\vec{r}$ is $|\vec{r}| = \sqrt{(9.0\text{ m})^2 + (6.0\text{ m})^2 + (-7.0\text{ m})^2} = 12.9\text{ m}$. The angle between $\vec{r}$ and the z-axis is given by

$$\cos\theta = \frac{\vec{r}\cdot\hat{\text{k}}}{|\vec{r}|} = \frac{-7.0\text{ m}}{12.9\text{ m}} = -0.543$$

which implies $\theta = 123°$.

(c) The component of $\vec{d}_1$ along the direction of $\vec{d}_2$ is given by $d_{\parallel} = \vec{d}_1 \cdot \hat{\text{u}} = d_1 \cos\varphi$ where φ is the angle between $\vec{d}_1$ and $\vec{d}_2$, and $\hat{\text{u}}$ is the unit vector in the direction of $\vec{d}_2$. Using the properties of the scalar (dot) product, we have

$$d_{\parallel} = d_1\left(\frac{\vec{d}_1\cdot\vec{d}_2}{d_1 d_2}\right) = \frac{\vec{d}_1\cdot\vec{d}_2}{d_2} = \frac{(4.0)(-1.0)+(5.0)(2.0)+(-6.0)(3.0)}{\sqrt{(-1.0)^2+(2.0)^2+(3.0)^2}} = \frac{-12}{\sqrt{14}} = -3.2\text{ m}.$$

(d) Now we are looking for $d_{\perp}$ such that $d_1^2 = (4.0)^2 + (5.0)^2 + (-6.0)^2 = 77 = d_{\parallel}^2 + d_{\perp}^2$. From (c), we have

$$d_{\perp} = \sqrt{77\text{ m}^2 - (-3.2\text{ m})^2} = 8.2\text{ m}.$$

This gives the magnitude of the perpendicular component (and is consistent with what one would get using Eq. 3-27), but if more information (such as the direction, or a full specification in terms of unit vectors) is sought then more computation is needed.

55. The two vectors are given by

$$\vec{A} = 8.00(\cos 130°\,\hat{\mathrm{i}} + \sin 130°\,\hat{\mathrm{j}}) = -5.14\,\hat{\mathrm{i}} + 6.13\,\hat{\mathrm{j}}$$
$$\vec{B} = B_x\hat{\mathrm{i}} + B_y\hat{\mathrm{j}} = -7.72\,\hat{\mathrm{i}} - 9.20\,\hat{\mathrm{j}}.$$

(a) The dot product of $5\vec{A}\cdot\vec{B}$ is

$$5\vec{A}\cdot\vec{B} = 5(-5.14\,\hat{\mathrm{i}} + 6.13\,\hat{\mathrm{j}})\cdot(-7.72\,\hat{\mathrm{i}} - 9.20\,\hat{\mathrm{j}}) = 5[(-5.14)(-7.72) + (6.13)(-9.20)]$$
$$= -83.4.$$

(b) In unit vector notation

$$4\vec{A}\times 3\vec{B} = 12\vec{A}\times\vec{B} = 12(-5.14\,\hat{\mathrm{i}} + 6.13\,\hat{\mathrm{j}})\times(-7.72\,\hat{\mathrm{i}} - 9.20\,\hat{\mathrm{j}}) = 12(94.6\,\hat{\mathrm{k}}) = 1.14\times 10^3\,\hat{\mathrm{k}}$$

(c) We note that the azimuthal angle is undefined for a vector along the *z* axis. Thus, our result is "1.14×10^3, θ not defined, and $\phi = 0°$."

(d) Since $\vec{A}$ is in the *xy* plane, and $\vec{A}\times\vec{B}$ is perpendicular to that plane, then the answer is 90°.

(e) Clearly, $\vec{A}$ + 3.00 $\hat{\mathrm{k}}$ = –5.14 $\hat{\mathrm{i}}$ + 6.13 $\hat{\mathrm{j}}$ + 3.00 $\hat{\mathrm{k}}$.

(f) The Pythagorean theorem yields magnitude $A = \sqrt{(5.14)^2 + (6.13)^2 + (3.00)^2} = 8.54$. The azimuthal angle is θ = 130°, just as it was in the problem statement ($\vec{A}$ <u>is</u> the projection onto to the *xy* plane of the new vector created in part (e)). The angle measured from the +*z* axis is $\phi = \cos^{-1}(3.00/8.54) = 69.4°$.

56. The two vectors $\vec{d}_1$ and $\vec{d}_2$ are given by $\vec{d}_1 = -d_1\,\hat{\mathrm{j}}$ and $\vec{d}_2 = d_2\,\hat{\mathrm{i}}$.

(a) The vector $\vec{d}_2 / 4 = (d_2 / 4)\,\hat{\mathrm{i}}$ points in the +*x* direction. The ¼ factor does not affect the result.

(b) The vector $\vec{d}_1 /(-4) = (d_1 / 4)\hat{\mathrm{j}}$ points in the +*y* direction. The minus sign (with the "–4") does affect the direction: –(–*y*) = + *y*.

(c) $\vec{d}_1 \cdot \vec{d}_2 = 0$ since $\hat{\mathrm{i}} \cdot \hat{\mathrm{j}} = 0$. The two vectors are perpendicular to each other.

(d) $\vec{d}_1 \cdot (\vec{d}_2 / 4) = (\vec{d}_1 \cdot \vec{d}_2)/4 = 0$, as in part (c).

(e) $\vec{d}_1 \times \vec{d}_2 = -d_1 d_2 (\hat{\mathrm{j}} \times \hat{\mathrm{i}}) = d_1 d_2\,\hat{\mathrm{k}}$, in the +*z*-direction.

(f) $\vec{d}_2 \times \vec{d}_1 = -d_2 d_1 (\hat{\mathrm{i}} \times \hat{\mathrm{j}}) = -d_1 d_2\,\hat{\mathrm{k}}$, in the –*z*-direction.

(g) The magnitude of the vector in (e) is $d_1 d_2$.

(h) The magnitude of the vector in (f) is $d_1 d_2$.

(i) Since $d_1 \times (\vec{d}_2 / 4) = (d_1 d_2 / 4)\hat{\mathrm{k}}$, the magnitude is $d_1 d_2 / 4$.

(j) The direction of $\vec{d}_1 \times (\vec{d}_2 / 4) = (d_1 d_2 / 4)\hat{\mathrm{k}}$ is in the +*z*-direction.

57. The three vectors are

$$\vec{d}_1 = -3.0\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}$$
$$\vec{d}_2 = -2.0\,\hat{\mathrm{i}} - 4.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}$$
$$\vec{d}_3 = 2.0\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} + 1.0\,\hat{\mathrm{k}}.$$

(a) Since $\vec{d}_2 + \vec{d}_3 = 0\,\hat{\mathrm{i}} - 1.0\,\hat{\mathrm{j}} + 3.0\,\hat{\mathrm{k}}$, we have

$$\begin{aligned}\vec{d}_1 \cdot (\vec{d}_2 + \vec{d}_3) &= (-3.0\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}) \cdot (0\,\hat{\mathrm{i}} - 1.0\,\hat{\mathrm{j}} + 3.0\,\hat{\mathrm{k}}) \\ &= 0 - 3.0 + 6.0 = 3.0\ \mathrm{m}^2.\end{aligned}$$

(b) Using Eq. 3-30, we obtain $\vec{d}_2 \times \vec{d}_3 = -10\,\hat{\mathrm{i}} + 6.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}.$ Thus,

$$\begin{aligned}\vec{d}_1 \cdot (\vec{d}_2 \times \vec{d}_3) &= (-3.0\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}) \cdot (-10\,\hat{\mathrm{i}} + 6.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}) \\ &= 30 + 18 + 4.0 = 52\ \mathrm{m}^3.\end{aligned}$$

(c) We found $\vec{d}_2 + \vec{d}_3$ in part (a). Use of Eq. 3-30 then leads to

$$\begin{aligned}\vec{d}_1 \times (\vec{d}_2 + \vec{d}_3) &= (-3.0\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}) \times (0\,\hat{\mathrm{i}} - 1.0\,\hat{\mathrm{j}} + 3.0\,\hat{\mathrm{k}}) \\ &= (11\,\hat{\mathrm{i}} + 9.0\,\hat{\mathrm{j}} + 3.0\,\hat{\mathrm{k}}\,)\ \mathrm{m}^2\end{aligned}$$

58. We choose $+x$ east and $+y$ north and measure all angles in the "standard" way (positive ones counterclockwise from $+x$, negative ones clockwise). Thus, vector $\vec{d}_1$ has magnitude d_1 = 3.66 (with the unit meter and three significant figures assumed) and direction $\theta_1 = 90°$. Also, $\vec{d}_2$ has magnitude d_2 = 1.83 and direction $\theta_2 = -45°$, and vector $\vec{d}_3$ has magnitude d_3 = 0.91 and direction $\theta_3 = -135°$. We add the x and y components, respectively:

$$x: \; d_1 \cos\theta_1 + d_2 \cos\theta_2 + d_3 \cos\theta_3 = 0.65 \text{ m}$$
$$y: \; d_1 \sin\theta_1 + d_2 \sin\theta_2 + d_3 \sin\theta_3 = 1.7 \text{ m}.$$

(a) The magnitude of the direct displacement (the vector sum $\vec{d}_1 + \vec{d}_2 + \vec{d}_3$) is $\sqrt{(0.65 \text{ m})^2 + (1.7 \text{ m})^2} = 1.8$ m.

(b) The angle (understood in the sense described above) is $\tan^{-1}(1.7/0.65) = 69°$. That is, the first putt must aim in the direction 69° north of east.

59. The vectors can be written as $\vec{a} = a\hat{\mathrm{i}}$ and $\vec{b} = b\hat{\mathrm{j}}$ where $a,\ b > 0$.

(a) We are asked to consider

$$\frac{\vec{b}}{d} = \left(\frac{b}{d}\right)\hat{\mathrm{j}}$$

in the case $d > 0$. Since the coefficient of $\hat{\mathrm{j}}$ is positive, then the vector points in the $+y$ direction.

(b) If, however, $d < 0$, then the coefficient is negative and the vector points in the $-y$ direction.

(c) Since cos 90° = 0, then $\vec{a} \cdot \vec{b} = 0$, using Eq. 3-20.

(d) Since $\vec{b}/d$ is along the y axis, then (by the same reasoning as in the previous part) $\vec{a} \cdot (\vec{b}/d) = 0$.

(e) By the right-hand rule, $\vec{a} \times \vec{b}$ points in the $+z$-direction.

(f) By the same rule, $\vec{b} \times \vec{a}$ points in the $-z$-direction. We note that $\vec{b} \times \vec{a} = -\vec{a} \times \vec{b}$ is true in this case and quite generally.

(g) Since sin 90° = 1, Eq. 3-27 gives $|\vec{a} \times \vec{b}| = ab$ where a is the magnitude of $\vec{a}$.

(h) Also, $|\vec{a} \times \vec{b}| = |\vec{b} \times \vec{a}| = ab$.

(i) With $d > 0$, we find that $\vec{a} \times (\vec{b}/d)$ has magnitude ab/d.

(j) The vector $\vec{a} \times (\vec{b}/d)$ points in the $+z$ direction.

60. The vector can be written as $\vec{d} = (2.5\ \text{m})\hat{\text{j}}$, where we have taken $\hat{\text{j}}$ to be the unit vector pointing north.

(a) The magnitude of the vector $\vec{a} = 4.0\,\vec{d}$ is (4.0)(2.5 m) = 10 m.

(b) The direction of the vector $\vec{a} = 4.0\vec{d}$ is the same as the direction of $\vec{d}$ (north).

(c) The magnitude of the vector $\vec{c} = -3.0\vec{d}$ is (3.0)(2.5 m) = 7.5 m.

(d) The direction of the vector $\vec{c} = -3.0\vec{d}$ is the opposite of the direction of $\vec{d}$. Thus, the direction of $\vec{c}$ is south.

61. We note that the set of choices for unit vector directions has correct orientation (for a right-handed coordinate system). Students sometimes confuse "north" with "up", so it might be necessary to emphasize that these are being treated as the mutually perpendicular directions of our real world, not just some "on the paper" or "on the blackboard" representation of it. Once the terminology is clear, these questions are basic to the definitions of the scalar (dot) and vector (cross) products.

(a) $\hat{\mathrm{i}} \cdot \hat{\mathrm{k}} = 0$ since $\hat{\mathrm{i}} \perp \hat{\mathrm{k}}$

(b) $(-\hat{\mathrm{k}}) \cdot (-\hat{\mathrm{j}}) = 0$ since $\hat{\mathrm{k}} \perp \hat{\mathrm{j}}$.

(c) $\hat{\mathrm{j}} \cdot (-\hat{\mathrm{j}}) = -1$.

(d) $\hat{\mathrm{k}} \times \hat{\mathrm{j}} = -\hat{\mathrm{i}}$ (west).

(e) $(-\hat{\mathrm{i}}) \times (-\hat{\mathrm{j}}) = +\hat{\mathrm{k}}$ (upward).

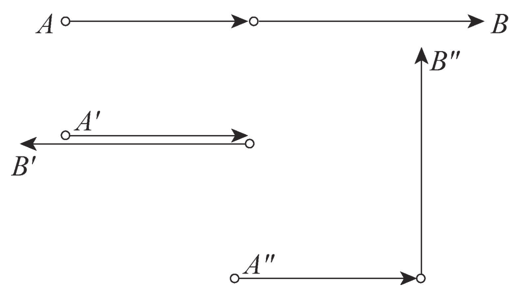
(f) $(-\hat{\mathrm{k}}) \times (-\hat{\mathrm{j}}) = -\hat{\mathrm{i}}$ (west).

62. (a) The vectors should be parallel to achieve a resultant 7 m long (the unprimed case shown below),

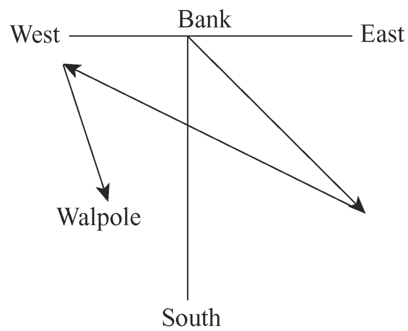
(b) anti-parallel (in opposite directions) to achieve a resultant 1 m long (primed case shown),

(c) and perpendicular to achieve a resultant $\sqrt{3^2+4^2}=5\,\text{m}$ long (the double-primed case shown).

In each sketch, the vectors are shown in a "head-to-tail" sketch but the resultant is not shown. The resultant would be a straight line drawn from beginning to end; the beginning is indicated by A (with or without primes, as the case may be) and the end is indicated by B.



63. A sketch of the displacements is shown. The resultant (not shown) would be a straight line from start (Bank) to finish (Walpole). With a careful drawing, one should find that the resultant vector has length 29.5 km at 35° west of south.



64. The point P is displaced vertically by $2R$, where R is the radius of the wheel. It is displaced horizontally by half the circumference of the wheel, or πR. Since $R = 0.450$ m, the horizontal component of the displacement is 1.414 m and the vertical component of the displacement is 0.900 m. If the x axis is horizontal and the y axis is vertical, the vector displacement (in meters) is $\vec{r} = \left(1.414\ \hat{\mathrm{i}} + 0.900\ \hat{\mathrm{j}}\right)$. The displacement has a magnitude of

$$|\vec{r}| = \sqrt{(\pi R)^2 + (2R)^2} = R\sqrt{\pi^2 + 4} = 1.68\,\mathrm{m}$$

and an angle of

$$\tan^{-1}\left(\frac{2R}{\pi R}\right) = \tan^{-1}\left(\frac{2}{\pi}\right) = 32.5°$$

above the floor. In physics there are no “exact” measurements, yet that angle computation seemed to yield something *exact*. However, there has to be some uncertainty in the observation that the wheel rolled half of a revolution, which introduces some indefiniteness in our result.

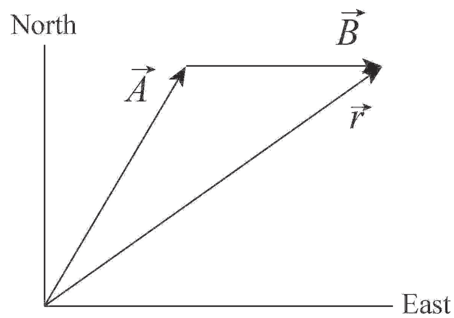
65. Reference to Figure 3-18 (and the accompanying material in that section) is helpful. If we convert $\vec{B}$ to the magnitude-angle notation (as $\vec{A}$ already is) we have $\vec{B} = (14.4 \angle 33.7°)$ (appropriate notation especially if we are using a vector capable calculator in polar mode). Where the length unit is not displayed in the solution, the unit meter should be understood. In the magnitude-angle notation, rotating the axis by +20° amounts to subtracting that angle from the angles previously specified. Thus, $\vec{A} = (12.0 \angle 40.0°)'$ and $\vec{B} = (14.4 \angle 13.7°)'$, where the 'prime' notation indicates that the description is in terms of the new coordinates. Converting these results to (x, y) representations, we obtain

(a) $\vec{A} = (9.19 \text{ m})\,\hat{\text{i}}' + (7.71 \text{ m})\,\hat{\text{j}}'$.

(b) Similarly, $\vec{B} = (14.0 \text{ m})\,\hat{\text{i}}' + (3.41 \text{ m})\,\hat{\text{j}}'$.

66. The diagram shows the displacement vectors for the two segments of her walk, labeled $\vec{A}$ and $\vec{B}$, and the total ("final") displacement vector, labeled $\vec{r}$. We take east to be the $+x$ direction and north to be the $+y$ direction. We observe that the angle between $\vec{A}$ and the x axis is 60°. Where the units are not explicitly shown, the distances are understood to be in meters. Thus, the components of $\vec{A}$ are $A_x = 250 \cos 60° = 125$ and $A_y = 250 \sin 60° = 216.5$. The components of $\vec{B}$ are $B_x = 175$ and $B_y = 0$. The components of the total displacement are

$$r_x = A_x + B_x = 125 + 175 = 300$$
$$r_y = A_y + B_y = 216.5 + 0 = 216.5.$$



(a) The magnitude of the resultant displacement is

$$|\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{(300 \text{ m})^2 + (216.5 \text{ m})^2} = 370 \text{ m}.$$

(b) The angle the resultant displacement makes with the $+x$ axis is

$$\tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{216.5 \text{ m}}{300 \text{ m}}\right) = 36°.$$

The direction is 36° north of due east.

(c) The total *distance* walked is $d = 250 \text{ m} + 175 \text{ m} = 425 \text{ m}$.

(d) The total distance walked is greater than the magnitude of the resultant displacement. The diagram shows why: $\vec{A}$ and $\vec{B}$ are not collinear.

67. The three vectors given are

$$\begin{aligned}\vec{a} &= 5.0\,\hat{\mathrm{i}} + 4.0\,\hat{\mathrm{j}} - 6.0\,\hat{\mathrm{k}}\\ \vec{b} &= -2.0\,\hat{\mathrm{i}} + 2.0\,\hat{\mathrm{j}} + 3.0\,\hat{\mathrm{k}}\\ \vec{c} &= 4.0\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}\end{aligned}$$

(a) The vector equation $\vec{r} = \vec{a} - \vec{b} + \vec{c}$ is

$$\begin{aligned}\vec{r} &= [5.0 - (-2.0) + 4.0]\hat{\mathrm{i}} + (4.0 - 2.0 + 3.0)\hat{\mathrm{j}} + (-6.0 - 3.0 + 2.0)\hat{\mathrm{k}}\\ &= 11\,\hat{\mathrm{i}} + 5.0\hat{\mathrm{j}} - 7.0\hat{\mathrm{k}}.\end{aligned}$$

(b) We find the angle from $+z$ by "dotting" (taking the scalar product) $\vec{r}$ with $\hat{\mathrm{k}}$. Noting that $r = |\vec{r}\,| = \sqrt{(11.0)^2 + (5.0)^2 + (-7.0)^2} = 14,$ Eq. 3-20 with Eq. 3-23 leads to

$$\vec{r} \cdot \vec{\mathrm{k}} = -7.0 = (14)(1)\cos\phi \quad \Rightarrow \quad \phi = 120°.$$

(c) To find the component of a vector in a certain direction, it is efficient to "dot" it (take the scalar product of it) with a unit-vector in that direction. In this case, we make the desired unit-vector by

$$\hat{b} = \frac{\vec{b}}{|\vec{b}\,|} = \frac{-2.0\hat{\mathrm{i}} + 2.0\hat{\mathrm{j}} + 3.0\hat{\mathrm{k}}}{\sqrt{(-2.0)^2 + (2.0)^2 + (3.0)^2}}.$$

We therefore obtain

$$a_b = \vec{a} \cdot \hat{b} = \frac{(5.0)(-2.0) + (4.0)(2.0) + (-6.0)(3.0)}{\sqrt{(-2.0)^2 + (2.0)^2 + (3.0)^2}} = -4.9.$$

(d) One approach (if all we require is the magnitude) is to use the vector cross product, as the problem suggests; another (which supplies more information) is to subtract the result in part (c) (multiplied by $\hat{b}$) from $\vec{a}$. We briefly illustrate both methods. We note that if $a \cos\theta$ (where θ is the angle between $\vec{a}$ and $\vec{b}$) gives a_b (the component along $\hat{b}$) then we expect $a \sin\theta$ to yield the orthogonal component:

$$a \sin\theta = \frac{|\vec{a} \times \vec{b}|}{b} = 7.3$$

(alternatively, one might compute θ form part (c) and proceed more directly). The second method proceeds as follows:

$$\begin{aligned}\vec{a} - a_b\hat{b} &= (5.0 - 2.35)\hat{\mathrm{i}} + (4.0 - (-2.35))\hat{\mathrm{j}} + ((-6.0) - (-3.53))\hat{\mathrm{k}} \\ &= 2.65\hat{\mathrm{i}} + 6.35\hat{\mathrm{j}} - 2.47\hat{\mathrm{k}}\end{aligned}$$

This describes the perpendicular part of $\vec{a}$ completely. To find the magnitude of this part, we compute

$$\sqrt{(2.65)^2 + (6.35)^2 + (-2.47)^2} = 7.3$$

which agrees with the first method.

68. The two vectors can be found be solving the simultaneous equations.

(a) If we add the equations, we obtain $2\vec{a} = 6\vec{c}$, which leads to $\vec{a} = 3\vec{c} = 9\hat{\text{i}} + 12\hat{\text{j}}$.

(b) Plugging this result back in, we find $\vec{b} = \vec{c} = 3\hat{\text{i}} + 4\hat{\text{j}}$.

69. (a) This is one example of an answer: $(-40\,\hat{\mathrm{i}} - 20\,\hat{\mathrm{j}} + 25\,\hat{\mathrm{k}})$ m, with $\hat{\mathrm{i}}$ directed anti-parallel to the first path, $\hat{\mathrm{j}}$ directed anti-parallel to the second path and $\hat{\mathrm{k}}$ directed upward (in order to have a right-handed coordinate system). Other examples are $(40\,\hat{\mathrm{i}} + 20\,\hat{\mathrm{j}} + 25\,\hat{\mathrm{k}})$ m and $(40\hat{\mathrm{i}} - 20\,\hat{\mathrm{j}} - 25\,\hat{\mathrm{k}})$ m (with slightly different interpretations for the unit vectors). Note that the product of the components is positive in each example.

(b) Using Pythagorean theorem, we have $\sqrt{(40\text{ m})^2 + (20\text{ m})^2} = 44.7\text{ m} \approx 45\text{ m}$.

70. The vector $\vec{d}$ (measured in meters) can be represented as $\vec{d} = (3.0\text{ m})(-\hat{\text{j}})$, where $-\hat{\text{j}}$ is the unit vector pointing south. Therefore,

$$5.0\vec{d} = 5.0(-3.0\text{ m }\hat{\text{j}}) = (-15\text{ m})\,\hat{\text{j}}.$$

(a) The positive scalar factor (5.0) affects the magnitude but not the direction. The magnitude of $5.0\vec{d}$ is 15 m.

(b) The new direction of $5\vec{d}$ is the same as the old: south.

The vector $-2.0\vec{d}$ can be written as $-2.0\vec{d} = (6.0\text{ m})\,\hat{\text{j}}$.

(c) The absolute value of the scalar factor (|–2.0| = 2.0) affects the magnitude. The new magnitude is 6.0 m.

(d) The minus sign carried by this scalar factor reverses the direction, so the new direction is $+\hat{\text{j}}$, or north.

71. Given: $\vec{A} + \vec{B} = 6.0\,\hat{\mathrm{i}} + 1.0\,\hat{\mathrm{j}}$ and $\vec{A} - \vec{B} = -4.0\,\hat{\mathrm{i}} + 7.0\,\hat{\mathrm{j}}$. Solving these simultaneously leads to $\vec{A} = 1.0\,\hat{\mathrm{i}} + 4.0\,\hat{\mathrm{j}}$. The Pythagorean theorem then leads to $A = \sqrt{(1.0)^2 + (4.0)^2} = 4.1$.

72. The ant's trip consists of three displacements:

$$\vec{d}_1 = (0.40 \text{ m})(\cos 225° \hat{\text{i}} + \sin 225° \hat{\text{j}}) = (-0.28 \text{ m})\hat{\text{i}} + (-0.28 \text{ m})\hat{\text{j}}$$
$$\vec{d}_2 = (0.50 \text{ m})\hat{\text{i}}$$
$$\vec{d}_3 = (0.60 \text{ m})(\cos 60° \hat{\text{i}} + \sin 60° \hat{\text{j}}) = (0.30 \text{ m})\hat{\text{i}} + (0.52 \text{ m})\hat{\text{j}},$$

where the angle is measured with respect to the positive x axis. We have taken the positive x and y directions to correspond to east and north, respectively.

(a) The x component of $\vec{d}_1$ is $d_{1x} = (0.40 \text{ m})\cos 225° = -0.28 \text{ m}$.

(b) The y component of $\vec{d}_1$ is $d_{1y} = (0.40 \text{ m})\sin 225° = -0.28 \text{ m}$.

(c) The x component of $\vec{d}_2$ is $d_{2x} = 0.50 \text{ m}$.

(d) The y component of $\vec{d}_2$ is $d_{2y} = 0 \text{ m}$.

(e) The x component of $\vec{d}_3$ is $d_{3x} = (0.60 \text{ m})\cos 60° = 0.30 \text{ m}$.

(f) The y component of $\vec{d}_3$ is $d_{3y} = (0.60 \text{ m})\sin 60° = 0.52 \text{ m}$.

(g) The x component of the net displacement $\vec{d}_{net}$ is

$$d_{net,x} = d_{1x} + d_{2x} + d_{3x} = (-0.28 \text{ m}) + (0.50 \text{ m}) + (0.30 \text{ m}) = 0.52 \text{ m}.$$

(h) The y component of the net displacement $\vec{d}_{net}$ is

$$d_{net,y} = d_{1y} + d_{2y} + d_{3y} = (-0.28 \text{ m}) + (0 \text{ m}) + (0.52 \text{ m}) = 0.24 \text{ m}.$$

(i) The magnitude of the net displacement is

$$d_{net} = \sqrt{d_{net,x}^2 + d_{net,y}^2} = \sqrt{(0.52 \text{ m})^2 + (0.24 \text{ m})^2} = 0.57 \text{ m}.$$

(j) The direction of the net displacement is

$$\theta = \tan^{-1}\left(\frac{d_{net,y}}{d_{net,x}}\right) = \tan^{-1}\left(\frac{0.24 \text{ m}}{0.52 \text{ m}}\right) = 25° \text{ (north of east)}$$

If the ant has to return directly to the starting point, the displacement would be $-\vec{d}_{net}$.
(k) The distance the ant has to travel is $|-\vec{d}_{net}| = 0.57$ m.

(l) The direction the ant has to travel is $25°$ (south of west).

Chapter 4

1. The initial position vector $\vec{r}_\text{o}$ satisfies $\vec{r} - \vec{r}_\text{o} = \Delta\vec{r}$, which results in

$$\vec{r}_\text{o} = \vec{r} - \Delta\vec{r} = (3.0\hat{\text{j}} - 4.0\hat{\text{k}})\text{m} - (2.0\hat{\text{i}} - 3.0\hat{\text{j}} + 6.0\,\hat{\text{k}})\text{m} = (-2.0\text{ m})\,\hat{\text{i}} + (6.0\text{ m})\,\hat{\text{j}} + (-10\text{ m})\,\hat{\text{k}}\,.$$

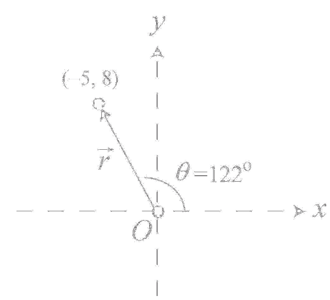
2. (a) The position vector, according to Eq. 4-1, is $\vec{r} = (-5.0 \text{ m})\,\hat{\text{i}} + (8.0 \text{ m})\hat{\text{j}}$.

(b) The magnitude is $|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = \sqrt{(-5.0 \text{ m})^2 + (8.0 \text{ m})^2 + (0 \text{ m})^2} = 9.4 \text{ m}$.

(c) Many calculators have polar ↔ rectangular conversion capabilities which make this computation more efficient than what is shown below. Noting that the vector lies in the *xy* plane and using Eq. 3-6, we obtain:

$$\theta = \tan^{-1}\left(\frac{8.0 \text{ m}}{-5.0 \text{ m}}\right) = -58° \text{ or } 122°$$

where the latter possibility (122° measured counterclockwise from the +*x* direction) is chosen since the signs of the components imply the vector is in the second quadrant.



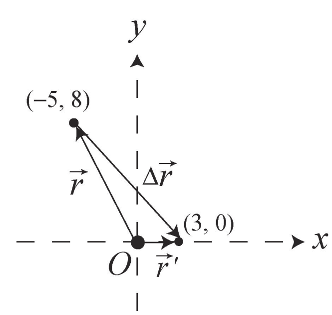
(d) The sketch is shown on the right. The vector is 122° counterclockwise from the +*x* direction.

(e) The displacement is $\Delta\vec{r} = \vec{r}\,' - \vec{r}$ where $\vec{r}$ is given in part (a) and $\vec{r}\,' = (3.0 \text{ m})\hat{\text{i}}$. Therefore, $\Delta\vec{r} = (8.0 \text{ m})\hat{\text{i}} - (8.0 \text{ m})\hat{\text{j}}$.

(f) The magnitude of the displacement is $|\Delta\vec{r}| = \sqrt{(8.0 \text{ m})^2 + (-8.0 \text{ m})^2} = 11 \text{ m}$.

(g) The angle for the displacement, using Eq. 3-6, is

$$\tan^{-1}\left(\frac{8.0 \text{ m}}{-8.0 \text{ m}}\right) = -45° \text{ or } 135°$$

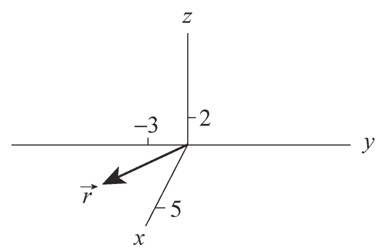


where we choose the former possibility (–45°, or 45° measured *clockwise* from +*x*) since the signs of the components imply the vector is in the fourth quadrant. A sketch of $\Delta\vec{r}$ is shown on the right.

3. (a) The magnitude of $\vec{r}$ is

$$|\vec{r}| = \sqrt{(5.0\text{ m})^2 + (-3.0\text{ m})^2 + (2.0\text{ m})^2} = 6.2\text{ m}.$$

(b) A sketch is shown. The coordinate values are in meters.



4. We choose a coordinate system with origin at the clock center and $+x$ rightward (towards the "3:00" position) and $+y$ upward (towards "12:00").

(a) In unit-vector notation, we have $\vec{r}_1 = (10\text{ cm})\hat{\text{i}}$ and $\vec{r}_2 = (-10\text{ cm})\hat{\text{j}}$. Thus, Eq. 4-2 gives

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (-10\text{ cm})\hat{\text{i}} + (-10\text{ cm})\hat{\text{j}}.$$

and the magnitude is given by $|\Delta\vec{r}| = \sqrt{(-10\text{ cm})^2 + (-10\text{ cm})^2} = 14\text{ cm}$.

(b) Using Eq. 3-6, the angle is

$$\theta = \tan^{-1}\left(\frac{-10\text{ cm}}{-10\text{ cm}}\right) = 45° \text{ or } -135°.$$

We choose $-135°$ since the desired angle is in the third quadrant. In terms of the magnitude-angle notation, one may write

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (-10\text{ cm})\hat{\text{i}} + (-10\text{ cm})\hat{\text{j}} \rightarrow (14\text{cm}\angle -135°).$$

(c) In this case, we have $\vec{r}_1 = (-10\text{ cm})\hat{\text{j}}$ and $\vec{r}_2 = (10\text{ cm})\hat{\text{j}}$, and $\Delta\vec{r} = (20\text{ cm})\hat{\text{j}}$. Thus, $|\Delta\vec{r}| = 20\text{ cm}$.

(d) Using Eq. 3-6, the angle is given by

$$\theta = \tan^{-1}\left(\frac{20\text{ cm}}{0\text{ cm}}\right) = 90°.$$

(e) In a full-hour sweep, the hand returns to its starting position, and the displacement is zero.

(f) The corresponding angle for a full-hour sweep is also zero.

5. Using Eq. 4-3 and Eq. 4-8, we have

$$\vec{v}_{\text{avg}} = \frac{(-2.0\hat{\text{i}} + 8.0\hat{\text{j}} - 2.0\hat{\text{k}})\ \text{m} - (5.0\hat{\text{i}} - 6.0\hat{\text{j}} + 2.0\hat{\text{k}})\ \text{m}}{10\ \text{s}} = (-0.70\hat{\text{i}} + 1.40\hat{\text{j}} - 0.40\hat{\text{k}})\ \text{m/s}.$$

6. To emphasize the fact that the velocity is a function of time, we adopt the notation $v(t)$ for dx/dt.

(a) Eq. 4-10 leads to

$$v(t) = \frac{d}{dt}\,(3.00t\hat{\text{i}} - 4.00t^2\hat{\text{j}} + 2.00\hat{\text{k}}) = (3.00 \text{ m/s})\hat{\text{i}} - (8.00t \text{ m/s})\,\hat{\text{j}}$$

(b) Evaluating this result at $t = 2.00$ s produces $\vec{v} = (3.00\hat{\text{i}} - 16.0\hat{\text{j}})$ m/s.

(c) The speed at $t = 2.00$ s is $v = |\vec{v}| = \sqrt{(3.00 \text{ m/s})^2 + (-16.0 \text{ m/s})^2} = 16.3$ m/s.

(d) The angle of $\vec{v}$ at that moment is

$$\tan^{-1}\left(\frac{-16.0 \text{ m/s}}{3.00 \text{ m/s}}\right) = -79.4° \text{ or } 101°$$

where we choose the first possibility (79.4° measured *clockwise* from the $+x$ direction, or 281° counterclockwise from $+x$) since the signs of the components imply the vector is in the fourth quadrant.

7. The average velocity is given by Eq. 4-8. The total displacement $\Delta\vec{r}$ is the sum of three displacements, each result of a (constant) velocity during a given time. We use a coordinate system with +*x* East and +*y* North.

(a) In unit-vector notation, the first displacement is given by

$$\Delta\vec{r}_1 = \left(60.0\ \frac{\text{km}}{\text{h}}\right)\left(\frac{40.0\ \text{min}}{60\ \text{min/h}}\right)\hat{\text{i}} = (40.0\ \text{km})\hat{\text{i}}.$$

The second displacement has a magnitude of $(60.0\ \frac{\text{km}}{\text{h}})\cdot(\frac{20.0\ \text{min}}{60\ \text{min/h}}) = 20.0\ \text{km}$, and its direction is 40° north of east. Therefore,

$$\Delta\vec{r}_2 = (20.0\ \text{km})\cos(40.0°)\,\hat{\text{i}} + (20.0\ \text{km})\sin(40.0°)\,\hat{\text{j}} = (15.3\ \text{km})\,\hat{\text{i}} + (12.9\ \text{km})\,\hat{\text{j}}.$$

And the third displacement is

$$\Delta\vec{r}_3 = -\left(60.0\ \frac{\text{km}}{\text{h}}\right)\left(\frac{50.0\ \text{min}}{60\ \text{min/h}}\right)\hat{\text{i}} = (-50.0\ \text{km})\,\hat{\text{i}}.$$

The total displacement is

$$\begin{aligned}\Delta\vec{r} &= \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 = (40.0\ \text{km})\hat{\text{i}} + (15.3\ \text{km})\,\hat{\text{i}} + (12.9\ \text{km})\,\hat{\text{j}} - (50.0\ \text{km})\,\hat{\text{i}} \\ &= (5.30\ \text{km})\,\hat{\text{i}} + (12.9\ \text{km})\,\hat{\text{j}}.\end{aligned}$$

The time for the trip is (40.0 + 20.0 + 50.0) min = 110 min, which is equivalent to 1.83 h. Eq. 4-8 then yields

$$\vec{v}_{\text{avg}} = \left(\frac{5.30\ \text{km}}{1.83\ \text{h}}\right)\hat{\text{i}} + \left(\frac{12.9\ \text{km}}{1.83\ \text{h}}\right)\hat{\text{j}} = (2.90\ \text{km/h})\,\hat{\text{i}} + (7.01\ \text{km/h})\,\hat{\text{j}}.$$

The magnitude is

$$|\vec{v}_{\text{avg}}| = \sqrt{(2.90\ \text{km/h})^2 + (7.01\ \text{km/h})^2} = 7.59\ \text{km/h}.$$

(b) The angle is given by

$$\theta = \tan^{-1}\left(\frac{7.01\ \text{km/h}}{2.90\ \text{km/h}}\right) = 67.5°\quad (\text{north of east}),$$

or $22.5°$ east of due north.

8. Our coordinate system has $\hat{\text{i}}$ pointed east and $\hat{\text{j}}$ pointed north. The first displacement is $\vec{r}_{AB} = (483 \text{ km})\hat{\text{i}}$ and the second is $\vec{r}_{BC} = (-966 \text{ km})\hat{\text{j}}$.

(a) The net displacement is

$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC} = (483 \text{ km})\hat{\text{i}} - (966 \text{ km})\hat{\text{j}}$$

which yields $|\vec{r}_{AC}| = \sqrt{(483 \text{ km})^2 + (-966 \text{ km})^2} = 1.08 \times 10^3 \text{ km}$.

(b) The angle is given by

$$\theta = \tan^{-1}\left(\frac{-966 \text{ km}}{483 \text{ km}}\right) = -63.4°.$$

We observe that the angle can be alternatively expressed as 63.4° south of east, or 26.6° east of south.

(c) Dividing the magnitude of $\vec{r}_{AC}$ by the total time (2.25 h) gives

$$\vec{v}_{\text{avg}} = \frac{(483 \text{ km})\hat{\text{i}} - (966 \text{ km})\hat{\text{j}}}{2.25 \text{ h}} = (215 \text{ km/h})\hat{\text{i}} - (429 \text{ km/h})\hat{\text{j}}.$$

with a magnitude $|\vec{v}_{\text{avg}}| = \sqrt{(215 \text{ km/h})^2 + (-429 \text{ km/h})^2} = 480 \text{ km/h}$.

(d) The direction of $\vec{v}_{\text{avg}}$ is 26.6° east of south, same as in part (b). In magnitude-angle notation, we would have $\vec{v}_{\text{avg}} = (480 \text{ km/h} \angle -63.4°)$.

(e) Assuming the *AB* trip was a straight one, and similarly for the *BC* trip, then $|\vec{r}_{AB}|$ is the distance traveled during the *AB* trip, and $|\vec{r}_{BC}|$ is the distance traveled during the *BC* trip. Since the average speed is the total distance divided by the total time, it equals

$$\frac{483 \text{ km} + 966 \text{ km}}{2.25 \text{ h}} = 644 \text{ km/h}.$$

9. The (x,y) coordinates (in meters) of the points are $A = (15, -15)$, $B = (30, -45)$, $C = (20, -15)$, and $D = (45, 45)$. The respective times are $t_A = 0$, $t_B = 300$ s, $t_C = 600$ s, and $t_D = 900$ s. Average velocity is defined by Eq. 4-8. Each displacement $\Delta\vec{r}$ is understood to originate at point A.

(a) The average velocity having the least magnitude (5.0 m/600 s) is for the displacement ending at point C: $|\vec{v}_{avg}| = 0.0083$ m/s.

(b) The direction of $\vec{v}_{avg}$ is 0° (measured counterclockwise from the $+x$ axis).

(c) The average velocity having the greatest magnitude ($\sqrt{(15\text{ m})^2 + (30\text{ m})^2}/300\text{ s}$) is for the displacement ending at point B: $|\vec{v}_{avg}| = 0.11\text{ m/s}$.

(d) The direction of $\vec{v}_{avg}$ is 297° (counterclockwise from $+x$) or −63° (which is equivalent to measuring 63° *clockwise* from the $+x$ axis).

10. We differentiate $\vec{r} = 5.00t\,\hat{\mathrm{i}} + (et + ft^2)\hat{\mathrm{j}}$.

(a) The particle's motion is indicated by the derivative of $\vec{r}$: $\vec{v} = 5.00\,\hat{\mathrm{i}} + (e + 2ft)\,\hat{\mathrm{j}}$. The angle of its direction of motion is consequently

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}[(e + 2ft)/5.00].$$

The graph indicates $\theta_o = 35.0°$ which determines the parameter e:

$$e = (5.00 \text{ m/s}) \tan(35.0°) = 3.50 \text{ m/s}.$$

(b) We note (from the graph) that $\theta = 0$ when $t = 14.0$ s. Thus, $e + 2ft = 0$ at that time. This determines the parameter f:

$$f = \frac{-e}{2t} = \frac{-3.5 \text{ m/s}}{2(14.0 \text{ s})} = -0.125 \text{ m/s}^2 .$$

11. We apply Eq. 4-10 and Eq. 4-16.

(a) Taking the derivative of the position vector with respect to time, we have, in SI units (m/s),

$$\vec{v} = \frac{d}{dt}(\hat{\mathrm{i}} + 4t^2\,\hat{\mathrm{j}} + t\,\hat{\mathrm{k}}) = 8t\,\hat{\mathrm{j}} + \hat{\mathrm{k}}\ .$$

(b) Taking another derivative with respect to time leads to, in SI units (m/s^2),

$$\vec{a} = \frac{d}{dt}\,(8t\,\hat{\mathrm{j}} + \hat{\mathrm{k}}) = 8\,\hat{\mathrm{j}}\ .$$

12. We use Eq. 4-15 with $\vec{v}_1$ designating the initial velocity and $\vec{v}_2$ designating the later one.

(a) The average acceleration during the $\Delta t = 4$ s interval is

$$\vec{a}_{\text{avg}} = \frac{(-2.0\,\hat{\text{i}} - 2.0\,\hat{\text{j}} + 5.0\,\hat{\text{k}})\text{ m/s} - (4.0\,\hat{\text{i}} - 22\,\hat{\text{j}} + 3.0\,\hat{\text{k}})\text{ m/s}}{4\text{ s}} = (-1.5\text{ m/s}^2)\,\hat{\text{i}} + (0.5\,\text{m/s}^2)\,\hat{\text{k}}.$$

(b) The magnitude of $\vec{a}_{\text{avg}}$ is $\sqrt{(-1.5\,\text{m/s}^2)^2 + (0.5\text{ m/s}^2)^2} = 1.6\,\text{m/s}^2$.

(c) Its angle in the *xz* plane (measured from the +*x* axis) is one of these possibilities:

$$\tan^{-1}\left(\frac{0.5\text{ m/s}^2}{-1.5\text{ m/s}^2}\right) = -18^\circ \text{ or } 162^\circ$$

where we settle on the second choice since the signs of its components imply that it is in the second quadrant.

13. In parts (b) and (c), we use Eq. 4-10 and Eq. 4-16. For part (d), we find the direction of the velocity computed in part (b), since that represents the asked-for tangent line.

(a) Plugging into the given expression, we obtain

$$\vec{r}\Big|_{t=2.00} = [2.00(8) - 5.00(2)]\hat{\mathrm{i}} + [6.00 - 7.00(16)]\hat{\mathrm{j}} = (6.00\,\hat{\mathrm{i}} - 106\,\hat{\mathrm{j}})\ \mathrm{m}$$

(b) Taking the derivative of the given expression produces

$$\vec{v}(t) = (6.00t^2 - 5.00)\,\hat{\mathrm{i}} - 28.0t^3\,\hat{\mathrm{j}}$$

where we have written $v(t)$ to emphasize its dependence on time. This becomes, at $t = 2.00$ s, $\vec{v} = (19.0\,\hat{\mathrm{i}} - 224\,\hat{\mathrm{j}})$ m/s.

(c) Differentiating the $\vec{v}(t)$ found above, with respect to t produces $12.0t\,\hat{\mathrm{i}} - 84.0t^2\,\hat{\mathrm{j}}$, which yields $\vec{a} = (24.0\,\hat{\mathrm{i}} - 336\,\hat{\mathrm{j}})$ m/s^2 at $t = 2.00$ s.

(d) The angle of $\vec{v}$, measured from $+x$, is either

$$\tan^{-1}\left(\frac{-224\ \mathrm{m/s}}{19.0\ \mathrm{m/s}}\right) = -85.2^\circ \ \text{ or } \ 94.8^\circ$$

where we settle on the first choice (–85.2°, which is equivalent to 275° measured counterclockwise from the $+x$ axis) since the signs of its components imply that it is in the fourth quadrant.

14. We adopt a coordinate system with $\hat{\mathrm{i}}$ pointed east and $\hat{\mathrm{j}}$ pointed north; the coordinate origin is the flagpole. We "translate" the given information into unit-vector notation as follows:

$$\vec{r}_o = (40.0\text{ m})\hat{\mathrm{i}} \quad \text{and} \quad \vec{v}_o = (-10.0\text{ m/s})\hat{\mathrm{j}}$$
$$\vec{r} = (40.0\text{ m})\hat{\mathrm{j}} \quad \text{and} \quad \vec{v} = (10.0\text{ m/s})\hat{\mathrm{i}}.$$

(a) Using Eq. 4-2, the displacement $\Delta\vec{r}$ is

$$\Delta\vec{r} = \vec{r} - \vec{r}_o = (-40.0\text{ m})\hat{\mathrm{i}} + (40.0\text{ m})\hat{\mathrm{j}}.$$

with a magnitude $|\Delta\vec{r}| = \sqrt{(-40.0\text{ m})^2 + (40.0\text{ m})^2} = 56.6\text{ m}$.

(b) The direction of $\Delta\vec{r}$ is

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{40.0\text{ m}}{-40.0\text{ m}}\right) = -45.0° \text{ or } 135°.$$

Since the desired angle is in the second quadrant, we pick $135°$ ($45°$ north of due west). Note that the displacement can be written as $\Delta\vec{r} = \vec{r} - \vec{r}_o = (56.6\angle 135°)$ in terms of the magnitude-angle notation.

(c) The magnitude of $\vec{v}_{\text{avg}}$ is simply the magnitude of the displacement divided by the time (Δt = 30.0 s). Thus, the average velocity has magnitude (56.6 m)/(30.0 s) = 1.89 m/s.

(d) Eq. 4-8 shows that $\vec{v}_{\text{avg}}$ points in the same direction as $\Delta\vec{r}$, i.e, $135°$ ($45°$ north of due west).

(e) Using Eq. 4-15, we have

$$\vec{a}_{\text{avg}} = \frac{\vec{v} - \vec{v}_o}{\Delta t} = (0.333\text{ m/s}^2)\hat{\mathrm{i}} + (0.333\text{ m/s}^2)\hat{\mathrm{j}}.$$

The magnitude of the average acceleration vector is therefore equal to $|\vec{a}_{\text{avg}}| = \sqrt{(0.333\text{ m/s}^2)^2 + (0.333\text{ m/s}^2)^2} = 0.471\text{ m/s}^2$.

(f) The direction of $\vec{a}_{\text{avg}}$ is

$$\theta = \tan^{-1}\left(\frac{0.333\text{ m/s}^2}{0.333\text{ m/s}^2}\right) = 45° \text{ or } -135°.$$

Since the desired angle is now in the first quadrant, we choose $45°$, and $\vec{a}_{avg}$ points north of due east.

15. We find t by applying Eq. 2-11 to motion along the y axis (with $v_y = 0$ characterizing $y = y_{\max}$):

$$0 = (12 \text{ m/s}) + (-2.0 \text{ m/s}^2)t \quad \Rightarrow \quad t = 6.0 \text{ s}.$$

Then, Eq. 2-11 applies to motion along the x axis to determine the answer:

$$v_x = (8.0 \text{ m/s}) + (4.0 \text{ m/s}^2)(6.0 \text{ s}) = 32 \text{ m/s}.$$

Therefore, the velocity of the cart, when it reaches $y = y_{\max}$, is $(32 \text{ m/s})\hat{\mathrm{i}}$.

16. We find t by solving $\Delta x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$:

$$12.0\ \text{m} = 0 + (4.00\ \text{m/s})t + \frac{1}{2}(5.00\ \text{m/s}^2)t^2$$

where we have used Δx = 12.0 m, v_x = 4.00 m/s, and a_x = 5.00 m/s^2. We use the quadratic formula and find t = 1.53 s. Then, Eq. 2-11 (actually, its analog in two dimensions) applies with this value of t. Therefore, its velocity (when Δx = 12.00 m) is

$$\begin{aligned}\vec{v} = \vec{v}_0 + \vec{a}t &= (4.00\ \text{m/s})\hat{\mathrm{i}} + (5.00\ \text{m/s}^2)(1.53\ \text{s})\hat{\mathrm{i}} + (7.00\ \text{m/s}^2)(1.53\ \text{s})\hat{\mathrm{j}} \\ &= (11.7\ \text{m/s})\,\hat{\mathrm{i}} + (10.7\ \text{m/s})\,\hat{\mathrm{j}}.\end{aligned}$$

Thus, the magnitude of $\vec{v}$ is $|\vec{v}| = \sqrt{(11.7\ \text{m/s})^2 + (10.7\ \text{m/s})^2} = 15.8\ \text{m/s}$.

(b) The angle of $\vec{v}$, measured from $+x$, is

$$\tan^{-1}\left(\frac{10.7\ \text{m/s}}{11.7\ \text{m/s}}\right) = 42.6°.$$

17. Constant acceleration in both directions (x and y) allows us to use Table 2-1 for the motion along each direction. This can be handled individually (for Δx and Δy) or together with the unit-vector notation (for Δr). Where units are not shown, SI units are to be understood.

(a) The velocity of the particle at any time t is given by $\vec{v} = \vec{v}_0 + \vec{a}t$, where $\vec{v}_0$ is the initial velocity and $\vec{a}$ is the (constant) acceleration. The x component is $v_x = v_{0x} + a_x t = 3.00 - 1.00t$, and the y component is

$$v_y = v_{0y} + a_y t = -0.500t$$

since $v_{0y} = 0$. When the particle reaches its maximum x coordinate at $t = t_m$, we must have $v_x = 0$. Therefore, $3.00 - 1.00t_m = 0$ or $t_m = 3.00$ s. The y component of the velocity at this time is

$$v_y = 0 - 0.500(3.00) = -1.50 \text{ m/s};$$

this is the only nonzero component of $\vec{v}$ at t_m.

(b) Since it started at the origin, the coordinates of the particle at any time t are given by $\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$. At $t = t_m$ this becomes

$$\vec{r} = \left(3.00\hat{\text{i}}\right)(3.00) + \frac{1}{2}\left(-1.00\,\hat{\text{i}} - 0.50\,\hat{\text{j}}\right)(3.00)^2 = (4.50\,\hat{\text{i}} - 2.25\,\hat{\text{j}}) \text{ m}.$$

18. We make use of Eq. 4-16.

(a) The acceleration as a function of time is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\left(6.0t - 4.0t^2\right)\hat{\mathrm{i}} + 8.0\,\hat{\mathrm{j}}\right) = \left(6.0 - 8.0t\right)\hat{\mathrm{i}}$$

in SI units. Specifically, we find the acceleration vector at $t = 3.0\,\mathrm{s}$ to be $\left(6.0 - 8.0(3.0)\right)\hat{\mathrm{i}} = (-18\ \mathrm{m/s^2})\hat{\mathrm{i}}.$

(b) The equation is $\vec{a} = \left(6.0 - 8.0t\right)\hat{\mathrm{i}} = 0$; we find $t = 0.75$ s.

(c) Since the y component of the velocity, $v_y = 8.0$ m/s, is never zero, the velocity cannot vanish.

(d) Since speed is the magnitude of the velocity, we have

$$v = |\,\vec{v}\,| = \sqrt{\left(6.0t - 4.0t^2\right)^2 + \left(8.0\right)^2} = 10$$

in SI units (m/s). To solve for t, we first square both sides of the above equation, followed by some rearrangement:

$$\left(6.0t - 4.0t^2\right)^2 + 64 = 100 \quad \Rightarrow \quad \left(6.0t - 4.0t^2\right)^2 = 36$$

Taking the square root of the new expression and making further simplification lead to

$$6.0t - 4.0t^2 = \pm 6.0 \quad \Rightarrow \quad 4.0t^2 - 6.0t \pm 6.0 = 0$$

Finally, using the quadratic formula, we obtain

$$t = \frac{6.0 \pm \sqrt{36 - 4\left(4.0\right)\left(\pm 6.0\right)}}{2\left(8.0\right)}$$

where the requirement of a real positive result leads to the unique answer: $t = 2.2$ s.

19. We make use of Eq. 4-16 and Eq. 4-10.

Using $\vec{a} = 3t\hat{\text{i}} + 4t\hat{\text{j}}$, we have (in m/s)

$$\vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a}\ dt = (5.00\hat{\text{i}} + 2.00\hat{\text{j}}) + \int_0^t (3t\hat{\text{i}} + 4t\hat{\text{j}})\ dt = \left(5.00 + 3t^2/2\right)\hat{\text{i}} + \left(2.00 + 2t^2\right)\hat{\text{j}}$$

Integrating using Eq. 4-10 then yields (in metes)

$$\begin{aligned}\vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}dt &= (20.0\hat{\text{i}} + 40.0\hat{\text{j}}) + \int_0^t [(5.00 + 3t^2/2)\hat{\text{i}} + (2.00 + 2t^2)\hat{\text{j}}]dt \\ &= (20.0\hat{\text{i}} + 40.0\hat{\text{j}}) + (5.00t + t^3/2)\hat{\text{i}} + (2.00t + 2t^3/3)\hat{\text{j}} \\ &= (20.0 + 5.00t + t^3/2)\hat{\text{i}} + (40.0 + 2.00t + 2t^3/3)\hat{\text{j}}\end{aligned}$$

(a) At $t = 4.00\ \text{s}$, we have $\vec{r}(t = 4.00\ \text{s}) = (72.0\ \text{m})\hat{\text{i}} + (90.7\ \text{m})\hat{\text{j}}$.

(b) $\vec{v}(t = 4.00\ \text{s}) = (29.0\ \text{m/s})\hat{\text{i}} + (34.0\ \text{m/s})\hat{\text{j}}$. Thus, the angle between the direction of travel and +x, measured counterclockwise, is $\theta = \tan^{-1}[(34.0\ \text{m/s})/(29.0\ \text{m/s})] = 49.5°$.

20. The acceleration is constant so that use of Table 2-1 (for both the x and y motions) is permitted. Where units are not shown, SI units are to be understood. Collision between particles A and B requires two things. First, the y motion of B must satisfy (using Eq. 2-15 and noting that θ is measured from the y axis)

$$y = \frac{1}{2}a_y t^2 \quad \Rightarrow \quad 30 \text{ m} = \frac{1}{2}\left[(0.40 \text{ m/s}^2)\cos\theta\right]t^2.$$

Second, the x motions of A and B must coincide:

$$vt = \frac{1}{2}a_x t^2 \quad \Rightarrow (3.0 \text{ m/s})t = \frac{1}{2}\left[(0.40 \text{ m/s}^2)\sin\theta\right]t^2.$$

We eliminate a factor of t in the last relationship and formally solve for time:

$$t = \frac{2v}{a_x} = \frac{2(3.0 \text{ m/s})}{(0.40 \text{ m/s}^2)\sin\theta}.$$

This is then plugged into the previous equation to produce

$$30 \text{ m} = \frac{1}{2}\left[(0.40 \text{ m/s}^2)\cos\theta\right]\left(\frac{2(3.0 \text{ m/s})}{(0.40 \text{ m/s}^2)\sin\theta}\right)^2$$

which, with the use of $\sin^2\theta = 1 - \cos^2\theta$, simplifies to

$$30 = \frac{9.0}{0.20}\,\frac{\cos\theta}{1-\cos^2\theta} \quad \Rightarrow \; 1-\cos^2\theta = \frac{9.0}{(0.20)(30)}\cos\theta.$$

We use the quadratic formula (choosing the positive root) to solve for $\cos\theta$:

$$\cos\theta = \frac{-1.5 + \sqrt{1.5^2 - 4(1.0)(-1.0)}}{2} = \frac{1}{2}$$

which yields $\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$.

21. (a) From Eq. 4-22 (with $\theta_0 = 0$), the time of flight is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(45.0\text{ m})}{9.80\text{ m/s}^2}} = 3.03\text{ s}.$$

(b) The horizontal distance traveled is given by Eq. 4-21:

$$\Delta x = v_0 t = (250\text{ m/s})(3.03\text{ s}) = 758\text{ m}.$$

(c) And from Eq. 4-23, we find

$$|v_y| = gt = (9.80\text{ m/s}^2)(3.03\text{ s}) = 29.7\text{ m/s}.$$

22. We use Eq. 4-26

$$R_{\max} = \left(\frac{v_0^2}{g} \sin 2\theta_0 \right)_{\max} = \frac{v_0^2}{g} = \frac{(9.50\,\mathrm{m/s})^2}{9.80\,\mathrm{m/s^2}} = 9.209\text{ m} \approx 9.21\mathrm{m}$$

to compare with Powell's long jump; the difference from $R_{\max}$ is only ΔR =(9.21m – 8.95m) = 0.259 m.

23. Using Eq. (4-26), the take-off speed of the jumper is

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{(9.80\ \text{m/s}^2)(77.0\ \text{m})}{\sin 2(12.0°)}} = 43.1\ \text{m/s}$$

24. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable.

(a) With the origin at the initial point (edge of table), the y coordinate of the ball is given by $y = -\frac{1}{2}gt^2$. If t is the time of flight and $y = -1.20$ m indicates the level at which the ball hits the floor, then

$$t = \sqrt{\frac{2(-1.20 \text{ m})}{-9.80 \text{ m/s}^2}} = 0.495 \text{s}.$$

(b) The initial (horizontal) velocity of the ball is $\vec{v} = v_0 \hat{\mathrm{i}}$. Since $x = 1.52$ m is the horizontal position of its impact point with the floor, we have $x = v_0 t$. Thus,

$$v_0 = \frac{x}{t} = \frac{1.52 \text{ m}}{0.495 \text{ s}} = 3.07 \text{ m/s}.$$

25. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and $v_{0x} = v_0 = 10$ m/s.

(a) With the origin at the initial point (where the dart leaves the thrower's hand), the y coordinate of the dart is given by $y = -\frac{1}{2} g t^2$, so that with $y = -PQ$ we have $PQ = \frac{1}{2}\left(9.8 \text{ m/s}^2\right)\left(0.19 \text{ s}\right)^2 = 0.18 \text{ m}$.

(b) From $x = v_0 t$ we obtain $x = (10 \text{ m/s})(0.19 \text{ s}) = 1.9$ m.

26. (a) Using the same coordinate system assumed in Eq. 4-22, we solve for $y = h$:

$$h = y_0 + v_0 \sin\theta_0 t - \frac{1}{2}gt^2$$

which yields $h = 51.8$ m for $y_0 = 0$, $v_0 = 42.0$ m/s, $\theta_0 = 60.0°$ and $t = 5.50$ s.

(b) The horizontal motion is steady, so $v_x = v_{0x} = v_0 \cos\theta_0$, but the vertical component of velocity varies according to Eq. 4-23. Thus, the speed at impact is

$$v = \sqrt{(v_0 \cos\theta_0)^2 + (v_0 \sin\theta_0 - gt)^2} = 27.4 \text{ m/s}.$$

(c) We use Eq. 4-24 with $v_y = 0$ and $y = H$:

$$H = \frac{(v_0 \sin\theta_0)^2}{2g} = 67.5 \text{ m}.$$

27. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write $\theta_0 = -30.0°$ since the angle shown in the figure is measured clockwise from horizontal. We note that the initial speed of the decoy is the plane's speed at the moment of release: v_0 = 290 km/h, which we convert to SI units: (290)(1000/3600) = 80.6 m/s.

(a) We use Eq. 4-12 to solve for the time:

$$\Delta x = (v_0 \cos\theta_0)\, t \quad \Rightarrow \quad t = \frac{700 \text{ m}}{(80.6 \text{ m/s})\cos(-30.0°)} = 10.0 \text{ s}.$$

(b) And we use Eq. 4-22 to solve for the initial height y_0:

$$y - y_0 = (v_0 \sin\theta_0)\, t - \frac{1}{2}gt^2 \quad \Rightarrow \quad 0 - y_0 = (-40.3 \text{ m/s})(10.0 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(10.0 \text{ s})^2$$

which yields y_0 = 897 m.

28. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is throwing point (the stone's initial position). The x component of its initial velocity is given by $v_{0x} = v_0 \cos\theta_0$ and the y component is given by $v_{0y} = v_0 \sin\theta_0$, where v_0 = 20 m/s is the initial speed and θ_0 = 40.0° is the launch angle.

(a) At t = 1.10 s, its x coordinate is

$$x = v_0 t \cos\theta_0 = (20.0 \text{ m/s})(1.10 \text{ s}) \cos 40.0° = 16.9 \text{ m}$$

(b) Its y coordinate at that instant is

$$y = v_0 t \sin\theta_0 - \frac{1}{2}gt^2 = (20.0\text{m/s})(1.10\text{s}) \sin 40.0° - \frac{1}{2}(9.80\text{m/s}^2)(1.10\text{s})^2 = 8.21\text{m}.$$

(c) At t' = 1.80 s, its x coordinate is $x = (20.0 \text{ m/s})(1.80 \text{ s}) \cos 40.0° = 27.6 \text{ m}.$

(d) Its y coordinate at t' is

$$y = (20.0\text{m/s})(1.80\text{s})\sin 40.0° - \frac{1}{2}(9.80\text{m/s}^2)(1.80\text{s}^2) = 7.26\text{m}.$$

(e) The stone hits the ground earlier than t = 5.0 s. To find the time when it hits the ground solve $y = v_0 t \sin\theta_0 - \frac{1}{2}gt^2 = 0$ for t. We find

$$t = \frac{2v_0}{g}\sin\theta_0 = \frac{2(20.0 \text{ m/s})}{9.8 \text{ m/s}^2}\sin 40° = 2.62 \text{ s}.$$

Its x coordinate on landing is

$$x = v_0 t \cos\theta_0 = (20.0 \text{ m/s})(2.62 \text{ s}) \cos 40° = 40.2 \text{ m}.$$

(f) Assuming it stays where it lands, its vertical component at t = 5.00 s is y = 0.

29. The initial velocity has no vertical component — only an x component equal to +2.00 m/s. Also, y_0 = +10.0 m if the water surface is established as y = 0.

(a) $x - x_0 = v_x t$ readily yields $x - x_0$ = 1.60 m.

(b) Using $y - y_0 = v_{0_y} t - \frac{1}{2} g t^2$, we obtain y = 6.86 m when t = 0.800 s and v_{0y}=0.

(c) Using the fact that y = 0 and y_0 = 10.0, the equation $y - y_0 = v_{0_y} t - \frac{1}{2} g t^2$ leads to

$$t = \sqrt{2(10.0 \text{ m})/9.80 \text{ m/s}^2} = 1.43 \text{ s}.$$

During this time, the x-displacement of the diver is $x - x_0$ = (2.00 m/s)(1.43 s) = 2.86 m.

30. (a) Since the y-component of the velocity of the stone at the top of its path is zero, its speed is

$$v=\sqrt{v_x^2+v_y^2}=v_x=v_0\cos\theta_0=(28.0\text{ m/s})\cos 40.0°=21.4\text{ m/s}.$$

(b) Using the fact that $v_y=0$ at the maximum height $y_{\max}$, the amount of time it takes for the stone to reach $y_{\max}$ is given by Eq. 4-23:

$$0=v_y=v_0\sin\theta_0-gt \quad\Rightarrow\; t=\frac{v_0\sin\theta_0}{g}.$$

Substituting the above expression into Eq. 4-22, we find the maximum height to be

$$y_{\max}=(v_0\sin\theta_0)\,t-\frac{1}{2}gt^2=v_0\sin\theta_0\left(\frac{v_0\sin\theta_0}{g}\right)-\frac{1}{2}g\left(\frac{v_0\sin\theta_0}{g}\right)^2=\frac{v_0^2\sin^2\theta_0}{2g}.$$

To find the time the stone descends to $y=y_{\max}/2$, we solve the quadratic equation given in Eq. 4-22:

$$y=\frac{1}{2}y_{\max}=\frac{v_0^2\sin^2\theta_0}{4g}=(v_0\sin\theta_0)\,t-\frac{1}{2}gt^2 \quad\Rightarrow\quad t_{\pm}=\frac{(2\pm\sqrt{2})v_0\sin\theta_0}{2g}.$$

Choosing $t=t_+$ (for descending), we have

$$v_x=v_0\cos\theta_0=(28.0\text{ m/s})\cos 40.0°=21.4\text{ m/s}$$
$$v_y=v_0\sin\theta_0-g\frac{(2+\sqrt{2})v_0\sin\theta_0}{2g}=-\frac{\sqrt{2}}{2}v_0\sin\theta_0=-\frac{\sqrt{2}}{2}(28.0\text{ m/s})\sin 40.0°=-12.7\text{ m/s}$$

Thus, the speed of the stone when $y=y_{\max}/2$ is

$$v=\sqrt{v_x^2+v_y^2}=\sqrt{(21.4\text{ m/s})^2+(-12.7\text{ m/s})^2}=24.9\text{ m/s}.$$

(c) The percentage difference is

$$\frac{24.9\text{ m/s}-21.4\text{ m/s}}{21.4\text{ m/s}}=0.163=16.3\%.$$

31. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write θ_0 = –37.0° for the angle measured from $+x$, since the angle given in the problem is measured from the $-y$ direction. We note that the initial speed of the projectile is the plane's speed at the moment of release.

(a) We use Eq. 4-22 to find v_0:

$$y - y_0 = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 \quad \Rightarrow \quad 0 - 730\text{ m} = v_0 \sin(-37.0°)(5.00\text{ s}) - \frac{1}{2}(9.80\text{ m/s}^2)(5.00\text{ s})^2$$

which yields v_0 = 202 m/s.

(b) The horizontal distance traveled is $x = v_0 t\cos\theta_0$ = (202 m/s)(5.00 s)cos(–37.0°) = 806 m.

(c) The x component of the velocity (just before impact) is

$$v_x = v_0\cos\theta_0 = (202\text{ m/s})\cos(-37.0°) = 161\text{ m/s}.$$

(d) The y component of the velocity (just before impact) is

$$v_y = v_0 \sin\theta_0 - gt = (202\text{ m/s}) \sin(-37.0°) - (9.80\text{ m/s}^2)(5.00\text{ s}) = -171\text{ m/s}.$$

32. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the point where the ball was hit by the racquet.

(a) We want to know how high the ball is above the court when it is at x = 12.0 m. First, Eq. 4-21 tells us the time it is over the fence:

$$t = \frac{x}{v_0 \cos\theta_0} = \frac{12.0 \text{ m}}{(23.6 \text{ m/s}) \cos 0°} = 0.508 \text{ s}.$$

At this moment, the ball is at a height (above the court) of

$$y = y_0 + (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 = 1.10 \text{m}$$

which implies it does indeed clear the 0.90 m high fence.

(b) At t = 0.508 s, the center of the ball is (1.10 m – 0.90 m) = 0.20 m above the net.

(c) Repeating the computation in part (a) with θ_0 = –5.0° results in t = 0.510 s and $y = 0.040$ m, which clearly indicates that it cannot clear the net.

(d) In the situation discussed in part (c), the distance between the top of the net and the center of the ball at t = 0.510 s is 0.90 m – 0.040 m = 0.86 m.

33. We first find the time it takes for the volleyball to hit the ground. Using Eq. 4-22, we have

$$y - y_0 = (v_0 \sin\theta_0)\, t - \frac{1}{2}gt^2 \;\Rightarrow\; 0 - 2.30\text{ m} = (-20.0\text{ m/s})\sin(18.0°)t - \frac{1}{2}(9.80\text{ m/s}^2)t^2$$

which gives $t = 0.30\text{ s}$. Thus, the range of the volleyball is

$$R = \left(v_0 \cos\theta_0\right)t = (20.0\text{ m/s})\cos 18.0°(0.30\text{ s}) = 5.71\text{ m}$$

On the other hand, when the angle is changed to $\theta_0' = 8.00°$, using the same procedure as shown above, we find

$$y - y_0 = (v_0 \sin\theta_0')\, t' - \frac{1}{2}gt'^2 \;\Rightarrow\; 0 - 2.30\text{ m} = (-20.0\text{ m/s})\sin(8.00°)t' - \frac{1}{2}(9.80\text{ m/s}^2)t'^2$$

which yields $t' = 0.46\text{ s}$, and the range is

$$R' = \left(v_0 \cos\theta_0\right)t' = (20.0\text{ m/s})\cos 18.0°(0.46\text{ s}) = 9.06\text{ m}$$

Thus, the ball travels an extra distance of

$$\Delta R = R' - R = 9.06\text{ m} - 5.71\text{ m} = 3.35\text{ m}$$

34. Although we could use Eq. 4-26 to find where it lands, we choose instead to work with Eq. 4-21 and Eq. 4-22 (for the soccer ball) since these will give information about where *and when* and these are also considered more fundamental than Eq. 4-26. With $\Delta y = 0$, we have

$$\Delta y = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 \;\Rightarrow\; t = \frac{(19.5 \text{ m/s}) \sin 45.0°}{(9.80 \text{ m/s}^2)/2} = 2.81 \text{ s}.$$

Then Eq. 4-21 yields $\Delta x = (v_0 \cos\theta_0)t = 38.7$ m. Thus, using Eq. 4-8, the player must have an average velocity of

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(38.7 \text{ m})\,\hat{\text{i}} - (55 \text{ m})\,\hat{\text{i}}}{2.81 \text{ s}} = (-5.8 \text{ m/s})\,\hat{\text{i}}$$

which means his average speed (assuming he ran in only one direction) is 5.8 m/s.

35. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at its initial position (where it is launched). At maximum height, we observe $v_y = 0$ and denote $v_x = v$ (which is also equal to v_{0x}). In this notation, we have $v_0 = 5v$. Next, we observe $v_0 \cos \theta_0 = v_{0x} = v$, so that we arrive at an equation (where $v \neq 0$ cancels) which can be solved for θ_0:

$$(5v)\cos\theta_0 = v \Rightarrow \theta_0 = \cos^{-1}\left(\frac{1}{5}\right) = 78.5°.$$

36. (a) Solving the quadratic equation Eq. 4-22:

$$y - y_0 = (v_0 \sin\theta_0)\ t - \frac{1}{2}gt^2 \ \Rightarrow\ 0 - 2.160\ \text{m} = (15.00\ \text{m/s})\sin(45.00°)t - \frac{1}{2}(9.800\ \text{m/s}^2)t^2$$

the total travel time of the shot in the air is found to be $t = 2.352\ \text{s}$. Therefore, the horizontal distance traveled is

$$R = \left(v_0 \cos\theta_0\right)t = (15.00\ \text{m/s})\cos 45.00°(2.352\ \text{s}) = 24.95\ \text{m}.$$

(b) Using the procedure outlined in (a) but for $\theta_0 = 42.00°$, we have

$$y - y_0 = (v_0 \sin\theta_0)\ t - \frac{1}{2}gt^2 \ \Rightarrow\ 0 - 2.160\ \text{m} = (15.00\ \text{m/s})\sin(42.00°)t - \frac{1}{2}(9.800\ \text{m/s}^2)t^2$$

and the total travel time is $t = 2.245\ \text{s}$. This gives

$$R = \left(v_0 \cos\theta_0\right)t = (15.00\ \text{m/s})\cos 42.00°(2.245\ \text{s}) = 25.02\ \text{m}.$$

37. We designate the given velocity $\vec{v} = (7.6\text{ m/s})\hat{\text{i}} + (6.1\text{ m/s})\hat{\text{j}}$ as $\vec{v}_1 -$ as opposed to the velocity when it reaches the max height $\vec{v}_2$ or the velocity when it returns to the ground $\vec{v}_3 -$ and take $\vec{v}_0$ as the launch velocity, as usual. The origin is at its launch point on the ground.

(a) Different approaches are available, but since it will be useful (for the rest of the problem) to first find the initial y velocity, that is how we will proceed. Using Eq. 2-16, we have

$$v_{1y}^2 = v_{0y}^2 - 2g\Delta y \;\Rightarrow\; (6.1\text{ m/s})^2 = v_{0y}^2 - 2(9.8\text{ m/s}^2)(9.1\text{ m})$$

which yields v_{0y} = 14.7 m/s. Knowing that v_{2y} must equal 0, we use Eq. 2-16 again but now with $\Delta y = h$ for the maximum height:

$$v_{2y}^2 = v_{0y}^2 - 2gh \;\Rightarrow\; 0 = (14.7\text{ m/s})^2 - 2(9.8\text{ m/s}^2)h$$

which yields h = 11 m.

(b) Recalling the derivation of Eq. 4-26, but using v_{0y} for $v_0 \sin\theta_0$ and v_{0x} for $v_0 \cos\theta_0$, we have

$$0 = v_{0y}t - \frac{1}{2}gt^2, \quad R = v_{0x}t$$

which leads to $R = 2v_{0x}v_{0y}/g$. Noting that $v_{0x} = v_{1x}$ = 7.6 m/s, we plug in values and obtain

$$R = 2(7.6\text{ m/s})(14.7\text{ m/s})/(9.8\text{ m/s}^2) = 23\text{ m}.$$

(c) Since $v_{3x} = v_{1x}$ = 7.6 m/s and $v_{3y} = -v_{0y}$ = –14.7 m/s, we have

$$v_3 = \sqrt{v_{3x}^2 + v_{3y}^2} = \sqrt{(7.6\text{ m/s})^2 + (-14.7\text{ m/s})^2} = 17\text{ m/s}.$$

(d) The angle (measured from horizontal) for $\vec{v}_3$ is one of these possibilities:

$$\tan^{-1}\left(\frac{-14.7\text{ m}}{7.6\text{ m}}\right) = -63° \;\text{ or }\; 117°$$

where we settle on the first choice (–63°, which is equivalent to 297°) since the signs of its components imply that it is in the fourth quadrant.

38. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the release point (the initial position for the ball as it begins projectile motion in the sense of §4-5), and we let θ_0 be the angle of throw (shown in the figure). Since the horizontal component of the velocity of the ball is $v_x = v_0 \cos 40.0°$, the time it takes for the ball to hit the wall is

$$t = \frac{\Delta x}{v_x} = \frac{22.0 \text{ m}}{(25.0 \text{ m/s}) \cos 40.0°} = 1.15 \text{ s}.$$

(a) The vertical distance is

$$\Delta y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 = (25.0 \text{ m/s}) \sin 40.0°(1.15 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(1.15 \text{ s})^2 = 12.0 \text{ m}.$$

(b) The horizontal component of the velocity when it strikes the wall does not change from its initial value: $v_x = v_0 \cos 40.0° = 19.2$ m/s.

(c) The vertical component becomes (using Eq. 4-23)

$$v_y = v_0 \sin\theta_0 - gt = (25.0 \text{ m/s}) \sin 40.0° - (9.80 \text{ m/s}^2)(1.15 \text{ s}) = 4.80 \text{ m/s}.$$

(d) Since $v_y > 0$ when the ball hits the wall, it has not reached the highest point yet.

39. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the end of the rifle (the initial point for the bullet as it begins projectile motion in the sense of § 4-5), and we let θ_0 be the firing angle. If the target is a distance d away, then its coordinates are $x = d$, $y = 0$. The projectile motion equations lead to $d = v_0 \cos\theta_0$ and $0 = v_0 t \sin\theta_0 - \frac{1}{2}gt^2$. Eliminating t leads to $2v_0^2 \sin\theta_0 \cos\theta_0 - gd = 0$. Using $\sin\theta_0 \cos\theta_0 = \frac{1}{2}\sin(2\theta_0)$, we obtain

$$v_0^2 \sin\ (2\theta_0) = gd \Rightarrow \sin(2\theta_0) = \frac{gd}{v_0^2} = \frac{(9.80\ \text{m/s}^2)(45.7\ \text{m})}{(460\ \text{m/s})^2}$$

which yields $\sin(2\theta_0) = 2.11\times10^{-3}$ and consequently $\theta_0 = 0.0606°$. If the gun is aimed at a point a distance ℓ above the target, then $\tan\theta_0 = \ell/d$ so that

$$\ell = d\tan\theta_0 = (45.7\ \text{m})\tan(0.0606°) = 0.0484\ \text{m} = 4.84\ \text{cm}.$$

40. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and $v_{0x} = v_0 = 161 \text{ km/h}$. Converting to SI units, this is v_0 = 44.7 m/s.

(a) With the origin at the initial point (where the ball leaves the pitcher's hand), the y coordinate of the ball is given by $y = -\frac{1}{2}gt^2$, and the x coordinate is given by $x = v_0 t$. From the latter equation, we have a simple proportionality between horizontal distance and time, which means the time to travel half the total distance is half the total time. Specifically, if x = 18.3/2 m, then t = (18.3/2 m)/(44.7 m/s) = 0.205 s.

(b) And the time to travel the next 18.3/2 m must also be 0.205 s. It can be useful to write the horizontal equation as $\Delta x = v_0 \Delta t$ in order that this result can be seen more clearly.

(c) From $y = -\frac{1}{2}gt^2$, we see that the ball has reached the height of $|-\frac{1}{2}\left(9.80 \text{ m/s}^2\right)\left(0.205 \text{ s}\right)^2| = 0.205 \text{ m}$ at the moment the ball is halfway to the batter.

(d) The ball's height when it reaches the batter is $-\frac{1}{2}\left(9.80 \text{ m/s}^2\right)\left(0.409 \text{ s}\right)^2 = -0.820 \text{m}$, which, when subtracted from the previous result, implies it has fallen another 0.615 m. Since the value of y is not simply proportional to t, we do not expect equal time-intervals to correspond to equal height-changes; in a physical sense, this is due to the fact that the initial y-velocity for the first half of the motion is not the same as the "initial" y-velocity for the second half of the motion.

41. Following the hint, we have the time-reversed problem with the ball thrown from the ground, towards the right, at 60° measured counterclockwise from a rightward axis. We see in this time-reversed situation that it is convenient to use the familiar coordinate system with $+x$ as *rightward* and with positive angles measured counterclockwise.

(a) The x-equation (with $x_0 = 0$ and $x = 25.0$ m) leads to

$$25.0 \text{ m} = (v_0 \cos 60.0°)(1.50 \text{ s}),$$

so that $v_0 = 33.3$ m/s. And with $y_0 = 0$, and $y = h > 0$ at $t = 1.50$ s, we have $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ where $v_{0y} = v_0 \sin 60.0°$. This leads to $h = 32.3$ m.

(b) We have

$$v_x = v_{0x} = (33.3 \text{ m/s})\cos 60.0° = 16.7 \text{ m/s}$$
$$v_y = v_{0y} - gt = (33.3 \text{ m/s})\sin 60.0° - (9.80 \text{ m/s}^2)(1.50 \text{ s}) = 14.2 \text{ m/s}.$$

The magnitude of $\vec{v}$ is given by

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(16.7 \text{ m/s})^2 + (14.2 \text{ m/s})^2} = 21.9 \text{ m/s}.$$

(c) The angle is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{14.2 \text{ m/s}}{16.7 \text{ m/s}}\right) = 40.4°.$$

(d) We interpret this result ("undoing" the time reversal) as an initial velocity (from the edge of the building) of magnitude 21.9 m/s with angle (down from leftward) of 40.4°.

42. In this projectile motion problem, we have $v_0 = v_x$ = constant, and what is plotted is $v=\sqrt{v_x^2+v_y^2}$. We infer from the plot that at $t = 2.5$ s, the ball reaches its maximum height, where $v_y = 0$. Therefore, we infer from the graph that $v_x = 19$ m/s.

(a) During $t = 5$ s, the horizontal motion is $x - x_0 = v_x t = 95$ m.

(b) Since $\sqrt{(19\text{ m/s})^2+v_{0y}^2}=31\text{ m/s}$ (the first point on the graph), we find $v_{0y}=24.5$ m/s. Thus, with t = 2.5 s, we can use $y_{\max}-y_0=v_{0y}t-\frac{1}{2}gt^2$ or $v_y^2=0=v_{0y}^2-2g(y_{\max}-y_0)$, or $y_{\max}-y_0=\frac{1}{2}\left(v_y+v_{0_y}\right)t$ to solve. Here we will use the latter:

$$y_{\max}-y_0=\frac{1}{2}(v_y+v_{0y})\,t \Rightarrow y_{\max}=\frac{1}{2}(0+24.5\text{m/s})(2.5\text{ s})=31\text{ m}$$

where we have taken $y_0 = 0$ as the ground level.

43. (a) Let $m = \frac{d_2}{d_1} = 0.600$ be the slope of the ramp, so $y = mx$ there. We choose our coordinate origin at the point of launch and use Eq. 4-25. Thus,

$$y = \tan(50.0°)x - \frac{(9.80 \text{ m/s}^2)x^2}{2(10.0 \text{ m/s})^2(\cos 50.0°)^2} = 0.600x$$

which yields $x = 4.99$ m. This is less than d_1 so the ball *does* land on the ramp.

(b) Using the value of x found in part (a), we obtain $y = mx = 2.99$ m. Thus, the Pythagorean theorem yields a displacement magnitude of $\sqrt{x^2 + y^2} = 5.82$ m.

(c) The angle is, of course, the angle of the ramp: $\tan^{-1}(m) = 31.0°$.

44. (a) Using the fact that the person (as the projectile) reaches the maximum height over the middle wheel located at $x = 23\text{ m} + (23/2)\text{ m} = 34.5\text{ m}$, we can deduce the initial launch speed from Eq. 4-26:

$$x = \frac{R}{2} = \frac{v_0^2 \sin 2\theta_0}{2g} \quad \Rightarrow \quad v_0 = \sqrt{\frac{2gx}{\sin 2\theta_0}} = \sqrt{\frac{2(9.8\text{ m/s}^2)(34.5\text{ m})}{\sin(2\cdot 53°)}} = 26.5\text{ m/s}.$$

Upon substituting the value to Eq. 4-25, we obtain

$$y = y_0 + x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} = 3.0\text{ m} + (23\text{ m})\tan 53° - \frac{(9.8\text{ m/s}^2)(23\text{ m})^2}{2(26.5\text{ m/s})^2(\cos 53°)^2} = 23.3\text{ m}.$$

Since the height of the wheel is $h_w = 18\text{ m}$, the clearance over the first wheel is $\Delta y = y - h_w = 23.3\text{ m} - 18\text{ m} = 5.3\text{ m}$.

(b) The height of the person when he is directly above the second wheel can be found by solving Eq. 4-24. With the second wheel located at $x = 23\text{ m} + (23/2)\text{ m} = 34.5\text{ m}$, we have

$$\begin{aligned} y &= y_0 + x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} = 3.0\text{ m} + (34.5\text{ m})\tan 53° - \frac{(9.8\text{ m/s}^2)(34.5\text{ m})^2}{2(26.52\text{ m/s})^2(\cos 53°)^2} \\ &= 25.9\text{ m}. \end{aligned}$$

Therefore, the clearance over the second wheel is $\Delta y = y - h_w = 25.9\text{ m} - 18\text{ m} = 7.9\text{ m}$.

(c) The location of the center of the net is given by

$$0 = y - y_0 = x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} \quad \Rightarrow \quad x = \frac{v_0^2\sin 2\theta_0}{g} = \frac{(26.52\text{ m/s})^2\sin(2\cdot 53°)}{9.8\text{ m/s}^2} = 69\text{ m}.$$

45. Using the information given, the position of the insect is given by (with the Archer fish at the origin)

$$x = d\cos\phi = (0.900\text{ m})\cos 36.0° = 0.728\text{ m}$$
$$y = d\sin\phi = (0.900\text{ m})\sin 36.0° = 0.529\text{ m}$$

Since y corresponds to the maximum height of the parabolic trajectory (see Problem 4-30): $y = y_{\max} = v_0^2 \sin^2\theta_0 / 2g$, the launch angle is found to be

$$\theta_0 = \sin^{-1}\left(\sqrt{\frac{2gy}{v_0^2}}\right) = \sin^{-1}\left(\sqrt{\frac{2(9.8\text{ m/s}^2)(0.529\text{ m})}{(3.56\text{ m/s})^2}}\right)\sin^{-1}(0.9044) = 64.8°$$

46. Following the hint, we have the time-reversed problem with the ball thrown from the roof, towards the left, at 60° measured clockwise from a leftward axis. We see in this time-reversed situation that it is convenient to take $+x$ as *leftward* with positive angles measured clockwise. Lengths are in meters and time is in seconds.

(a) With y_0 = 20.0 m, and y = 0 at t = 4.00 s, we have $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ where $v_{0y} = v_0 \sin 60°$. This leads to v_0 = 16.9 m/s. This plugs into the x-equation $x - x_0 = v_{0x}t$ (with x_0 = 0 and $x = d$) to produce d = (16.9 m/s)cos 60°(4.00 s) = 33.7 m.

(b)We have

$$\begin{aligned} v_x &= v_{0x} = (16.9 \text{ m/s}) \cos 60.0° = 8.43 \text{ m/s} \\ v_y &= v_{0y} - gt = (16.9 \text{ m/s}) \sin 60.0° - (9.80 \text{m/s}^2)(4.00 \text{ s}) = -24.6 \text{ m/s}. \end{aligned}$$

The magnitude of $\vec{v}$ is $|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(8.43 \text{ m/s})^2 + (-24.6 \text{ m/s})^2} = 26.0 \text{ m/s}$.

(c) The angle relative to horizontal is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-24.6 \text{ m/s}}{8.43 \text{ m/s}}\right) = -71.1°.$$

We may convert the result from rectangular components to magnitude-angle representation:

$$\vec{v} = (8.43, -24.6) \rightarrow (26.0 \angle -71.1°)$$

and we now interpret our result (“undoing” the time reversal) as an initial velocity of magnitude 26.0 m/s with angle (up from rightward) of 71.1°.

47. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below impact point between bat and ball. The *Hint* given in the problem is important, since it provides us with enough information to find v_0 directly from Eq. 4-26.

(a) We want to know how high the ball is from the ground when it is at $x = 97.5$ m, which requires knowing the initial velocity. Using the range information and $\theta_0 = 45°$, we use Eq. 4-26 to solve for v_0:

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{(9.8\ \text{m/s}^2)(107\ \text{m})}{1}} = 32.4\ \text{m/s}.$$

Thus, Eq. 4-21 tells us the time it is over the fence:

$$t = \frac{x}{v_0 \cos\theta_0} = \frac{97.5\ \text{m}}{(32.4\ \text{m/s})\cos 45°} = 4.26\ \text{s}.$$

At this moment, the ball is at a height (above the ground) of

$$y = y_0 + (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 = 9.88\ \text{m}$$

which implies it does indeed clear the 7.32 m high fence.

(b) At $t = 4.26$ s, the center of the ball is 9.88 m – 7.32 m = 2.56 m above the fence.

48. Using the fact that $v_y = 0$ when the player is at the maximum height $y_{\max}$, the amount of time it takes to reach $y_{\max}$ can be solved by using Eq. 4-23:

$$0 = v_y = v_0 \sin\theta_0 - gt \quad \Rightarrow \; t_{\max} = \frac{v_0 \sin\theta_0}{g}.$$

Substituting the above expression into Eq. 4-22, we find the maximum height to be

$$y_{\max} = (v_0 \sin\theta_0)\, t_{\max} - \frac{1}{2} g t_{\max}^2 = v_0 \sin\theta_0 \left(\frac{v_0 \sin\theta_0}{g}\right) - \frac{1}{2} g \left(\frac{v_0 \sin\theta_0}{g}\right)^2 = \frac{v_0^2 \sin^2\theta_0}{2g}.$$

To find the time when the player is at $y = y_{\max}/2$, we solve the quadratic equation given in Eq. 4-22:

$$y = \frac{1}{2} y_{\max} = \frac{v_0^2 \sin^2\theta_0}{4g} = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 \quad \Rightarrow \quad t_{\pm} = \frac{(2 \pm \sqrt{2}) v_0 \sin\theta_0}{2g}.$$

With $t = t_-$ (for ascending), the amount of time the player spends at a height $y \geq y_{\max}/2$ is

$$\Delta t = t_{\max} - t_- = \frac{v_0 \sin\theta_0}{g} - \frac{(2 - \sqrt{2}) v_0 \sin\theta_0}{2g} = \frac{v_0 \sin\theta_0}{\sqrt{2} g} = \frac{t_{\max}}{\sqrt{2}} \quad \Rightarrow \quad \frac{\Delta t}{t_{\max}} = \frac{1}{\sqrt{2}} = 0.707.$$

Therefore, the player spends about 70.7% of the time in the upper half of the jump. Note that the ratio $\Delta t / t_{\max}$ is independent of v_0 and θ_0, even though Δt and $t_{\max}$ depend on these quantities.

49. (a) The skier jumps up at an angle of $\theta_0 = 9.0°$ up from the horizontal and thus returns to the launch level with his velocity vector 9.0° below the horizontal. With the snow surface making an angle of $\alpha = 11.3°$ (downward) with the horizontal, the angle between the slope and the velocity vector is $\phi = \alpha - \theta_0 = 11.3° - 9.0° = 2.3°$.

(b) Suppose the skier lands at a distance d down the slope. Using Eq. 4-25 with $x = d\cos\alpha$ and $y = -d\sin\alpha$ (the edge of the track being the origin), we have

$$-d\sin\alpha = d\cos\alpha\tan\theta_0 - \frac{g(d\cos\alpha)^2}{2v_0^2\cos^2\theta_0}.$$

Solving for d, we obtain

$$\begin{aligned} d &= \frac{2v_0^2\cos^2\theta_0}{g\cos^2\alpha}(\cos\alpha\tan\theta_0 + \sin\alpha) = \frac{2v_0^2\cos\theta_0}{g\cos^2\alpha}(\cos\alpha\sin\theta_0 + \cos\theta_0\sin\alpha) \\ &= \frac{2v_0^2\cos\theta_0}{g\cos^2\alpha}\sin(\theta_0 + \alpha). \end{aligned}$$

Substituting the values given, we find

$$d = \frac{2(10\text{ m/s})^2\cos(9.0°)}{(9.8\text{ m/s}^2)\cos^2(11.3°)}\sin(9.0° + 11.3°) = 7.27\text{ m}.$$

which gives

$$y = -d\sin\alpha = -(7.27\text{ m})\sin(11.3°) = -1.42\text{ m}.$$

Therefore, at landing the skier is approximately 1.4 m below the launch level.

(c) The time it takes for the skier to land is

$$t = \frac{x}{v_x} = \frac{d\cos\alpha}{v_0\cos\theta_0} = \frac{(7.27\text{ m})\cos(11.3°)}{(10\text{ m/s})\cos(9.0°)} = 0.72\text{ s}.$$

Using Eq. 4-23, the x-and y-components of the velocity at landing are

$$\begin{aligned} v_x &= v_0\cos\theta_0 = (10\text{ m/s})\cos(9.0°) = 9.9\text{ m/s} \\ v_y &= v_0\sin\theta_0 - gt = (10\text{ m/s})\sin(9.0°) - (9.8\text{ m/s}^2)(0.72\text{ s}) = -5.5\text{ m/s} \end{aligned}$$

Thus, the direction of travel at landing is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-5.5\text{ m/s}}{9.9\text{ m/s}}\right) = -29.1°.$$

or 29.1° below the horizontal. The result implies that the angle between the skier's path and the slope is $\phi = 29.1° - 11.3° = 17.8°$, or approximately 18° to two significant figures.

50. From Eq. 4-21, we find $t = x / v_{0x}$. Then Eq. 4-23 leads to

$$v_y = v_{0y} - gt = v_{0y} - \frac{gx}{v_{0x}}.$$

Since the slope of the graph is –0.500, we conclude $\frac{g}{v_{ox}} = \frac{1}{2} \Rightarrow v_{ox} = 19.6$ m/s. And from the "*y* intercept" of the graph, we find $v_{oy} = 5.00$ m/s. Consequently, $\theta_o = \tan^{-1}(v_{oy} / v_{ox}) = 14.3°$.

51. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the point where the ball is kicked. We use x and y to denote the coordinates of ball at the goalpost, and try to find the kicking angle(s) θ_0 so that y = 3.44 m when x = 50 m. Writing the kinematic equations for projectile motion:

$$x = v_0 \cos\theta_0, \quad y = v_0 t \sin\,\theta_0 - \tfrac{1}{2} g t^2,$$

we see the first equation gives $t = x/v_0 \cos\,\theta_0$, and when this is substituted into the second the result is

$$y = x\,\tan\theta_0 - \frac{g x^2}{2 v_0^2 \cos^2\theta_0}.$$

One may solve this by trial and error: systematically trying values of θ_0 until you find the two that satisfy the equation. A little manipulation, however, will give an algebraic solution: Using the trigonometric identity $1 / \cos^2\theta_0 = 1 + \tan^2\theta_0$, we obtain

$$\frac{1}{2}\frac{g x^2}{v_0^2}\tan^2\theta_0 - x\,\tan\theta_0 + y + \frac{1}{2}\frac{g x^2}{v_0^2} = 0$$

which is a second-order equation for $\tan\,\theta_0$. To simplify writing the solution, we denote

$$c = \tfrac{1}{2} g x^2 / v_0^2 = \tfrac{1}{2}\left(9.80\text{ m/s}^2\right)\left(50\text{ m}\right)^2 / \left(25\text{ m/s}\right)^2 = 19.6\text{m}.$$

Then the second-order equation becomes $c\,\tan^2\theta_0 - x\,\tan\,\theta_0 + y + c = 0$. Using the quadratic formula, we obtain its solution(s).

$$\tan\,\theta_0 = \frac{x \pm \sqrt{x^2 - 4(y+c)c}}{2c} = \frac{50\text{ m} \pm \sqrt{(50\text{ m})^2 - 4(3.44\text{ m} + 19.6\text{ m})(19.6\text{ m})}}{2(19.6\text{ m})}.$$

The two solutions are given by $\tan\,\theta_0 = 1.95$ and $\tan\,\theta_0 = 0.605$. The corresponding (first-quadrant) angles are $\theta_0 = 63°$ and $\theta_0 = 31°$. Thus,

(a) The smallest elevation angle is $\theta_0 = 31°$, and

(b) The greatest elevation angle is $\theta_0 = 63°$.

If kicked at any angle between these two, the ball will travel above the cross bar on the goalposts.

52. For $\Delta y = 0$, Eq. 4-22 leads to $t = 2v_o\sin\theta_o/g$, which immediately implies $t_{max} = 2v_o/g$ (which occurs for the "straight up" case: $\theta_o = 90°$). Thus,

$$\frac{1}{2}t_{max} = v_o/g \;\;\Rightarrow\; \frac{1}{2} = \sin\theta_o.$$

Therefore, the half-maximum-time flight is at angle $\theta_o = 30.0°$. Since the least speed occurs at the top of the trajectory, which is where the velocity is simply the x-component of the initial velocity ($v_o\cos\theta_o = v_o\cos30°$ for the half-maximum-time flight), then we need to refer to the graph in order to find v_o – in order that we may complete the solution. In the graph, we note that the range is 240 m when $\theta_o = 45.0°$. Eq. 4-26 then leads to v_o = 48.5 m/s. The answer is thus (48.5 m/s)cos30.0° = 42.0 m/s.

53. We denote h as the height of a step and w as the width. To hit step n, the ball must fall a distance nh and travel horizontally a distance between $(n - 1)w$ and nw. We take the origin of a coordinate system to be at the point where the ball leaves the top of the stairway, and we choose the y axis to be positive in the upward direction. The coordinates of the ball at time t are given by $x = v_{0x}t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0y} = 0$). We equate y to $-nh$ and solve for the time to reach the level of step n:

$$t = \sqrt{\frac{2nh}{g}}.$$

The x coordinate then is

$$x = v_{0x}\sqrt{\frac{2nh}{g}} = (1.52 \text{ m/s})\sqrt{\frac{2n(0.203 \text{ m})}{9.8 \text{ m/s}^2}} = (0.309 \text{ m})\sqrt{n}.$$

The method is to try values of n until we find one for which x/w is less than n but greater than $n - 1$. For $n = 1$, $x = 0.309$ m and $x/w = 1.52$, which is greater than n. For $n = 2$, $x = 0.437$ m and $x/w = 2.15$, which is also greater than n. For $n = 3$, $x = 0.535$ m and $x/w = 2.64$. Now, this is less than n and greater than $n - 1$, so the ball hits the third step.

54. We apply Eq. 4-21, Eq. 4-22 and Eq. 4-23.

(a) From $\Delta x = v_{0x} t$, we find $v_{0x} = 40 \text{ m}/2 \text{ s} = 20$ m/s.

(b) From $\Delta y = v_{0y} t - \frac{1}{2} g t^2$, we find $v_{0y} = \left(53 \text{ m} + \frac{1}{2}(9.8 \text{ m/s}^2)(2 \text{ s})^2\right)/2 = 36 \text{ m/s}$.

(c) From $v_y = v_{0y} - g t'$ with v_y = 0 as the condition for maximum height, we obtain $t' = (36 \text{ m/s})/(9.8 \text{ m/s}^2) = 3.7$ s. During that time the x-motion is constant, so $x' - x_0 = (20 \text{ m/s})(3.7 \text{ s}) = 74$ m.

55. Let $y_0 = h_0 = 1.00$ m at $x_0 = 0$ when the ball is hit. Let $y_1 = h$ (the height of the wall) and x_1 describe the point where it first rises above the wall one second after being hit; similarly, $y_2 = h$ and x_2 describe the point where it passes back down behind the wall four seconds later. And $y_f = 1.00$ m at $x_f = R$ is where it is caught. Lengths are in meters and time is in seconds.

(a) Keeping in mind that v_x is constant, we have $x_2 - x_1 = 50.0 \text{ m} = v_{1x}$ (4.00 s), which leads to $v_{1x} = 12.5$ m/s. Thus, applied to the full six seconds of motion:

$$x_f - x_0 = R = v_x(6.00 \text{ s}) = 75.0 \text{ m}.$$

(b) We apply $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ to the motion above the wall,

$$y_2 - y_1 = 0 = v_{1y}\left(4.00 \text{ s}\right) - \frac{1}{2}g\left(4.00 \text{ s}\right)^2$$

and obtain $v_{1y} = 19.6$ m/s. One second earlier, using $v_{1y} = v_{0y} - g(1.00 \text{ s})$, we find $v_{0y} = 29.4 \text{ m/s}$. Therefore, the velocity of the ball just after being hit is

$$\vec{v} = v_{0x}\hat{\mathrm{i}} + v_{0y}\hat{\mathrm{j}} = (12.5 \text{ m/s})\ \hat{\mathrm{i}} + \ (29.4 \text{ m/s})\,\hat{\mathrm{j}}$$

Its magnitude is $|\vec{v}| = \sqrt{(12.5 \text{ m/s})^2 + (29.4 \text{ m/s})^2} = 31.9 \text{ m/s}.$

(c) The angle is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{29.4 \text{ m/s}}{12.5 \text{ m/s}}\right) = 67.0°.$$

We interpret this result as a velocity of magnitude 31.9 m/s, with angle (up from rightward) of 67.0°.

(d) During the first 1.00 s of motion, $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ yields

$$h = 1.0 \text{ m} + \left(29.4 \text{ m/s}\right)\left(1.00 \text{ s}\right) - \tfrac{1}{2}\left(9.8 \text{ m/s}^2\right)\left(1.00 \text{ s}\right)^2 = 25.5 \text{ m}.$$

56. (a) During constant-speed circular motion, the velocity vector is perpendicular to the acceleration vector at every instant. Thus, $\vec{v} \cdot \vec{a} = 0$.

(b) The acceleration in this vector, at every instant, points towards the center of the circle, whereas the position vector points from the center of the circle to the object in motion. Thus, the angle between $\vec{r}$ and $\vec{a}$ is 180° at every instant, so $\vec{r} \times \vec{a} = 0$.

57. (a) Since the wheel completes 5 turns each minute, its period is one-fifth of a minute, or 12 s.

(b) The magnitude of the centripetal acceleration is given by $a = v^2/R$, where R is the radius of the wheel, and v is the speed of the passenger. Since the passenger goes a distance $2\pi R$ for each revolution, his speed is

$$v = \frac{2\pi(15\text{ m})}{12\text{ s}} = 7.85\text{ m/s}$$

and his centripetal acceleration is $a = \dfrac{(7.85\text{ m/s})^2}{15\text{ m}} = 4.1\text{ m/s}^2$.

(c) When the passenger is at the highest point, his centripetal acceleration is downward, toward the center of the orbit.

(d) At the lowest point, the centripetal acceleration is $a = 4.1\text{ m/s}^2$, same as part (b).

(e) The direction is up, toward the center of the orbit.

58. The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(10\ \mathrm{m/s})^2}{25\ \mathrm{m}} = 4.0\ \mathrm{m/s^2}.$$

59. We apply Eq. 4-35 to solve for speed v and Eq. 4-34 to find centripetal acceleration a.

(a) $v = 2\pi r/T = 2\pi(20 \text{ km})/1.0 \text{ s} = 126 \text{ km/s} = 1.3 \times 10^5$ m/s.

(b) The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(126 \text{ km/s})^2}{20 \text{ km}} = 7.9 \times 10^5 \text{ m/s}^2.$$

(c) Clearly, both v and a will increase if T is reduced.

60. We apply Eq. 4-35 to solve for speed v and Eq. 4-34 to find acceleration a.

(a) Since the radius of Earth is 6.37×10^6 m, the radius of the satellite orbit is

$$r = (6.37 \times 10^6 + 640 \times 10^3) \text{ m} = 7.01 \times 10^6 \text{ m}.$$

Therefore, the speed of the satellite is

$$v = \frac{2\pi r}{T} = \frac{2\pi\left(7.01 \times 10^6 \text{ m}\right)}{(98.0 \text{ min})(60 \text{ s/min})} = 7.49 \times 10^3 \text{ m/s}.$$

(b) The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{\left(7.49 \times 10^3 \text{ m/s}\right)^2}{7.01 \times 10^6 \text{ m}} = 8.00 \text{ m/s}^2.$$

61. The magnitude of centripetal acceleration ($a = v^2/r$) and its direction (towards the center of the circle) form the basis of this problem.

(a) If a passenger at this location experiences $\vec{a} = 1.83 \text{ m/s}^2$ east, then the center of the circle is *east* of this location. The distance is $r = v^2/a = (3.66 \text{ m/s})^2/(1.83 \text{ m/s}^2) = 7.32$ m.

(b) Thus, relative to the center, the passenger at that moment is located 7.32 m toward the west.

(c) If the direction of $\vec{a}$ experienced by the passenger is now *south*—indicating that the center of the merry-go-round is south of him, then relative to the center, the passenger at that moment is located 7.32 m toward the north.

62. (a) The circumference is $c = 2\pi r = 2\pi(0.15\text{ m}) = 0.94\text{ m}$.

(b) With $T = (60\text{ s})/1200 = 0.050\text{ s}$, the speed is $v = c/T = (0.94\text{ m})/(0.050\text{ s}) = 19\text{ m/s}$. This is equivalent to using Eq. 4-35.

(c) The magnitude of the acceleration is $a = v^2/r = (19\text{ m/s})^2/(0.15\text{ m}) = 2.4 \times 10^3\text{ m/s}^2$.

(d) The period of revolution is $(1200\text{ rev/min})^{-1} = 8.3 \times 10^{-4}$ min which becomes, in SI units, $T = 0.050\text{ s} = 50\text{ ms}$.

63. Since the period of a uniform circular motion is $T = 2\pi r / v$, where r is the radius and v is the speed, the centripetal acceleration can be written as

$$a = \frac{v^2}{r} = \frac{1}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 r}{T^2}.$$

Based on this expression, we compare the (magnitudes) of the wallet and purse accelerations, and find their ratio is the ratio of r values. Therefore, $a_{\text{wallet}} = 1.50\ a_{\text{purse}}$. Thus, the wallet acceleration vector is

$$a = 1.50[(2.00\ \text{m/s}^2)\hat{\text{i}} + (4.00\ \text{m/s}^2)\hat{\text{j}}] = (3.00\ \text{m/s}^2)\hat{\text{i}} + (6.00\ \text{m/s}^2)\hat{\text{j}}.$$

64. The fact that the velocity is in the $+y$ direction, and the acceleration is in the $+x$ direction at t_1= 4.00 s implies that the motion is clockwise. The position corresponds to the "9:00 position." On the other hand, the position at t_2=10.0 s is in the "6:00 position" since the velocity points in the $-x$ direction and the acceleration is in the $+y$ direction. The time interval $\Delta t = 10.0\text{ s} - 4.00\text{ s} = 6.00\text{ s}$ is equal to 3/4 of a period:

$$6.00\text{ s} = \frac{3}{4}T \quad \Rightarrow \quad T = 8.00\text{ s}.$$

Eq. 4-35 then yields

$$r = \frac{vT}{2\pi} = \frac{(3.00\text{ m/s})(8.00\text{ s})}{2\pi} = 3.82\text{ m}.$$

(a) The x coordinate of the center of the circular path is $x = 5.00\text{ m} + 3.82\text{ m} = 8.82\text{ m}$.

(b) The y coordinate of the center of the circular path is $y = 6.00\text{ m}$.

In other words, the center of the circle is at (x,y) = (8.82 m, 6.00 m).

65. We first note that $\vec{a}_1$ (the acceleration at t_1 = 2.00 s) is perpendicular to $\vec{a}_2$ (the acceleration at t_2=5.00 s), by taking their scalar (dot) product.:

$$\vec{a}_1 \cdot \vec{a}_2 = [(6.00 \text{ m/s}^2)\hat{\text{i}} + (4.00 \text{ m/s}^2)\hat{\text{j}}] \cdot [(4.00 \text{ m/s}^2)\hat{\text{i}} + (-6.00 \text{ m/s}^2)\hat{\text{j}}] = 0.$$

Since the acceleration vectors are in the (negative) radial directions, then the two positions (at t_1 and t_2) are a quarter-circle apart (or three-quarters of a circle, depending on whether one measures clockwise or counterclockwise). A quick sketch leads to the conclusion that if the particle is moving counterclockwise (as the problem states) then it travels three-quarters of a circumference in moving from the position at time t_1 to the position at time t_2 . Letting T stand for the period, then $t_2 - t_1$ = 3.00 s = $3T/4$. This gives T = 4.00 s. The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(6.00 \text{ m/s}^2)^2 + (4.00 \text{ m/s})^2} = 7.21 \text{ m/s}^2.$$

Using Eq. 4-34 and 4-35, we have $a = 4\pi^2 r / T^2$, which yields

$$r = \frac{aT^2}{4\pi^2} = \frac{(7.21 \text{ m/s}^2)(4.00 \text{ s})^2}{4\pi^2} = 2.92 \text{ m}.$$

66. When traveling in circular motion with constant speed, the instantaneous acceleration vector necessarily points towards the center. Thus, the center is "straight up" from the cited point.

(a) Since the center is "straight up" from (4.00 m, 4.00 m), the x coordinate of the center is 4.00 m.

(b) To find out "how far up" we need to know the radius. Using Eq. 4-34 we find

$$r = \frac{v^2}{a} = \frac{(5.00\text{ m/s})^2}{12.5\text{ m/s}^2} = 2.00\text{ m.}$$

Thus, the y coordinate of the center is 2.00 m + 4.00 m = 6.00 m. Thus, the center may be written as (x, y) = (4.00 m, 6.00 m).

67. To calculate the centripetal acceleration of the stone, we need to know its speed during its circular motion (this is also its initial speed when it flies off). We use the kinematic equations of projectile motion (discussed in §4-6) to find that speed. Taking the $+y$ direction to be upward and placing the origin at the point where the stone leaves its circular orbit, then the coordinates of the stone during its motion as a projectile are given by $x = v_0t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0y} = 0$). It hits the ground at $x = 10$ m and $y = -2.0$ m. Formally solving the second equation for the time, we obtain $t = \sqrt{-2y/g}$, which we substitute into the first equation:

$$v_0 = x\sqrt{-\frac{g}{2y}} = (10 \text{ m})\sqrt{-\frac{9.8 \text{ m/s}^2}{2(-2.0 \text{ m})}} = 15.7 \text{ m/s}.$$

Therefore, the magnitude of the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{(15.7 \text{ m/s})^2}{1.5 \text{ m}} = 160 \text{ m/s}^2.$$

68. We note that after three seconds have elapsed ($t_2 - t_1 = 3.00$ s) the velocity (for this object in circular motion of period T) is reversed; we infer that it takes three seconds to reach the opposite side of the circle. Thus, $T = 2(3.00 \text{ s}) = 6.00$ s.

(a) Using Eq. 4-35, $r = vT/2\pi$, where $v = \sqrt{(3.00 \text{ m/s})^2 + (4.00 \text{ m/s})^2} = 5.00 \text{ m/s}$, we obtain $r = 4.77 \text{ m}$. The magnitude of the object's centripetal acceleration is therefore $a = v^2/r = 5.24 \text{ m/s}^2$.

(b) The average acceleration is given by Eq. 4-15:

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{(-3.00\hat{\text{i}} - 4.00\hat{\text{j}}) \text{ m/s} - (3.00\hat{\text{i}} + 4.00\hat{\text{j}}) \text{ m/s}}{5.00 \text{ s} - 2.00 \text{ s}} = (-2.00 \text{ m/s}^2)\hat{\text{i}} + (-2.67 \text{ m/s}^2)\hat{\text{j}}$$

which implies $|\vec{a}_{\text{avg}}| = \sqrt{(-2.00 \text{ m/s}^2)^2 + (-2.67 \text{ m/s}^2)^2} = 3.33 \text{ m/s}^2$.

69. We use Eq. 4-15 first using velocities relative to the truck (subscript t) and then using velocities relative to the ground (subscript g). We work with SI units, so $20 \text{ km/h} \to 5.6 \text{ m/s}$, $30 \text{ km/h} \to 8.3 \text{ m/s}$, and $45 \text{ km/h} \to 12.5 \text{ m/s}$. We choose east as the $+\hat{\text{i}}$ direction.

(a) The velocity of the cheetah (subscript c) at the end of the 2.0 s interval is (from Eq. 4-44)

$$\vec{v}_{\text{c t}} = \vec{v}_{\text{c g}} - \vec{v}_{\text{t g}} = (12.5 \text{ m/s})\ \hat{\text{i}} - (-5.6 \text{ m/s})\ \hat{\text{i}} = (18.1 \text{ m/s})\ \hat{\text{i}}$$

relative to the truck. Since the velocity of the cheetah relative to the truck at the beginning of the 2.0 s interval is $(-8.3 \text{ m/s})\hat{\text{i}}$, the (average) acceleration vector relative to the cameraman (in the truck) is

$$\vec{a}_{\text{avg}} = \frac{(18.1 \text{ m/s})\hat{\text{i}} - (-8.3 \text{ m/s})\hat{\text{i}}}{2.0 \text{ s}} = (13 \text{ m/s}^2)\hat{\text{i}},$$

or $|\vec{a}_{\text{avg}}| = 13 \text{ m/s}^2$.

(b) The direction of $\vec{a}_{avg}$ is $+\hat{\text{i}}$, or eastward.

(c) The velocity of the cheetah at the start of the 2.0 s interval is (from Eq. 4-44)

$$\vec{v}_{0\text{cg}} = \vec{v}_{0\text{ct}} + \vec{v}_{0\text{tg}} = (-8.3 \text{ m/s})\hat{\text{i}} + (-5.6 \text{ m/s})\hat{\text{i}} = (-13.9 \text{ m/s})\hat{\text{i}}$$

relative to the ground. The (average) acceleration vector relative to the crew member (on the ground) is

$$\vec{a}_{\text{avg}} = \frac{(12.5 \text{ m/s})\hat{\text{i}} - (-13.9 \text{ m/s})\hat{\text{i}}}{2.0 \text{ s}} = (13 \text{ m/s}^2)\hat{\text{i}}, \quad |\vec{a}_{\text{avg}}| = 13 \text{ m/s}^2$$

identical to the result of part (a).

70. We use Eq. 4-44, noting that the upstream corresponds to the $+\hat{\text{i}}$ direction.

(a) The subscript b is for the boat, w is for the water, and g is for the ground.

$$\vec{v}_{\text{bg}} = \vec{v}_{\text{bw}} + \vec{v}_{\text{wg}} = (14 \text{ km/h})\ \hat{\text{i}} + (-9 \text{ km/h})\ \hat{\text{i}} = (5 \text{ km/h})\ \hat{\text{i}}.$$

Thus, the magnitude is $|\vec{v}_{\text{bg}}| = 5 \text{ km/h}$.

(b) The direction of $\vec{v}_{\text{bg}}$ is $+x$, or upstream.

(c) We use the subscript c for the child, and obtain

$$\vec{v}_{\text{c g}} = \vec{v}_{\text{c b}} + \vec{v}_{\text{b g}} = (-6 \text{ km / h})\ \hat{\text{i}} + (5 \text{ km / h})\ \hat{\text{i}} = (-1 \text{ km / h})\ \hat{\text{i}}\ .$$

The magnitude is $|\vec{v}_{\text{cg}}| = 1 \text{ km/h}$.

(d) The direction of $\vec{v}_{\text{cg}}$ is $-x$, or downstream.

71. While moving in the same direction as the sidewalk's motion (covering a distance d relative to the ground in time t_1 = 2.50 s), Eq. 4-44 leads to

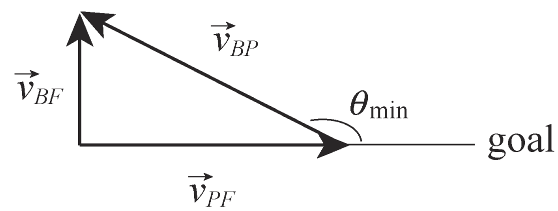
$$v_{\text{sidewalk}} + v_{\text{man running}} = \frac{d}{t_1} \ .$$

While he runs back (taking time t_2 = 10.0 s) we have

$$v_{\text{sidewalk}} - v_{\text{man running}} = -\frac{d}{t_2} \ .$$

Dividing these equations and solving for the desired ratio, we get $\frac{12.5}{7.5} = \frac{5}{3}$ = 1.67.

72. We denote the velocity of the player with $\vec{v}_{PF}$ and the relative velocity between the player and the ball be $\vec{v}_{BP}$. Then the velocity $\vec{v}_{BF}$ of the ball relative to the field is given by $\vec{v}_{BF} = \vec{v}_{PF} + \vec{v}_{BP}$. The smallest angle $\theta_{\min}$ corresponds to the case when $\vec{v}_{BF} \perp \vec{v}_{PF}$. Hence,



$$\theta_{\min} = 180° - \cos^{-1}\left(\frac{|\vec{v}_{PF}|}{|\vec{v}_{BP}|}\right) = 180° - \cos^{-1}\left(\frac{4.0 \text{ m/s}}{6.0 \text{ m/s}}\right) = 130°.$$

73. The velocity vectors (relative to the shore) for ships *A* and *B* are given by

$$\vec{v}_A = -\,(v_A \cos 45°)\,\hat{\text{i}} + (v_A \sin 45°)\,\hat{\text{j}}$$
$$\vec{v}_B = -\,(v_B \sin 40°)\,\hat{\text{i}} - (v_B \cos 40°)\,\hat{\text{j}},$$

with v_A = 24 knots and v_B = 28 knots. We take east as $+\,\hat{\text{i}}$ and north as $\hat{\text{j}}$.

(a) Their relative velocity is

$$\vec{v}_{A\,B} = \vec{v}_A - \vec{v}_B = \;(v_B \sin 40° - v_A \cos 45°)\,\hat{\text{i}} + (v_B \cos 40° + v_A \sin 45°)\,\hat{\text{j}}$$

the magnitude of which is $|\,\vec{v}_{A\,B}\,| = \sqrt{(1.03 \text{ knots})^2 + (38.4 \text{ knots})^2} \approx 38$ knots.

(b) The angle θ which $\vec{v}_{A\,B}$ makes with north is given by

$$\theta = \tan^{-1}\left(\frac{v_{AB,x}}{v_{AB,y}}\right) = \tan^{-1}\left(\frac{1.03 \text{ knots}}{38.4 \text{ knots}}\right) = 1.5°$$

which is to say that $\vec{v}_{A\,B}$ points 1.5° east of north.

(c) Since they started at the same time, their relative velocity describes at what rate the distance between them is increasing. Because the rate is steady, we have

$$t = \frac{|\,\Delta r_{AB}\,|}{|\,\vec{v}_{AB}\,|} = \frac{160}{38.4} = 4.2 \text{ h}.$$

(d) The velocity $\vec{v}_{A\,B}$ does not change with time in this problem, and $\vec{r}_{A\,B}$ is in the same direction as $\vec{v}_{A\,B}$ since they started at the same time. Reversing the points of view, we have $\vec{v}_{A\,B} = -\vec{v}_{B\,A}$ so that $\vec{r}_{A\,B} = -\vec{r}_{B\,A}$ (i.e., they are 180° opposite to each other). Hence, we conclude that *B* stays at a bearing of 1.5° west of south relative to *A* during the journey (neglecting the curvature of Earth).

74. The destination is $\vec{D} = 800 \text{ km}\,\hat{\text{j}}$ where we orient axes so that $+y$ points north and $+x$ points east. This takes two hours, so the (constant) velocity of the plane (relative to the ground) is $\vec{v}_{pg} = (400 \text{ km/h})\,\hat{\text{j}}$. This must be the vector sum of the plane's velocity with respect to the air which has (x,y) components (500cos70º, 500sin70º) and the velocity of the air (*wind*) relative to the ground $\vec{v}_{ag}$. Thus,

$$(400 \text{ km/h})\,\hat{\text{j}} = (500 \text{ km/h})\cos 70^\circ\,\hat{\text{i}} + (500 \text{ km/h})\sin 70^\circ\,\hat{\text{j}} + \vec{v}_{ag}$$

which yields

$$\vec{v}_{ag} = (-171 \text{ km/h})\hat{\text{i}} - (70.0 \text{ km/h})\hat{\text{j}}\,.$$

(a) The magnitude of $\vec{v}_{ag}$ is $|\vec{v}_{ag}| = \sqrt{(-171 \text{ km/h})^2 + (-70.0 \text{ km/h})^2} = 185 \text{ km/h}.$

(b) The direction of $\vec{v}_{ag}$ is

$$\theta = \tan^{-1}\left(\frac{-70.0 \text{ km/h}}{-171 \text{ km/h}}\right) = 22.3^\circ \quad \text{(south of west)}.$$

75. Relative to the car the velocity of the snowflakes has a vertical component of 8.0 m/s and a horizontal component of 50 km/h = 13.9 m/s. The angle θ from the vertical is found from

$$\tan\theta = \frac{v_h}{v_v} = \frac{13.9 \text{ m/s}}{8.0 \text{ m/s}} = 1.74$$

which yields $\theta = 60°$.

76. Velocities are taken to be constant; thus, the velocity of the plane relative to the ground is $\vec{v}_{PG} = (55\ \text{km})/(1/4\ \text{hour})\ \hat{\text{j}} = (220\ \text{km/h})\hat{\text{j}}$. In addition,

$$\vec{v}_{AG} = (42\ \text{km/h})(\cos 20°\hat{\text{i}} - \sin 20°\hat{\text{j}}) = (39\ \text{km/h})\hat{\text{i}} - (14\ \text{km/h})\hat{\text{j}}.$$

Using $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$, we have

$$\vec{v}_{PA} = \vec{v}_{PG} - \vec{v}_{AG} = -(39\ \text{km/h})\hat{\text{i}} + (234\ \text{km/h})\hat{\text{j}}.$$

which implies $|\vec{v}_{PA}| = 237\ \text{km/h}$, or 240 km/h (to two significant figures.)

77. Since the raindrops fall vertically relative to the train, the horizontal component of the velocity of a raindrop is v_h = 30 m/s, the same as the speed of the train. If v_v is the vertical component of the velocity and θ is the angle between the direction of motion and the vertical, then tan $\theta = v_h/v_v$. Thus $v_v = v_h/\tan\theta$ = (30 m/s)/tan 70° = 10.9 m/s. The speed of a raindrop is

$$v = \sqrt{v_h^2 + v_v^2} = \sqrt{(30\text{ m/s})^2 + (10.9\text{ m/s})^2} = 32\text{ m/s}.$$

78. This is a classic problem involving two-dimensional relative motion. We align our coordinates so that *east* corresponds to $+x$ and *north* corresponds to $+y$. We write the vector addition equation as $\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$. We have $\vec{v}_{WG} = (2.0\angle 0°)$ in the magnitude-angle notation (with the unit m/s understood), or $\vec{v}_{WG} = 2.0\hat{\text{i}}$ in unit-vector notation. We also have $\vec{v}_{BW} = (8.0\angle 120°)$ where we have been careful to phrase the angle in the 'standard' way (measured counterclockwise from the $+x$ axis), or $\vec{v}_{BW} = (-4.0\hat{\text{i}}+6.9\hat{\text{j}})$ m/s.

(a) We can solve the vector addition equation for $\vec{v}_{BG}$:

$$\vec{v}_{BG} = v_{BW} + \vec{v}_{WG} = (2.0\text{ m/s})\hat{\text{i}} + (-4.0\hat{\text{i}}+6.9\hat{\text{j}})\text{ m/s} = (-2.0\text{ m/s})\hat{\text{i}} + (6.9\text{ m/s})\hat{\text{j}}.$$

Thus, we find $|\vec{v}_{BG}| = 7.2$ m/s.

(b) The direction of $\vec{v}_{BG}$ is $\theta = \tan^{-1}[(6.9\text{ m/s})/(-2.0\text{ m/s})] = 106°$ (measured counterclockwise from the $+x$ axis), or 16° west of north.

(c) The velocity is constant, and we apply $y - y_0 = v_y t$ in a reference frame. Thus, in the *ground* reference frame, we have $(200\text{ m}) = (7.2\text{ m/s})\sin(106°)t \rightarrow t = 29$ s. Note: if a student obtains "28 s", then the student has probably neglected to take the y component properly (a common mistake).

79. We denote the police and the motorist with subscripts p and m, respectively. The coordinate system is indicated in Fig. 4-49.

(a) The velocity of the motorist with respect to the police car is

$$\vec{v}_{m\,p} = \vec{v}_m - \vec{v}_p = (-60 \text{ km/h})\hat{\text{j}} - (-80 \text{ km/h})\hat{\text{i}} = (80 \text{ km/h})\hat{\text{i}} - (60 \text{ km/h})\hat{\text{j}}.$$

(b) $\vec{v}_{m\,p}$ does happen to be along the line of sight. Referring to Fig. 4-49, we find the vector pointing from one car to another is $\vec{r} = (800 \text{ m})\hat{\text{i}} - (600 \text{ m})\,\hat{\text{j}}$ (from M to P). Since the ratio of components in $\vec{r}$ is the same as in $\vec{v}_{m\,p}$, they must point the same direction.

(c) No, they remain unchanged.

80. We make use of Eq. 4-44 and Eq. 4-45.

The velocity of Jeep P relative to A at the instant is

$$\vec{v}_{PA} = (40.0 \text{ m/s})(\cos 60°\hat{\text{i}} + \sin 60°\hat{\text{j}}) = (20.0 \text{ m/s})\hat{\text{i}} + (34.6 \text{ m/s})\hat{\text{j}}.$$

Similarly, the velocity of Jeep B relative to A at the instant is

$$\vec{v}_{BA} = (20.0 \text{ m/s})(\cos 30°\hat{\text{i}} + \sin 30°\hat{\text{j}}) = (17.3 \text{ m/s})\hat{\text{i}} + (10.0 \text{ m/s})\hat{\text{j}}.$$

Thus, the velocity of P relative to B is

$$\vec{v}_{PB} = \vec{v}_{PA} - \vec{v}_{BA} = (20.0\hat{\text{i}} + 34.6\hat{\text{j}}) \text{ m/s} - (17.3\hat{\text{i}} + 10.0\hat{\text{j}}) \text{ m/s} = (2.68 \text{ m/s})\hat{\text{i}} + (24.6 \text{ m/s})\hat{\text{j}}.$$

(a) The magnitude of $\vec{v}_{PB}$ is $|\vec{v}_{PB}| = \sqrt{(2.68 \text{ m/s})^2 + (24.6 \text{ m/s})^2} = 24.8 \text{ m/s}.$

(b) The direction of $\vec{v}_{PB}$ is $\theta = \tan^{-1}[(24.6 \text{ m/s})/(2.68 \text{ m/s})] = 83.8°$ north of east (or 6.2º east of north).

(c) The acceleration of P is

$$\vec{a}_{PA} = (0.400 \text{ m/s}^2)(\cos 60.0°\hat{\text{i}} + \sin 60.0°\hat{\text{j}}) = (0.200 \text{ m/s}^2)\hat{\text{i}} + (0.346 \text{ m/s}^2)\hat{\text{j}},$$

and $\vec{a}_{PA} = \vec{a}_{PB}$. Thus, we have $|\vec{a}_{PB}| = 0.400 \text{ m/s}^2$.

(d) The direction is 60.0° north of east (or 30.0° east of north).

81. Here, the subscript W refers to the water. Our coordinates are chosen with $+x$ being *east* and $+y$ being *north*. In these terms, the angle specifying *east* would be 0° and the angle specifying *south* would be –90° or 270°. Where the length unit is not displayed, km is to be understood.

(a) We have $\vec{v}_{A\,W} = \vec{v}_{A\,B} + \vec{v}_{B\,W}$, so that

$$\vec{v}_{A\,B} = (22 \angle -90°) - (40 \angle 37°) = (56 \angle -125°)$$

in the magnitude-angle notation (conveniently done with a vector-capable calculator in polar mode). Converting to rectangular components, we obtain

$$\vec{v}_{A\,B} = (-32\text{km/h})\,\hat{\text{i}} - (46\text{ km/h})\,\hat{\text{j}}\;.$$

Of course, this could have been done in unit-vector notation from the outset.

(b) Since the velocity-components are constant, integrating them to obtain the position is straightforward ($\vec{r} - \vec{r}_0 = \int \vec{v}\,dt$)

$$\vec{r} = (2.5 - 32t)\,\hat{\text{i}} + (4.0 - 46t)\,\hat{\text{j}}$$

with lengths in kilometers and time in hours.

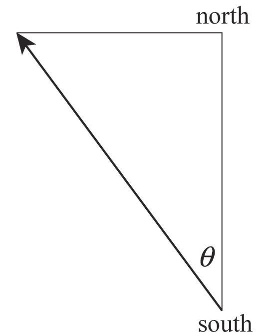
(c) The magnitude of this $\vec{r}$ is $r = \sqrt{(2.5-32t)^2 + (4.0-46t)^2}$. We minimize this by taking a derivative and requiring it to equal zero — which leaves us with an equation for t

$$\frac{dr}{dt} = \frac{1}{2}\frac{6286t - 528}{\sqrt{(2.5-32t)^2 + (4.0-46t)^2}} = 0$$

which yields $t = 0.084$ h.

(d) Plugging this value of t back into the expression for the distance between the ships (r), we obtain $r = 0.2$ km. Of course, the calculator offers more digits ($r = 0.225\ldots$), but they are not significant; in fact, the uncertainties implicit in the given data, here, should make the ship captains worry.

82. We construct a right triangle starting from the clearing on the south bank, drawing a line (200 m long) due north (*upward* in our sketch) across the river, and then a line due west (upstream, leftward in our sketch) along the north bank for a distance $(82\text{ m}) + (1.1\text{ m/s})t$, where the t-dependent contribution is the distance that the river will carry the boat downstream during time t.



The hypotenuse of this right triangle (the arrow in our sketch) also depends on t and on the boat's speed (relative to the water), and we set it equal to the Pythagorean "sum" of the triangle's sides:

$$(4.0)t = \sqrt{200^2 + (82 + 1.1t)^2}$$

which leads to a quadratic equation for t

$$46724 + 180.4t - 14.8t^2 = 0.$$

We solve this and find a positive value: t = 62.6 s.

The angle between the northward (200 m) leg of the triangle and the hypotenuse (which is measured "west of north") is then given by

$$\theta = \tan^{-1}\left(\frac{82 + 1.1t}{200}\right) = \tan^{-1}\left(\frac{151}{200}\right) = 37°.$$

83. Using displacement = velocity × time (for each constant-velocity part of the trip), along with the fact that 1 hour = 60 minutes, we have the following vector addition exercise (using notation appropriate to many vector capable calculators):

$(1667 \text{ m} \angle 0^\circ) + (1333 \text{ m} \angle -90^\circ) + (333 \text{ m} \angle 180^\circ) + (833 \text{ m} \angle -90^\circ) + (667 \text{ m} \angle 180^\circ) + (417 \text{ m} \angle -90^\circ) = (2668 \text{ m} \angle -76^\circ)$.

(a) Thus, the magnitude of the net displacement is 2.7 km.

(b) Its direction is 76° clockwise (relative to the initial direction of motion).

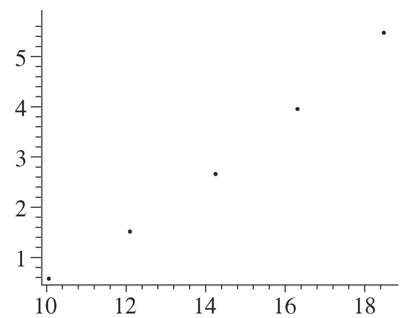
84. We compute the coordinate pairs (x, y) from $x = (v_0 \cos\theta)t$ and $y = v_0 \sin\theta t - \frac{1}{2}gt^2$ for $t = 20$ s and the speeds and angles given in the problem.

(a) We obtain

$$(x_A, y_A) = (10.1\text{ km}, 0.56\text{ km}) \qquad (x_B, y_B) = (12.1\text{ km}, 1.51\text{ km})$$
$$(x_C, y_C) = (14.3\text{ km}, 2.68\text{ km}) \qquad (x_D, y_D) = (16.4\text{ km}, 3.99\text{ km})$$

and $(x_E, y_E) = (18.5\text{ km}, 5.53\text{ km})$ which we plot in the next part.

(b) The vertical (y) and horizontal (x) axes are in kilometers. The graph does not start at the origin. The curve to "fit" the data is not shown, but is easily imagined (forming the "curtain of death").



85. Let $v_o = 2\pi(0.200 \text{ m})/(0.00500 \text{ s}) \approx 251$ m/s (using Eq. 4-35) be the speed it had in circular motion and $\theta_o = (1 \text{ hr})(360°/12 \text{ hr [for full rotation]}) = 30.0°$. Then Eq. 4-25 leads to

$$y = (2.50 \text{ m}) \tan 30.0° - \frac{(9.8 \text{ m/s}^2)(2.50 \text{ m})^2}{2(251 \text{ m/s})^2 (\cos 30.0°)^2} \approx 1.44 \text{ m}$$

which means its height above the floor is 1.44 m + 1.20 m = 2.64 m.

86. For circular motion, we must have $\vec{v}$ with direction perpendicular to $\vec{r}$ and (since the speed is constant) magnitude $v = 2\pi r/T$ where $r = \sqrt{(2.00\text{ m})^2 + (-3.00\text{ m})^2}$ and $T = 7.00\text{ s}$. The $\vec{r}$ (given in the problem statement) specifies a point in the fourth quadrant, and since the motion is clockwise then the velocity must have both components negative. Our result, satisfying these three conditions, (using unit-vector notation which makes it easy to double-check that $\vec{r}\cdot\vec{v} = 0$) for $\vec{v} = (-2.69\text{ m/s})\hat{\text{i}} + (-1.80\text{ m/s})\hat{\text{j}}$.

87. Using Eq. 2-16, we obtain $v^2 = v_0^2 - 2gh$, or $h = (v_0^2 - v^2)/2g$.

(a) Since $v = 0$ at the maximum height of an upward motion, with $v_0 = 7.00$ m/s, we have $h = (7.00 \text{ m/s})^2 / 2(9.80 \text{ m/s}^2) = 2.50$ m.

(b) The relative speed is $v_r = v_0 - v_c = 7.00 \text{ m/s} - 3.00 \text{ m/s} = 4.00 \text{ m/s}$ with respect to the floor. Using the above equation we obtain $h = (4.00 \text{ m/s})^2 / 2(9.80 \text{ m/s}^2) = 0.82$ m.

(c) The acceleration, or the rate of change of speed of the ball with respect to the ground is 9.80 m/s^2 (downward).

(d) Since the elevator cab moves at constant velocity, the rate of change of speed of the ball with respect to the cab floor is also 9.80 m/s^2 (downward).

88. Relative to the sled, the launch velocity is $\vec{v}_{o\,rel} = v_{ox}\,\hat{i} + v_{oy}\,\hat{j}$. Since the sled's motion is in the negative direction with speed v_s (note that we are treating v_s as a positive number, so the sled's velocity is actually $-v_s\,\hat{i}$), then the launch velocity relative to the ground is $\vec{v}_o = (v_{ox} - v_s)\,\hat{i} + v_{oy}\,\hat{j}$. The horizontal and vertical displacement (relative to the ground) are therefore

$$x_{\text{land}} - x_{\text{launch}} = \Delta x_{bg} = (v_{ox} - v_s)\, t_{\text{flight}}$$

$$y_{\text{land}} - y_{\text{launch}} = 0 = v_{oy}\, t_{\text{flight}} + \tfrac{1}{2}(-g)(t_{\text{flight}})^2 .$$

Combining these equations leads to

$$\Delta x_{bg} = \frac{2\, v_{ox}\, v_{oy}}{g} - \left(\frac{2v_{oy}}{g}\right) v_{s}.$$

The first term corresponds to the "y intercept" on the graph, and the second term (in parentheses) corresponds to the magnitude of the "slope." From Figure 4-54, we have

$$\Delta x_{bg} = 40 - 4v_s .$$

This implies $v_{oy} = (4.0\text{ s})(9.8\text{ m/s}^2)/2 = 19.6$ m/s, and that furnishes enough information to determine v_{ox}.

(a) $v_{ox} = 40g/2v_{oy} = (40\text{ m})(9.8\text{ m/s}^2)/(39.2\text{ m/s}) = 10$ m/s.

(b) As noted above, $v_{oy} = 19.6$ m/s.

(c) Relative to the sled, the displacement Δx_{bs} does not depend on the sled's speed, so $\Delta x_{bs} = v_{ox}\, t_{\text{flight}} = 40$ m.

(d) As in (c), relative to the sled, the displacement Δx_{bs} does not depend on the sled's speed, and $\Delta x_{bs} = v_{ox}\, t_{\text{flight}} = 40$ m.

89. We establish coordinates with $\hat{\mathrm{i}}$ pointing to the far side of the river (perpendicular to the current) and $\hat{\mathrm{j}}$ pointing in the direction of the current. We are told that the magnitude (presumed constant) of the velocity of the boat relative to the water is $|\vec{v}_{bw}|$ = 6.4 km/h. Its angle, relative to the x axis is θ. With km and h as the understood units, the velocity of the water (relative to the ground) is $\vec{v}_{wg} = (3.2 \text{ km/h})\hat{\mathrm{j}}$.

(a) To reach a point "directly opposite" means that the velocity of her boat relative to ground must be $\vec{v}_{bg} = v_{bg}\hat{\mathrm{i}}$ where $v_{bg} > 0$ is unknown. Thus, all $\hat{\mathrm{j}}$ components must cancel in the vector sum $\vec{v}_{bw} + \vec{v}_{wg} = \vec{v}_{bg}$, which means the $\vec{v}_{bw} \sin\theta = (-3.2 \text{ km/h})\ \hat{\mathrm{j}}$, so

$$\theta = \sin^{-1}[(-3.2 \text{ km/h})/(6.4 \text{ km/h})] = -30°.$$

(b) Using the result from part (a), we find $v_{bg} = v_{bw}\cos\theta = 5.5$ km/h. Thus, traveling a distance of $\ell = 6.4$ km requires a time of (6.4 km)/(5.5 km/h) = 1.15 h or 69 min.

(c) If her motion is completely along the y axis (as the problem implies) then with v_{wg} = 3.2 km/h (the water speed) we have

$$t_{\text{total}} = \frac{D}{v_{bw} + v_{wg}} + \frac{D}{v_{bw} - v_{wg}} = 1.33 \text{ h}$$

where D = 3.2 km. This is equivalent to 80 min.

(d) Since

$$\frac{D}{v_{bw}+v_{wg}} + \frac{D}{v_{bw}-v_{wg}} = \frac{D}{v_{bw}-v_{wg}} + \frac{D}{v_{bw}+v_{wg}}$$

the answer is the same as in the previous part, i.e., $t_{\text{total}} = 80 \text{ min}$.

(e) The shortest-time path should have $\theta = 0°$. This can also be shown by noting that the case of general θ leads to

$$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg} = v_{bw}\cos\theta\ \hat{\mathrm{i}} + (v_{bw}\sin\theta + v_{wg})\ \hat{\mathrm{j}}$$

where the x component of $\vec{v}_{bg}$ must equal l/t. Thus,

$$t = \frac{l}{v_{bw}\cos\theta}$$

which can be minimized using $dt/d\theta = 0$.

(f) The above expression leads to t = (6.4 km)/(6.4 km/h) = 1.0 h, or 60 min.

90. We use a coordinate system with $+x$ eastward and $+y$ upward.

(a) We note that 123° is the angle between the initial position and later position vectors, so that the angle from $+x$ to the later position vector is 40° + 123° = 163°. In unit-vector notation, the position vectors are

$$\vec{r}_1 = (360\text{ m})\cos(40°)\,\hat{\text{i}} + (360\text{ m})\sin(40°)\,\hat{\text{j}} = (276\text{ m})\hat{\text{i}} + (231\text{ m})\,\hat{\text{j}}$$
$$\vec{r}_2 = (790\text{ m})\cos(163°)\,\hat{\text{i}} + (790\text{ m})\sin(163°)\,\hat{\text{j}} = (-755\text{ m})\hat{\text{i}} + (231\text{ m})\hat{\text{j}}$$

respectively. Consequently, we plug into Eq. 4-3

$$\Delta\vec{r} = [(-755\text{ m}) - (276\text{ m})]\hat{\text{i}} + (231\text{ m} - 231\text{ m})\,\hat{\text{j}} = -(1031\text{ m})\,\hat{\text{i}}.$$

The magnitude of the displacement $\Delta\vec{r}$ is $|\Delta\vec{r}| = 1031\text{ m}$.

(b) The direction of $\Delta\vec{r}$ is $-\hat{\text{i}}$, or westward.

91. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable.

(a) With the origin at the firing point, the y coordinate of the bullet is given by $y = -\frac{1}{2}gt^2$. If t is the time of flight and y = – 0.019 m indicates where the bullet hits the target, then

$$t = \sqrt{\frac{2(0.019 \text{ m})}{9.8 \text{ m/s}^2}} = 6.2\times10^{-2} \text{ s}.$$

(b) The muzzle velocity is the initial (horizontal) velocity of the bullet. Since x = 30 m is the horizontal position of the target, we have $x = v_0t$. Thus,

$$v_0 = \frac{x}{t} = \frac{30 \text{ m}}{6.3\times10^{-2} \text{ s}} = 4.8\times10^{2} \text{ m/s}.$$

92. Eq. 4-34 describes an inverse proportionality between r and a, so that a large acceleration results from a small radius. Thus, an upper limit for a corresponds to a lower limit for r.

(a) The minimum turning radius of the train is given by

$$r_{\text{min}} = \frac{v^2}{a_{\text{max}}} = \frac{(216\ \text{km/h})^2}{(0.050)(9.8\ \text{m/s}^2)} = 7.3 \times 10^3\ \text{m}.$$

(b) The speed of the train must be reduced to no more than

$$v = \sqrt{a_{\text{max}} r} = \sqrt{0.050\left(9.8\ \text{m/s}^2\right)\left(1.00 \times 10^3\ \text{m}\right)} = 22\ \text{m/s}$$

which is roughly 80 km/h.

93. (a) With $r = 0.15$ m and $a = 3.0 \times 10^{14}$ m/s^2, Eq. 4-34 gives

$$v = \sqrt{ra} = 6.7 \times 10^6 \text{ m/s}.$$

(b) The period is given by Eq. 4-35:

$$T = \frac{2\pi r}{v} = 1.4 \times 10^{-7} \text{ s}.$$

94. We use Eq. 4-2 and Eq. 4-3.

(a) With the initial position vector as $\vec{r}_1$ and the later vector as $\vec{r}_2$, Eq. 4-3 yields

$$\Delta r = [(-2.0\text{ m}) - 5.0\text{ m}]\hat{\mathrm{i}} + [(6.0\text{ m}) - (-6.0\text{ m})]\hat{\mathrm{j}} + (2.0\text{ m} - 2.0\text{ m})\hat{\mathrm{k}} = (-7.0\text{ m})\hat{\mathrm{i}} + (12\text{ m})\hat{\mathrm{j}}$$

for the displacement vector in unit-vector notation.

(b) Since there is no *z* component (that is, the coefficient of $\hat{\mathrm{k}}$ is zero), the displacement vector is in the *xy* plane.

95. We write our magnitude-angle results in the form $(R \angle \theta)$ with SI units for the magnitude understood (m for distances, m/s for speeds, m/s^2 for accelerations). All angles θ are measured counterclockwise from $+x$, but we will occasionally refer to angles ϕ which are measured counterclockwise from the vertical line between the circle-center and the coordinate origin and the line drawn from the circle-center to the particle location (see r in the figure). We note that the speed of the particle is $v = 2\pi r/T$ where r = 3.00 m and T = 20.0 s; thus, v = 0.942 m/s. The particle is moving counterclockwise in Fig. 4-56.

(a) At t = 5.0 s, the particle has traveled a fraction of

$$\frac{t}{T} = \frac{5.00 \text{ s}}{20.0 \text{ s}} = \frac{1}{4}$$

of a full revolution around the circle (starting at the origin). Thus, relative to the circle-center, the particle is at

$$\phi = \frac{1}{4}(360°) = 90°$$

measured from vertical (as explained above). Referring to Fig. 4-56, we see that this position (which is the "3 o'clock" position on the circle) corresponds to x = 3.0 m and y = 3.0 m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R \angle \theta) = (4.2 \angle 45°)$. Although this position is easy to analyze without resorting to trigonometric relations, it is useful (for the computations below) to note that these values of x and y relative to coordinate origin can be gotten from the angle ϕ from the relations

$$x = r\sin\phi, \quad y = r - r\cos\phi .$$

Of course, $R = \sqrt{x^2 + y^2}$ and θ comes from choosing the appropriate possibility from $\tan^{-1}(y/x)$ (or by using particular functions of vector-capable calculators).

(b) At t = 7.5 s, the particle has traveled a fraction of 7.5/20 = 3/8 of a revolution around the circle (starting at the origin). Relative to the circle-center, the particle is therefore at ϕ = 3/8 (360°) = 135° measured from vertical in the manner discussed above. Referring to Fig. 4-56, we compute that this position corresponds to

$$x = (3.00 \text{ m})\sin 135° = 2.1 \text{ m}$$
$$y = (3.0 \text{ m}) - (3.0 \text{ m})\cos 135° = 5.1 \text{ m}$$

relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R \angle \theta) = (5.5 \angle 68°)$.

(c) At t = 10.0 s, the particle has traveled a fraction of 10/20 = 1/2 of a revolution around the circle. Relative to the circle-center, the particle is at ϕ = 180° measured from vertical (see explanation, above). Referring to Fig. 4-56, we see that this position corresponds to x = 0 and y = 6.0 m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R\angle\theta) = (6.0\angle 90°)$.

(d) We subtract the position vector in part (a) from the position vector in part (c):

$$(6.0\angle 90°) - (4.2\angle 45°) = (4.2\angle 135°)$$

using magnitude-angle notation (convenient when using vector-capable calculators). If we wish instead to use unit-vector notation, we write

$$\Delta\vec{R} = (0 - 3.0 \text{ m})\,\hat{\text{i}} + (6.0 \text{ m} - 3.0 \text{ m})\,\hat{\text{j}} = (-3.0 \text{ m})\hat{\text{i}} + (3.0 \text{ m})\hat{\text{j}}$$

which leads to $|\Delta\vec{R}| = 4.2 \text{ m}$ and θ = 135°.

(e) From Eq. 4-8, we have $\vec{v}_{\text{avg}} = \Delta\vec{R}/\Delta t$. With $\Delta t = 5.0$ s, we have

$$\vec{v}_{\text{avg}} = (-0.60 \text{ m/s})\,\hat{\text{i}} + (0.60 \text{ m/s})\,\hat{\text{j}}$$

in unit-vector notation or (0.85 $\angle$ 135°) in magnitude-angle notation.

(f) The speed has already been noted (v = 0.94 m/s), but its direction is best seen by referring again to Fig. 4-56. The velocity vector is tangent to the circle at its "3 o'clock position" (see part (a)), which means $\vec{v}$ is vertical. Thus, our result is $(0.94 \angle 90°)$.

(g) Again, the speed has been noted above (v = 0.94 m/s), but its direction is best seen by referring to Fig. 4-56. The velocity vector is tangent to the circle at its "12 o'clock position" (see part (c)), which means $\vec{v}$ is horizontal. Thus, our result is $(0.94 \angle 180°)$.

(h) The acceleration has magnitude $a = v^2/r = 0.30 \text{ m/s}^2$, and at this instant (see part (a)) it is horizontal (towards the center of the circle). Thus, our result is $(0.30 \angle 180°)$.

(i) Again, $a = v^2/r = 0.30 \text{ m/s}^2$, but at this instant (see part (c)) it is vertical (towards the center of the circle). Thus, our result is $(0.30 \angle 270°)$.

96. Noting that $\vec{v}_2 = 0$, then, using Eq. 4-15, the average acceleration is

$$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{0 - \left(6.30\,\hat{\text{i}} - 8.42\,\hat{\text{j}}\right)\text{ m/s}}{3\text{ s}} = \left(-2.1\,\hat{\text{i}} + 2.8\,\hat{\text{j}}\right)\text{ m/s}^2$$

97. (a) The magnitude of the displacement vector $\Delta\vec{r}$ is given by

$$|\Delta\vec{r}| = \sqrt{(21.5 \text{ km})^2 + (9.7 \text{ km})^2 + (2.88 \text{ km})^2} = 23.8 \text{ km}.$$

Thus,

$$|\vec{v}_{\text{avg}}| = \frac{|\Delta\vec{r}|}{\Delta t} = \frac{23.8 \text{ km}}{3.50 \text{ h}} = 6.79 \text{ km/h}.$$

(b) The angle θ in question is given by

$$\theta = \tan^{-1}\left(\frac{2.88 \text{ km}}{\sqrt{(21.5 \text{ km})^2 + (9.7 \text{ km})^2}}\right) = 6.96°.$$

98. The initial velocity has magnitude v_0 and because it is horizontal, it is equal to v_x the horizontal component of velocity at impact. Thus, the speed at impact is

$$\sqrt{v_0^2 + v_y^2} = 3v_0$$

where $v_y = \sqrt{2gh}$ and we have used Eq. 2-16 with Δx replaced with h = 20 m. Squaring both sides of the first equality and substituting from the second, we find

$$v_0^2 + 2gh = (3v_0)^2$$

which leads to $gh = 4v_0^2$ and therefore to $v_0 = \sqrt{(9.8 \text{ m/s}^2)(20 \text{ m})} / 2 = 7.0 \text{ m/s}$.

99. We choose horizontal x and vertical y axes such that both components of $\vec{v}_0$ are positive. Positive angles are counterclockwise from $+x$ and negative angles are clockwise from it. In unit-vector notation, the velocity at each instant during the projectile motion is

$$\vec{v} = v_0 \cos\theta_0 \,\hat{\mathrm{i}} + \left(v_0 \sin\theta_0 - gt\right)\hat{\mathrm{j}}.$$

(a) With v_0 = 30 m/s and θ_0 = 60°, we obtain $\vec{v} = (15\hat{\mathrm{i}} + 6.4\hat{\mathrm{j}})$ m/s, for t = 2.0 s. The magnitude of $\vec{v}$ is $|\vec{v}| = \sqrt{(15\text{ m/s})^2 + (6.4\text{ m/s})^2} = 16$ m/s.

(b) The direction of $\vec{v}$ is

$$\theta = \tan^{-1}[(6.4\text{ m/s})/(15\text{ m/s})] = 23°,$$

measured counterclockwise from $+x$.

(c) Since the angle is positive, it is above the horizontal.

(d) With t = 5.0 s, we find $\vec{v} = (15\hat{\mathrm{i}} - 23\hat{\mathrm{j}})$ m/s, which yields

$$|\vec{v}| = \sqrt{(15\text{ m/s})^2 + (-23\text{ m/s})^2} = 27\text{ m/s}.$$

(e) The direction of $\vec{v}$ is $\theta = \tan^{-1}[(-23\text{ m/s})/(15\text{ m/s})] = -57°$, or 57° measured *clockwise* from $+x$.

(f) Since the angle is negative, it is below the horizontal.

100. The velocity of Larry is v_1 and that of Curly is v_2. Also, we denote the length of the corridor by L. Now, Larry's time of passage is t_1 = 150 s (which must equal L/v_1), and Curly's time of passage is t_2 = 70 s (which must equal L/v_2). The time Moe takes is therefore

$$t = \frac{L}{v_1 + v_2} = \frac{1}{v_1 / L + v_2 / L} = \frac{1}{\frac{1}{150\,\text{s}} + \frac{1}{70\,\text{s}}} = 48\,\text{s}.$$

101. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the initial position for the football as it begins projectile motion in the sense of §4-5), and we let θ_0 be the angle of its initial velocity measured from the $+x$ axis.

(a) $x = 46$ m and $y = -1.5$ m are the coordinates for the landing point; it lands at time $t = 4.5$ s. Since $x = v_{0x}t$,

$$v_{0x} = \frac{x}{t} = \frac{46 \text{ m}}{4.5 \text{ s}} = 10.2 \text{ m/s}.$$

Since $y = v_{0y}t - \frac{1}{2}gt^2$,

$$v_{0y} = \frac{y + \frac{1}{2}gt^2}{t} = \frac{(-1.5 \text{ m}) + \frac{1}{2}(9.8 \text{ m/s}^2)(4.5 \text{ s})^2}{4.5 \text{ s}} = 21.7 \text{ m/s}.$$

The magnitude of the initial velocity is

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{(10.2 \text{ m/s})^2 + (21.7 \text{ m/s})^2} = 24 \text{ m/s}.$$

(b) The initial angle satisfies $\tan\theta_0 = v_{0y}/v_{0x}$. Thus, $\theta_0 = \tan^{-1}$ [(21.7 m/s)/(10.2 m/s)]= 65°.

102. We assume the ball's initial velocity is perpendicular to the plane of the net. We choose coordinates so that $(x_0, y_0) = (0, 3.0)$ m, and $v_x > 0$ (note that $v_{0y} = 0$).

(a) To (barely) clear the net, we have

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 \;\;\Rightarrow\;\; 2.24\text{ m} - 3.0\text{ m} = 0 - \frac{1}{2}\left(9.8\text{ m/s}^2\right)t^2$$

which gives $t = 0.39$ s for the time it is passing over the net. This is plugged into the x-equation to yield the (minimum) initial velocity $v_x = (8.0\text{ m})/(0.39\text{ s}) = 20.3$ m/s.

(b) We require $y = 0$ and find t from $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$. This value $\left(t = \sqrt{2(3.0\text{ m})/(9.8\text{ m/s}^2)} = 0.78\text{ s}\right)$ is plugged into the x-equation to yield the (maximum) initial velocity $v_x = (17.0\text{ m})/(0.78\text{ s}) = 21.7$ m/s.

103. (a) With $\Delta x = 8.0$ m, $t = \Delta t_1$, $a = a_x$, and $v_{ox} = 0$, Eq. 2-15 gives

$$8.0 \text{ m} = \tfrac{1}{2} a_x (\Delta t_1)^2 ,$$

and the corresponding expression for motion along the y axis leads to

$$\Delta y = 12 \text{ m} = \tfrac{1}{2} a_y (\Delta t_1)^2 .$$

Dividing the second expression by the first leads to $a_y / a_x = 3/2 = 1.5$.

(b) Letting $t = 2\Delta t_1$, then Eq. 2-15 leads to $\Delta x = (8.0 \text{ m})(2)^2 = 32$ m, which implies that its x coordinate is now $(4.0 + 32)$ m = 36 m. Similarly, $\Delta y = (12 \text{ m})(2)^2 = 48$ m, which means its y coordinate has become $(6.0 + 48)$ m = 54 m.

104. We apply Eq. 4-34 to solve for speed v and Eq. 4-35 to find the period T.

(a) We obtain

$$v = \sqrt{ra} = \sqrt{(5.0 \text{ m})(7.0)(9.8 \text{ m/s}^2)} = 19 \text{ m/s}.$$

(b) The time to go around once (the period) is $T = 2\pi r/v = 1.7$ s. Therefore, in one minute ($t = 60$ s), the astronaut executes

$$\frac{t}{T} = \frac{60 \text{ s}}{1.7 \text{ s}} = 35$$

revolutions. Thus, 35 rev/min is needed to produce a centripetal acceleration of $7g$ when the radius is 5.0 m.

(c) As noted above, $T = 1.7$ s.

105. The radius of Earth may be found in Appendix C.

(a) The speed of an object at Earth's equator is $v = 2\pi R/T$, where R is the radius of Earth (6.37×10^6 m) and T is the length of a day (8.64×10^4 s):

$$v = 2\pi(6.37 \times 10^6 \text{ m})/(8.64 \times 10^4 \text{ s}) = 463 \text{ m/s}.$$

The magnitude of the acceleration is given by

$$a = \frac{v^2}{R} = \frac{(463 \text{ m/s})^2}{6.37 \times 10^6 \text{ m}} = 0.034 \text{ m/s}^2.$$

(b) If T is the period, then $v = 2\pi R/T$ is the speed and the magnitude of the acceleration is

$$a = \frac{v^2}{R} = \frac{(2\pi R/T)^2}{R} = \frac{4\pi^2 R}{T^2}.$$

Thus,

$$T = 2\pi\sqrt{\frac{R}{a}} = 2\pi\sqrt{\frac{6.37 \times 10^6 \text{ m}}{9.8 \text{ m/s}^2}} = 5.1 \times 10^3 \text{ s} = 84 \text{ min}.$$

106. When the escalator is stalled the speed of the person is $v_p = \ell/t$, where ℓ is the length of the escalator and t is the time the person takes to walk up it. This is v_p = (15 m)/(90 s) = 0.167 m/s. The escalator moves at v_e = (15 m)/(60 s) = 0.250 m/s. The speed of the person walking up the moving escalator is

$$v = v_p + v_e = 0.167 \text{ m/s} + 0.250 \text{ m/s} = 0.417 \text{ m/s}$$

and the time taken to move the length of the escalator is

$$t = \ell / v = (15 \text{ m}) / (0.417 \text{ m/s}) = 36 \text{ s}.$$

If the various times given are independent of the escalator length, then the answer does not depend on that length either. In terms of ℓ (in meters) the speed (in meters per second) of the person walking on the stalled escalator is $\ell/90$, the speed of the moving escalator is $\ell/60$, and the speed of the person walking on the moving escalator is $v = (\ell/90) + (\ell/60) = 0.0278\ell$. The time taken is $t = \ell/v = \ell/0.0278\ell = 36\,\text{s}$ and is independent of ℓ.

107. (a) Eq. 2-15 can be applied to the vertical (y axis) motion related to reaching the maximum height (when $t = 3.0$ s and $v_y = 0$):

$$y_{\max} - y_0 = v_y t - \frac{1}{2} g t^2 \ .$$

With ground level chosen so $y_0 = 0$, this equation gives the result $y_{\max} = \frac{1}{2} g (3.0 \text{ s})^2 = 44$ m.

(b) After the moment it reached maximum height, it is falling; at $t = 2.5$ s, it will have fallen an amount given by Eq. 2-18:

$$y_{\text{fence}} - y_{\max} = (0)(2.5 \text{ s}) - \frac{1}{2} g (2.5 \text{ s})^2$$

which leads to $y_{\text{fence}} = 13$ m.

(c) Either the *range* formula, Eq. 4-26, can be used or one can note that after passing the fence, it will strike the ground in 0.5 s (so that the total "fall-time" equals the "rise-time"). Since the horizontal component of velocity in a projectile-motion problem is constant (neglecting air friction), we find the original x-component from 97.5 m = v_{0x}(5.5 s) and then apply it to that final 0.5 s. Thus, we find $v_{0x} = 17.7$ m/s and that after the fence

$$\Delta x = (17.7 \text{ m/s})(0.5 \text{ s}) = 8.9 \text{ m}.$$

108. With $g_B = 9.8128 \text{ m/s}^2$ and $g_M = 9.7999 \text{ m/s}^2$, we apply Eq. 4-26:

$$R_M - R_B = \frac{v_0^2 \sin 2\theta_0}{g_M} - \frac{v_0^2 \sin 2\theta_0}{g_B} = \frac{v_0^2 \sin 2\theta_0}{g_B}\left(\frac{g_B}{g_M} - 1\right)$$

which becomes

$$R_M - R_B = R_B \left(\frac{9.8128 \text{ m/s}^2}{9.7999 \text{ m/s}^2} - 1\right)$$

and yields (upon substituting $R_B = 8.09$ m) $R_M - R_B = 0.01$ m = 1 cm.

109. We make use of Eq. 4-25.

(a) By rearranging Eq. 4-25, we obtain the initial speed:

$$v_0 = \frac{x}{\cos\theta_0}\sqrt{\frac{g}{2(x\tan\theta_0 - y)}}$$

which yields $v_0 = 255.5 \approx 2.6 \times 10^2$ m/s for $x = 9400$ m, $y = -3300$ m, and $\theta_0 = 35°$.

(b) From Eq. 4-21, we obtain the time of flight:

$$t = \frac{x}{v_0\cos\theta_0} = \frac{9400\text{ m}}{(255.5\text{ m/s})\cos 35°} = 45\text{ s}.$$

(c) We expect the air to provide resistance but no appreciable lift to the rock, so we would need a greater launching speed to reach the same target.

110. When moving in the same direction as the jet stream (of speed v_s), the time is

$$t_1 = \frac{d}{v_{ja} + v_s},$$

where d = 4000 km is the distance and v_{ja} is the speed of the jet relative to the air (1000 km/h). When moving against the jet stream, the time is

$$t_2 = \frac{d}{v_{ja} - v_s},$$

where $t_2 - t_1 = \frac{70}{60}\text{ h}$. Combining these equations and using the quadratic formula to solve gives v_s = 143 km/h.

111. Since the x and y components of the acceleration are constants, we can use Table 2-1 for the motion along both axes. This can be handled individually (for Δx and Δy) or together with the unit-vector notation (for Δr). Where units are not shown, SI units are to be understood.

(a) Since $\vec{r}_0 = 0$, the position vector of the particle is (adapting Eq. 2-15)

$$\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 = \left(8.0\,\hat{\mathrm{j}}\right)t + \frac{1}{2}\left(4.0\,\hat{\mathrm{i}} + 2.0\,\hat{\mathrm{j}}\right)t^2 = \left(2.0t^2\right)\hat{\mathrm{i}} + \left(8.0t + 1.0t^2\right)\hat{\mathrm{j}}.$$

Therefore, we find when x = 29 m, by solving $2.0t^2 = 29$, which leads to t = 3.8 s. The y coordinate at that time is y = (8.0 m/s)(3.8 s) + (1.0 m/s^2)(3.8 s)^2 = 45 m.

(b) Adapting Eq. 2-11, the velocity of the particle is given by

$$\vec{v} = \vec{v}_0 + \vec{a}t.$$

Thus, at t = 3.8 s, the velocity is

$$\vec{v} = (8.0 \text{ m/s})\,\hat{\mathrm{j}} + \left((4.0 \text{ m/s}^2)\,\hat{\mathrm{i}} + (2.0 \text{ m/s}^2)\,\hat{\mathrm{j}}\right)(3.8 \text{ s}) = (15.2 \text{ m/s})\,\hat{\mathrm{i}} + (15.6 \text{ m/s})\,\hat{\mathrm{j}}$$

which has a magnitude of

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.2 \text{ m/s})^2 + (15.6 \text{ m/s})^2} = 22 \text{ m/s}.$$

112. We make use of Eq. 4-34 and Eq. 4-35.

(a) The track radius is given by

$$r = \frac{v^2}{a} = \frac{(9.2 \text{ m/s})^2}{3.8 \text{ m/s}^2} = 22 \text{ m}.$$

(b) The period of the circular motion is $T = 2\pi(22 \text{ m})/(9.2 \text{ m/s}) = 15$ s.

113. Since this problem involves constant downward acceleration of magnitude *a*, similar to the projectile motion situation, we use the equations of §4-6 as long as we substitute *a* for *g*. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and

$$v_{0x} = v_0 = 1.00\times10^9 \text{ cm/s}.$$

(a) If ℓ is the length of a plate and *t* is the time an electron is between the plates, then $\ell = v_0 t$, where v_0 is the initial speed. Thus

$$t = \frac{\ell}{v_0} = \frac{2.00\,\text{cm}}{1.00\times10^9\,\text{cm/s}} = 2.00\times10^{-9}\text{ s}.$$

(b) The vertical displacement of the electron is

$$y = -\frac{1}{2}at^2 = -\frac{1}{2}\left(1.00\times10^{17}\text{ cm/s}^2\right)\left(2.00\times10^{-9}\text{ s}\right)^2 = -0.20\,\text{cm} = -2.00\text{ mm},$$

or $|\,y\,| = 2.00$ mm.

(c) The *x* component of velocity does not change: $v_x = v_0 = 1.00\times10^9$ cm/s $= 1.00\times10^7$ m/s.

(d) The *y* component of the velocity is

$$v_y = a_y t = \left(1.00\times10^{17}\text{ cm/s}^2\right)\left(2.00\times10^{-9}\text{ s}\right) = 2.00\times10^8\text{ cm/s} = 2.00\times10^6\text{ m/s}.$$

114. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion of the shot ball. We are allowed to use Table 2-1 (with Δy replacing Δx) because the ball has constant acceleration motion. We use primed variables (except t) with the constant-velocity elevator (so $v' = 10$ m/s), and unprimed variables with the ball (with initial velocity $v_0 = v' + 20 = 30$ m/s, relative to the ground). SI units are used throughout.

(a) Taking the time to be zero at the instant the ball is shot, we compute its maximum height y (relative to the ground) with $v^2 = v_0^2 - 2g(y - y_0)$, where the highest point is characterized by $v = 0$. Thus,

$$y = y_o + \frac{v_0^2}{2g} = 76 \text{ m}$$

where $y_o = y'_o + 2 = 30$ m (where $y'_o = 28$ m is given in the problem) and $v_0 = 30$ m/s relative to the ground as noted above.

(b) There are a variety of approaches to this question. One is to continue working in the frame of reference adopted in part (a) (which treats the ground as motionless and "fixes" the coordinate origin to it); in this case, one describes the elevator motion with $y' = y'_o + v't$ and the ball motion with Eq. 2-15, and solves them for the case where they reach the same point at the same time. Another is to work in the frame of reference of the elevator (the boy in the elevator might be oblivious to the fact the elevator is moving since it isn't accelerating), which is what we show here in detail:

$$\Delta y_e = v_{0_e} t - \frac{1}{2} g t^2 \quad \Rightarrow \quad t = \frac{v_{0_e} + \sqrt{v_{0e}^2 - 2g\Delta y_e}}{g}$$

where $v_{0e} = 20$ m/s is the initial velocity of the ball relative to the elevator and $\Delta y_e = -2.0$ m is the ball's displacement relative to the floor of the elevator. The positive root is chosen to yield a positive value for t; the result is $t = 4.2$ s.

115. (a) With $v = c/10 = 3\times10^7$ m/s and $a = 20g = 196\ \text{m/s}^2$, Eq. 4-34 gives

$$r = v^2/a = 4.6\times10^{12}\ \text{m}.$$

(b) The period is given by Eq. 4-35: $T = 2\pi r/v = 9.6\times10^5$ s. Thus, the time to make a quarter-turn is $T/4 = 2.4 \times 10^5$ s or about 2.8 days.

116. Using the same coordinate system assumed in Eq. 4-25, we rearrange that equation to solve for the initial speed:

$$v_0 = \frac{x}{\cos\theta_0}\sqrt{\frac{g}{2\,(x\tan\theta_0 - y)}}$$

which yields $v_0 = 23$ ft/s for $g = 32$ ft/s^2, $x = 13$ ft, $y = 3$ ft and $\theta_0 = 55°$.

117. The (box)car has velocity $\vec{v}_{c\,g} = v_1\,\hat{\mathrm{i}}$ relative to the ground, and the bullet has velocity

$$\vec{v}_{0b\,g} = v_2\cos\theta\,\hat{\mathrm{i}} + v_2\sin\theta\,\hat{\mathrm{j}}$$

relative to the ground before entering the car (we are neglecting the effects of gravity on the bullet). While in the car, its velocity relative to the outside ground is $\vec{v}_{bg} = 0.8v_2\cos\theta\,\hat{\mathrm{i}} + 0.8v_2\sin\theta\,\hat{\mathrm{j}}$ (due to the 20% reduction mentioned in the problem). The problem indicates that the velocity of the bullet in the car *relative to the car* is (with v_3 unspecified) $\vec{v}_{b\,c} = v_3\,\hat{\mathrm{j}}$. Now, Eq. 4-44 provides the condition

$$\begin{aligned}\vec{v}_{b\,g} &= \vec{v}_{b\,c} + \vec{v}_{c\,g}\\ 0.8v_2\cos\theta\;\hat{\mathrm{i}} + 0.8v_2\sin\theta\;\hat{\mathrm{j}} &= v_3\;\hat{\mathrm{j}} + v_1\;\hat{\mathrm{i}}\end{aligned}$$

so that equating x components allows us to find θ. If one wished to find v_3 one could also equate the y components, and from this, if the car width were given, one could find the time spent by the bullet in the car, but this information is not asked for (which is why the width is irrelevant). Therefore, examining the x components in SI units leads to

$$\theta = \cos^{-1}\left(\frac{v_1}{0.8v_2}\right) = \cos^{-1}\left(\frac{85\ \text{km/h}\left(\frac{1000\ \text{m/km}}{3600\ \text{s/h}}\right)}{0.8\ (650\ \text{m/s})}\right)$$

which yields 87° for the direction of $\vec{v}_{b\,g}$ (measured from $\hat{\mathrm{i}}$, which is the direction of motion of the car). The problem asks, “from what direction was it fired?” — which means the answer is not 87° but rather its supplement 93° (measured from the direction of motion). Stating this more carefully, in the coordinate system we have adopted in our solution, the bullet velocity vector is in the first quadrant, at 87° measured counterclockwise from the $+x$ direction (the direction of train motion), which means that the direction from which the bullet came (where the sniper is) is in the third quadrant, at –93° (that is, 93° measured clockwise from $+x$).

118. Since $v_y^2 = v_{0y}^2 - 2g\Delta y$, and v_y=0 at the target, we obtain

$$v_{0y} = \sqrt{2\left(9.80 \text{ m/s}^2\right)\left(5.00 \text{ m}\right)} = 9.90 \text{ m/s}$$

(a) Since $v_0 \sin \theta_0 = v_{0y}$, with $v_0 = 12.0$ m/s, we find $\theta_0 = 55.6°$.

(b) Now, $v_y = v_{0y} - gt$ gives $t = (9.90 \text{ m/s})/(9.80 \text{ m/s}^2) = 1.01$ s. Thus, $\Delta x = (v_0 \cos \theta_0)t$ = 6.85 m.

(c) The velocity at the target has only the v_x component, which is equal to $v_{0x} = v_0 \cos \theta_0$ = 6.78 m/s.

119. From the figure, the three displacements can be written as

$$\vec{d}_1 = d_1(\cos\theta_1\hat{\text{i}} + \sin\theta_1\hat{\text{j}}) = (5.00\text{ m})(\cos 30°\hat{\text{i}} + \sin 30°\hat{\text{j}}) = (4.33\text{ m})\hat{\text{i}} + (2.50\text{ m})\hat{\text{j}}$$

$$\begin{aligned}\vec{d}_2 &= d_2[\cos(180° + \theta_1 - \theta_2)\hat{\text{i}} + \sin(180° + \theta_1 - \theta_2)\hat{\text{j}}] = (8.00\text{ m})(\cos 160°\hat{\text{i}} + \sin 160°\hat{\text{j}}) \\ &= (-7.52\text{ m})\hat{\text{i}} + (2.74\text{ m})\hat{\text{j}}\end{aligned}$$

$$\begin{aligned}\vec{d}_3 &= d_3[\cos(360° - \theta_3 - \theta_2 + \theta_1)\hat{\text{i}} + \sin(360° - \theta_3 - \theta_2 + \theta_1)\hat{\text{j}}] = (12.0\text{ m})(\cos 260°\hat{\text{i}} + \sin 260°\hat{\text{j}}) \\ &= (-2.08\text{ m})\hat{\text{i}} - (11.8\text{ m})\hat{\text{j}}\end{aligned}$$

where the angles are measured from the $+x$ axis. The net displacement is

$$\vec{d} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = (-5.27\text{ m})\hat{\text{i}} - (6.58\text{ m})\hat{\text{j}}.$$

(a) The magnitude of the net displacement is

$$|\vec{d}| = \sqrt{(-5.27\text{ m})^2 + (-6.58\text{ m})^2} = 8.43\text{ m}.$$

(b) The direction of $\vec{d}$ is

$$\theta = \tan^{-1}\left(\frac{d_y}{d_x}\right) = \tan^{-1}\left(\frac{-6.58\text{ m}}{-5.27\text{ m}}\right) = 51.3° \text{ or } 231°.$$

We choose $231°$ (measured counterclockwise from $+x$) since the desired angle is in the third quadrant. An equivalent answer is $-129°$ (measured clockwise from $+x$).

120. With $v_0 = 30.0$ m/s and $R = 20.0$ m, Eq. 4-26 gives

$$\sin 2\theta_0 = \frac{gR}{v_0^2} = 0.218.$$

Because $\sin\phi = \sin(180° - \phi)$, there are two roots of the above equation:

$$2\theta_0 = \sin^{-1}(0.218) = 12.58° \text{ and } 167.4°.$$

which correspond to the two possible launch angles that will hit the target (in the absence of air friction and related effects).

(a) The smallest angle is $\theta_0 = 6.29°$.

(b) The greatest angle is and $\theta_0 = 83.7°$.

An alternative approach to this problem in terms of Eq. 4-25 (with $y = 0$ and $1/\cos^2 = 1 + \tan^2$) is possible — and leads to a quadratic equation for $\tan\theta_0$ with the roots providing these two possible θ_0 values.

121. On the one hand, we could perform the vector addition of the displacements with a vector-capable calculator in polar mode $((75 \angle 37°) + (65 \angle -90°) = (63 \angle -18°))$, but in keeping with Eq. 3-5 and Eq. 3-6 we will show the details in unit-vector notation. We use a 'standard' coordinate system with $+x$ East and $+y$ North. Lengths are in kilometers and times are in hours.

(a) We perform the vector addition of individual displacements to find the net displacement of the camel.

$$\Delta\vec{r}_1 = (75\text{ km})\cos(37°)\,\hat{\text{i}} + (75\text{ km})\sin(37°)\,\hat{\text{j}}$$
$$\Delta\vec{r}_2 = (-65\text{ km})\,\hat{\text{j}}$$
$$\Delta\vec{r} = \Delta\vec{r}_1 + \Delta\vec{r}_2 = (60\text{ km})\,\hat{\text{i}} - (20\text{ km})\,\hat{\text{j}}\ .$$

If it is desired to express this in magnitude-angle notation, then this is equivalent to a vector of length $|\Delta\vec{r}| = \sqrt{(60\text{ km})^2 + (-20\text{ km})^2} = 63\text{ km}$.

(b) The direction of $\Delta\vec{r}$ is $\theta = \tan^{-1}[(-20\text{ km})/(60\text{ km})] = -18°$, or $18°$ south of east.

(c) We use the result from part (a) in Eq. 4-8 along with the fact that $\Delta t = 90$ h. In unit vector notation, we obtain

$$\vec{v}_{\text{avg}} = \frac{(60\,\hat{\text{i}} - 20\,\hat{\text{j}})\text{ km}}{90\text{ h}} = (0.67\,\hat{\text{i}} - 0.22\,\hat{\text{j}})\text{ km/h}.$$

This leads to $|\vec{v}_{\text{avg}}| = 0.70\text{ km/h}$.

(d) The direction of $\vec{v}_{avg}$ is $\theta = \tan^{-1}[(-0.22\text{ km/h})/(0.67\text{ km/h})] = -18°$, or $18°$ south of east.

(e) The average speed is distinguished from the magnitude of average velocity in that it depends on the total distance as opposed to the net displacement. Since the camel travels 140 km, we obtain (140 km)/(90 h) = 1.56 km/h ≈ 1.6 km/h .

(f) The net displacement is required to be the 90 km East from A to B. The displacement from the resting place to B is denoted $\Delta\vec{r}_3$. Thus, we must have

$$\Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 = (90\text{ km})\,\hat{\text{i}}$$

which produces $\Delta\vec{r}_3 = (30\text{ km})\hat{\text{i}} + (20\text{ km})\hat{\text{j}}$ in unit-vector notation, or $(36 \angle 33°)$ in magnitude-angle notation. Therefore, using Eq. 4-8 we obtain

$$|\vec{v}_{\text{avg}}| = \frac{36\text{ km}}{(120-90)\text{ h}} = 1.2\text{ km/h}.$$

(g) The direction of $\vec{v}_{avg}$ is the same as $\vec{r}_3$ (that is, 33° north of east).

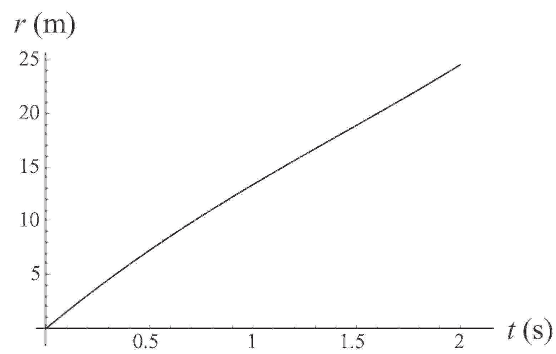
122. We make use of Eq. 4-21 and Eq.4-22.

(a) With v_o = 16 m/s, we square Eq. 4-21 and Eq. 4-22 and add them, then (using Pythagoras' theorem) take the square root to obtain r:

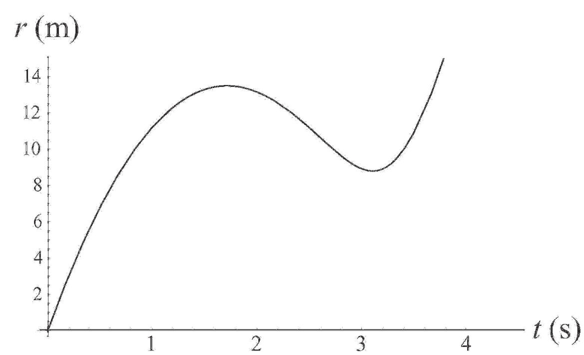
$$r=\sqrt{(x-x_0)^2+(y-y_0)^2}=\sqrt{(v_0\cos\theta_0 t)^2+(v_0\sin\theta_0 t-gt^2/2)^2}$$

$$=t\sqrt{v_0^2-v_0 g\sin\theta_0 t+g^2t^2/4}$$

Below we plot r as a function of time for θ_o = 40.0º:



(b) For this next graph for r versus t we set θ_o = 80.0º.



(c) Differentiating r with respect to t, we obtain

$$\frac{dr}{dt}=\frac{v_0^2-3v_0 gt\sin\theta_0/2+g^2t^2/2}{\sqrt{v_0^2-v_0 g\sin\theta_0 t+g^2t^2/4}}$$

Setting $dr/dt=0$, with $v_0=16.0$ m/s and $\theta_0=40.0°$, we have $256-151t+48t^2=0$. The equation has no real solution. This means that the maximum is reached at the end of the flight, with

$$t_{total}=2v_0\sin\theta_0/g=2(16.0\text{ m/s})\sin(40.0°)/(9.80\text{ m/s}^2)=2.10\text{ s}.$$

(d) The value of r is given by

$$r = (2.10)\sqrt{(16.0)^2 - (16.0)(9.80)\sin 40.0°(2.10) + (9.80)^2(2.10)^2/4} = 25.7 \text{ m}.$$

(e) The horizontal distance is $r_x = v_0 \cos\theta_0 t = (16.0 \text{ m/s})\cos 40.0°(2.10 \text{ s}) = 25.7 \text{ m}.$

(f) The vertical distance is $r_y = 0$.

(g) For the θ_0 = 80º launch, the condition for maximum r is $256 - 232t + 48t^2 = 0$, or $t = 1.71$ s (the other solution, t = 3.13 s, corresponds to a minimum.)

(h) The distance traveled is

$$r = (1.71)\sqrt{(16.0)^2 - (16.0)(9.80)\sin 80.0°(1.71) + (9.80)^2(1.71)^2/4} = 13.5 \text{ m}.$$

(i) The horizontal distance is

$$r_x = v_0 \cos\theta_0 t = (16.0 \text{ m/s})\cos 80.0°(1.71 \text{ s}) = 4.75 \text{ m}.$$

(j) The vertical distance is

$$r_y = v_0 \sin\theta_0 t - \frac{gt^2}{2} = (16.0 \text{ m/s})\sin 80°(1.71 \text{ s}) - \frac{(9.80 \text{ m/s}^2)(1.71 \text{ s})^2}{2} = 12.6 \text{ m}.$$

123. Using the same coordinate system assumed in Eq. 4-25, we find x for the elevated cannon from

$$y = x\tan\theta_0 - \frac{gx^2}{2(v_0\cos\theta_0)^2} \quad \text{where } y = -30 \text{ m.}$$

Using the quadratic formula (choosing the positive root), we find

$$x = v_0\cos\theta_0\left(\frac{v_0\sin\theta_0 + \sqrt{(v_0\sin\theta_0)^2 - 2gy}}{g}\right)$$

which yields x = 715 m for v_0 = 82 m/s and θ_0 = 45°. This is 29 m longer than the 686 m found in that Sample Problem. Since the "9" in 29 m is not reliable, due to the low level of precision in the given data, we write the answer as 3×10^1 m.

124. (a) Using the same coordinate system assumed in Eq. 4-25, we find

$$y = x\tan\theta_0 - \frac{gx^2}{2(v_0\cos\theta_0)^2} = -\frac{gx^2}{2v_0^2} \quad \text{if } \theta_0 = 0.$$

Thus, with $v_0 = 3.0 \times 10^6$ m/s and $x = 1.0$ m, we obtain $y = -5.4 \times 10^{-13}$ m which is not practical to measure (and suggests why gravitational processes play such a small role in the fields of atomic and subatomic physics).

(b) It is clear from the above expression that $|y|$ decreases as v_0 is increased.

125. At maximum height, the y-component of a projectile's velocity vanishes, so the given 10 m/s is the (constant) x-component of velocity.

(a) Using v_{0y} to denote the y-velocity 1.0 s before reaching the maximum height, then (with $v_y = 0$) the equation $v_y = v_{0y} - gt$ leads to $v_{0y} = 9.8$ m/s. The magnitude of the velocity vector (or *speed*) at that moment is therefore

$$\sqrt{v_x^2 + v_{0y}^2} = \sqrt{(10\text{ m/s})^2 + (9.8\text{ m/s})^2} = 14\text{ m/s}.$$

(b) It is clear from the symmetry of the problem that the speed is the same 1.0 s after reaching the top, as it was 1.0 s before (14 m/s again). This may be verified by using $v_y = v_{0y} - gt$ again but now "starting the clock" at the highest point so that $v_{0y} = 0$ (and $t = 1.0\text{ s}$). This leads to $v_y = -9.8$ m/s and $\sqrt{(10\text{ m/s})^2 + \left(-9.8\text{ m/s}\right)^2} = 14\text{ m/s}$.

(c) The x_0 value may be obtained from $x = 0 = x_0 + (10\text{ m/s})(1.0\text{s})$, which yields $x_0 = -10\,\text{m}$.

(d) With $v_{0y} = 9.8$ m/s denoting the y-component of velocity one second before the top of the trajectory, then we have $y = 0 = y_0 + v_{0y}t - \frac{1}{2}gt^2$ where $t = 1.0$ s. This yields $y_0 = -4.9$ m.

(e) By using $x - x_0 = (10\text{ m/s})(1.0\text{ s})$ where $x_0 = 0$, we obtain $x = 10$ m.

(f) Let $t = 0$ at the top with $y_0 = v_{0y} = 0$. From $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$, we have, for $t = 1.0$ s,

$$y = -(9.8\text{ m/s}^2)(1.0\text{ s})^2/2 = -4.9\text{ m}.$$

126. With no acceleration in the x direction yet a constant acceleration of 1.4 m/s^2 in the y direction, the position (in meters) as a function of time (in seconds) must be

$$\vec{r} = (6.0t)\hat{\mathrm{i}} + \left(\frac{1}{2}(1.4)t^2\right)\hat{\mathrm{j}}$$

and $\vec{v}$ is its derivative with respect to t.

(a) At $t = 3.0$ s, therefore, $\vec{v} = (6.0\hat{\mathrm{i}} + 4.2\hat{\mathrm{j}})$ m/s.

(b) At $t = 3.0$ s, the position is $\vec{r} = (18\hat{\mathrm{i}} + 6.3\hat{\mathrm{j}})$ m.

127. We note that

$$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$$

describes a right triangle, with one leg being $\vec{v}_{PG}$ (east), another leg being $\vec{v}_{AG}$ (magnitude = 20, direction = south), and the hypotenuse being $\vec{v}_{PA}$ (magnitude = 70). Lengths are in kilometers and time is in hours. Using the Pythagorean theorem, we have

$$\left|\vec{v}_{PA}\right| = \sqrt{|\vec{v}_{PG}|^2 + |\vec{v}_{AG}|^2} \;\Rightarrow\; 70 \text{ km/h} = \sqrt{|\vec{v}_{PG}|^2 + (20 \text{ km/h})^2}$$

which is easily solved for the ground speed: $|\vec{v}_{PG}| = 67 \text{ km/h}$.

128. The figure offers many interesting points to analyze, and others are easily inferred (such as the point of maximum height). The focus here, to begin with, will be the final point shown (1.25 s after the ball is released) which is when the ball returns to its original height. In English units, $g = 32$ ft/s^2.

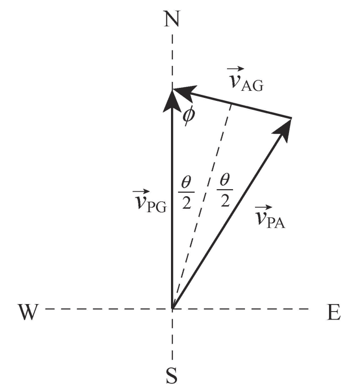
(a) Using $x - x_0 = v_x t$ we obtain $v_x = (40 \text{ ft})/(1.25 \text{ s}) = 32$ ft/s. And $y - y_0 = 0 = v_{0y}t - \frac{1}{2}gt^2$ yields $v_{0y} = \frac{1}{2}\left(32 \text{ ft/s}^2\right)\left(1.25 \text{ s}\right) = 20 \text{ ft/s}$. Thus, the initial speed is

$$v_0 = |\vec{v}_0| = \sqrt{(32 \text{ ft/s})^2 + (20 \text{ ft/s})^2} = 38 \text{ ft/s}.$$

(b) Since $v_y = 0$ at the maximum height and the horizontal velocity stays constant, then the speed at the top is the same as $v_x = 32$ ft/s.

(c) We can infer from the figure (or compute from $v_y = 0 = v_{0y} - gt$) that the time to reach the top is 0.625 s. With this, we can use $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ to obtain 9.3 ft (where $y_0 =$ 3 ft has been used). An alternative approach is to use $v_y^2 = v_{0y}^2 - 2g\left(y - y_0\right)$.

129. We denote $\vec{v}_{PG}$ as the velocity of the plane relative to the ground, $\vec{v}_{AG}$ as the velocity of the air relative to the ground, and $\vec{v}_{PA}$ as the velocity of the plane relative to the air.



(a) The vector diagram is shown on the right: $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$. Since the magnitudes v_{PG} and v_{PA} are equal the triangle is isosceles, with two sides of equal length.

Consider either of the right triangles formed when the bisector of θ is drawn (the dashed line). It bisects $\vec{v}_{AG}$, so

$$\sin(\theta/2) = \frac{v_{AG}}{2v_{PG}} = \frac{70.0\text{ mi/h}}{2(135\text{ mi/h})}$$

which leads to θ = 30.1°. Now $\vec{v}_{AG}$ makes the same angle with the E-W line as the dashed line does with the N-S line. The wind is blowing in the direction 15.0° north of west. Thus, it is blowing *from* 75.0° east of south.

(b) The plane is headed along $\vec{v}_{PA}$, in the direction 30.0° east of north. There is another solution, with the plane headed 30.0° west of north and the wind blowing 15° north of east (that is, from 75° west of south).

130. Taking derivatives of $\vec{r} = 2t\hat{\mathrm{i}} + 2\sin(\pi t/4)\hat{\mathrm{j}}$ (with lengths in meters, time in seconds and angles in radians) provides expressions for velocity and acceleration:

$$\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{\mathrm{i}} + \frac{\pi}{2}\cos\left(\frac{\pi t}{4}\right)\hat{\mathrm{j}}$$
$$\vec{a} = \frac{d\vec{v}}{dt} = -\frac{\pi^2}{8}\sin\left(\frac{\pi t}{4}\right)\hat{\mathrm{j}}.$$

Thus, we obtain:

time t			0.0	1.0	2.0	3.0	4.0
(a)	$\vec{r}$ position	x	0.0	2.0	4.0	6.0	8.0
		y	0.0	1.4	2.0	1.4	0.0
(b)	$\vec{v}$ velocity	v_x		2.0	2.0	2.0	
		v_y		1.1	0.0	−1.1	
(c)	$\vec{a}$ acceleration	a_x		0.0	0.0	0.0	
		a_y		−0.87	−1.2	−0.87	

And the path of the particle in the xy plane is shown in the following graph. The arrows indicating the velocities are not shown here, but they would appear as tangent-lines, as expected.

131. We make use of Eq. 4-24 and Eq. 4-25.

(a) With x = 180 m, θ_o = 30°, and v_o = 43 m/s, we obtain

$$y = \tan(30°)(180\text{ m}) - \frac{(9.8\text{ m/s}^2)(180\text{ m})^2}{2(43\text{ m/s})^2(\cos 30°)^2} = -11\text{ m}$$

or $|y| = 11\text{ m}$. This implies the rise is roughly eleven meters above the fairway.

(b) The horizontal component (in the absence of air friction) is unchanged, but the vertical component increases (see Eq. 4-24). The Pythagorean theorem then gives the magnitude of final velocity (right before striking the ground): 45 m/s.

132. We let g_p denote the magnitude of the gravitational acceleration on the planet. A number of the points on the graph (including some "inferred" points — such as the max height point at $x = 12.5$ m and $t = 1.25$ s) can be analyzed profitably; for future reference, we label (with subscripts) the first ($(x_0, y_0) = (0, 2)$ at $t_0 = 0$) and last ("final") points ($(x_f, y_f) = (25, 2)$ at $t_f = 2.5$), with lengths in meters and time in seconds.

(a) The x-component of the initial velocity is found from $x_f - x_0 = v_{0x}\, t_f$. Therefore, $v_{0x} = 25/2.5 = 10$ m/s. And we try to obtain the y-component from $y_f - y_0 = 0 = v_{0y} t_f - \frac{1}{2} g_p t_f^2$. This gives us $v_{0y} = 1.25 g_p$, and we see we need another equation (by analyzing another point, say, the next-to-last one) $y - y_0 = v_{0y} t - \frac{1}{2} g_p t^2$ with $y = 6$ and $t = 2$; this produces our second equation $v_{0y} = 2 + g_p$. Simultaneous solution of these two equations produces results for v_{0y} and g_p (relevant to part (b)). Thus, our complete answer for the initial velocity is $\vec{v} = (10 \text{ m/s})\hat{\text{i}} + (10 \text{ m/s})\hat{\text{j}}$.

(b) As a by-product of the part (a) computations, we have $g_p = 8.0$ m/s^2.

(c) Solving for t_g (the time to reach the ground) in $y_g = 0 = y_0 + v_{0y} t_g - \frac{1}{2} g_p t_g^2$ leads to a positive answer: $t_g = 2.7$ s.

(d) With $g = 9.8$ m/s^2, the method employed in part (c) would produce the quadratic equation $-4.9 t_g^2 + 10 t_g + 2 = 0$ and then the positive result $t_g = 2.2$ s.

Chapter 5

1. We apply Newton's second law (specifically, Eq. 5-2).

(a) We find the x component of the force is

$$F_x = ma_x = ma \cos 20.0° = (1.00\,\text{kg})\left(2.00\,\text{m/s}^2\right) \cos 20.0° = 1.88\,\text{N}.$$

(b) The y component of the force is

$$F_y = ma_y = ma \sin 20.0° = (1.0\,\text{kg})\left(2.00\,\text{m/s}^2\right) \sin 20.0° = 0.684\,\text{N}.$$

(c) In unit-vector notation, the force vector is

$$\vec{F} = F_x\hat{\text{i}} + F_y\hat{\text{j}} = (1.88\text{ N})\hat{\text{i}} + (0.684\text{ N})\hat{\text{j}}\,.$$

2. We apply Newton's second law (Eq. 5-1 or, equivalently, Eq. 5-2). The net force applied on the chopping block is $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2$, where the vector addition is done using unit-vector notation. The acceleration of the block is given by $\vec{a} = \left(\vec{F}_1 + \vec{F}_2\right) / m$.

(a) In the first case

$$\vec{F}_1 + \vec{F}_2 = \left[(3.0\text{N})\hat{\text{i}} + (4.0\text{N})\hat{\text{j}}\right] + \left[(-3.0\text{N})\hat{\text{i}} + (-4.0\text{N})\hat{\text{j}}\right] = 0$$

so $\vec{a} = 0$.

(b) In the second case, the acceleration $\vec{a}$ equals

$$\frac{\vec{F}_1 + \vec{F}_2}{m} = \frac{\left((3.0\text{N})\hat{\text{i}} + (4.0\text{N})\hat{\text{j}}\right) + \left((-3.0\text{N})\hat{\text{i}} + (4.0\text{N})\hat{\text{j}}\right)}{2.0\text{kg}} = (4.0\,\text{m/s}^2)\hat{\text{j}}.$$

(c) In this final situation, $\vec{a}$ is

$$\frac{\vec{F}_1 + \vec{F}_2}{m} = \frac{\left((3.0\text{N})\hat{\text{i}} + (4.0\text{N})\hat{\text{j}}\right) + \left((3.0\text{N})\hat{\text{i}} + (-4.0\text{N})\hat{\text{j}}\right)}{2.0\,\text{kg}} = (3.0\,\text{m/s}^2)\hat{\text{i}}.$$

3. We are only concerned with horizontal forces in this problem (gravity plays no direct role). We take East as the $+x$ direction and North as $+y$. This calculation is efficiently implemented on a vector-capable calculator, using magnitude-angle notation (with SI units understood).

$$\vec{a} = \frac{\vec{F}}{m} = \frac{(9.0 \angle 0°) + (8.0 \angle 118°)}{3.0} = (2.9 \angle 53°)$$

Therefore, the acceleration has a magnitude of 2.9 m/s^2.

4. We note that $m\vec{a} = (-16\text{ N})\,\hat{\text{i}} + (12\text{ N})\,\hat{\text{j}}$. With the other forces as specified in the problem, then Newton's second law gives the third force as

$$\vec{F}_3 = m\vec{a} - \vec{F}_1 - \vec{F}_2 = (-34\text{ N})\,\hat{\text{i}} - (12\text{ N})\,\hat{\text{j}}.$$

5. We denote the two forces $\vec{F}_1$ and $\vec{F}_2$. According to Newton's second law, $\vec{F}_1 + \vec{F}_2 = m\vec{a}$, so $\vec{F}_2 = m\vec{a} - \vec{F}_1$.

(a) In unit vector notation $\vec{F}_1 = (20.0 \text{ N})\hat{\text{i}}$ and

$$\vec{a} = -\left(12.0 \sin 30.0° \text{m/s}^2\right)\hat{\text{i}} - \left(12.0 \cos 30.0° \text{m/s}^2\right)\hat{\text{j}} = -\left(6.00 \text{ m/s}^2\right)\hat{\text{i}} - \left(10.4 \text{m/s}^2\right)\hat{\text{j}}.$$

Therefore,

$$\begin{aligned}\vec{F}_2 &= (2.00 \text{kg})\left(-6.00 \text{m/s}^2\right)\hat{\text{i}} + (2.00 \text{ kg})\left(-10.4 \text{ m/s}^2\right)\hat{\text{j}} - (20.0 \text{ N})\hat{\text{i}} \\ &= (-32.0 \text{ N})\hat{\text{i}} - (20.8 \text{ N})\hat{\text{j}}.\end{aligned}$$

(b) The magnitude of $\vec{F}_2$ is

$$|\vec{F}_2| = \sqrt{F_{2x}^2 + F_{2y}^2} = \sqrt{(-32.0 \text{ N})^2 + (-20.8 \text{ N})^2} = 38.2 \text{ N}.$$

(c) The angle that $\vec{F}_2$ makes with the positive x axis is found from

$$\tan \theta = (F_{2y}/F_{2x}) = [(-20.8 \text{ N})/(-32.0 \text{ N})] = 0.656.$$

Consequently, the angle is either 33.0° or 33.0° + 180° = 213°. Since both the x and y components are negative, the correct result is 213°. An alternative answer is $213° - 360° = -147°$.

6. Since $\vec{v}$ = constant, we have $\vec{a} = 0$, which implies

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = m\vec{a} = 0 .$$

Thus, the other force must be

$$\vec{F}_2 = -\vec{F}_1 = (-2\text{ N})\,\hat{\text{i}} + (6\text{ N})\,\hat{\text{j}} .$$

7. The net force applied on the chopping block is $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$, where the vector addition is done using unit-vector notation. The acceleration of the block is given by $\vec{a} = \left(\vec{F}_1 + \vec{F}_2 + \vec{F}_3\right)/m$.

(a) The forces exerted by the three astronauts can be expressed in unit-vector notation as follows:

$$\begin{aligned}
\vec{F}_1 &= (32\text{ N})\left(\cos 30°\,\hat{\text{i}} + \sin 30°\,\hat{\text{j}}\right) = (27.7\text{ N})\,\hat{\text{i}} + (16\text{ N})\,\hat{\text{j}} \\
\vec{F}_2 &= (55\text{ N})\left(\cos 0°\,\hat{\text{i}} + \sin 0°\,\hat{\text{j}}\right) = (55\text{ N})\,\hat{\text{i}} \\
\vec{F}_3 &= (41\text{ N})\left(\cos(-60°)\,\hat{\text{i}} + \sin(-60°)\,\hat{\text{j}}\right) = (20.5\text{ N})\,\hat{\text{i}} - (35.5\text{ N})\,\hat{\text{j}}.
\end{aligned}$$

The resultant acceleration of the asteroid of mass m = 120 kg is therefore

$$\vec{a} = \frac{\left(27.7\hat{\text{i}} + 16\hat{\text{j}}\right)\text{N} + \left(55\hat{\text{i}}\right)\text{N} + \left(20.5\hat{\text{i}} - 35.5\hat{\text{j}}\right)\text{N}}{120\text{ kg}} = (0.86\text{m/s}^2)\hat{\text{i}} - (0.16\text{m/s}^2)\hat{\text{j}}\,.$$

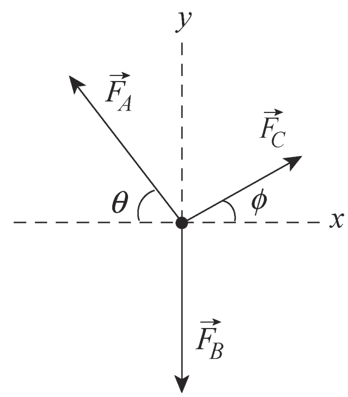
(b) The magnitude of the acceleration vector is

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2} = \sqrt{(0.86\text{ m/s}^2)^2 + \left(-0.16\text{ m/s}^2\right)^2} = 0.88\text{ m/s}^2\,.$$

(c) The vector $\vec{a}$ makes an angle θ with the $+x$ axis, where

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-0.16\text{ m/s}^2}{0.86\text{ m/s}^2}\right) = -11°\,.$$

8. Since the tire remains stationary, by Newton's second law, the net force must be zero:



$$\vec{F}_{\text{net}} = \vec{F}_A + \vec{F}_B + \vec{F}_C = m\vec{a} = 0.$$

From the free-body diagram shown on the right, we have

$$0 = \sum F_{\text{net},x} = F_C \cos\phi - F_A \cos\theta$$
$$0 = \sum F_{\text{net},y} = F_A \sin\theta + F_C \sin\phi - F_B$$

To solve for F_B, we first compute ϕ. With $F_A = 220\text{ N}$, $F_C = 170\text{ N}$ and $\theta = 47°$, we get

$$\cos\phi = \frac{F_A \cos\theta}{F_C} = \frac{(220\text{ N})\cos 47.0°}{170\text{ N}} = 0.883 \quad \Rightarrow \ \phi = 28.0°$$

Substituting the value into the second force equation, we find

$$F_B = F_A \sin\theta + F_C \sin\phi = (220\text{ N})\sin 47.0° + (170\text{ N})\sin 28.0 = 241\text{ N}.$$

9. The velocity is the derivative (with respect to time) of given function x, and the acceleration is the derivative of the velocity. Thus, $a = 2c - 3(2.0)(2.0)t$, which we use in Newton's second law: $F = (2.0 \text{ kg})a = 4.0c - 24t$ (with SI units understood). At $t = 3.0$ s, we are told that $F = -36$ N. Thus, $-36 = 4.0c - 24(3.0)$ can be used to solve for c. The result is $c = +9.0 \text{ m/s}^2$.

10. To solve the problem, we note that acceleration is the second time derivative of the position function, and the net force is related to the acceleration via Newton's second law. Thus, differentiating

$$x(t) = -13.00 + 2.00t + 4.00t^2 - 3.00t^3$$

twice with respect to t, we get

$$\frac{dx}{dt} = 2.00 + 8.00t - 9.00t^2, \quad \frac{d^2x}{dt^2} = 8.00 - 18.0t$$

The net force acting on the particle at $t = 3.40 \text{ s}$ is

$$\vec{F} = m\frac{d^2x}{dt^2}\hat{\mathrm{i}} = (0.150)\left[8.00 - 18.0(3.40)\right]\hat{\mathrm{i}} = (-7.98 \text{ N})\hat{\mathrm{i}}$$

11. To solve the problem, we note that acceleration is the second time derivative of the position function; it is a vector and can be determined from its components. The net force is related to the acceleration via Newton's second law. Thus, differentiating $x(t) = -15.0 + 2.00t + 4.00t^3$ twice with respect to t, we get

$$\frac{dx}{dt} = 2.00 - 12.0t^2, \quad \frac{d^2x}{dt^2} = -24.0t$$

Similarly, differentiating $y(t) = 25.0 + 7.00t - 9.00t^2$ twice with respect to t yields

$$\frac{dy}{dt} = 7.00 - 18.0t, \quad \frac{d^2y}{dt^2} = -18.0$$

(a) The acceleration is

$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} = \frac{d^2x}{dt^2}\hat{\mathrm{i}} + \frac{d^2y}{dt^2}\hat{\mathrm{j}} = (-24.0t)\hat{\mathrm{i}} + (-18.0)\hat{\mathrm{j}}.$$

At $t = 0.700\ \mathrm{s}$, we have $\vec{a} = (-16.8)\hat{\mathrm{i}} + (-18.0)\hat{\mathrm{j}}$ with a magnitude of

$$a = |\vec{a}| = \sqrt{(-16.8)^2 + (-18.0)^2} = 24.6\ \mathrm{m/s^2}.$$

Thus, the magnitude of the force is $F = ma = (0.34\ \mathrm{kg})(24.6\ \mathrm{m/s^2}) = 8.37\ \mathrm{N}.$

(b) The angle $\vec{F}$ or $\vec{a} = \vec{F}/m$ makes with $+x$ is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-18.0\ \mathrm{m/s^2}}{-16.8\ \mathrm{m/s^2}}\right) = 47.0° \text{ or } -133°.$$

We choose the latter ($-133°$) since $\vec{F}$ is in the third quadrant.

(c) The direction of travel is the direction of a tangent to the path, which is the direction of the velocity vector:

$$\vec{v}(t) = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} = \frac{dx}{dt}\hat{\mathrm{i}} + \frac{dy}{dt}\hat{\mathrm{j}} = (2.00 - 12.0t^2)\hat{\mathrm{i}} + (7.00 - 18.0t)\hat{\mathrm{j}}.$$

At $t = 0.700\ \mathrm{s}$, we have $\vec{v}(t = 0.700\ \mathrm{s}) = (-3.88\ \mathrm{m/s})\hat{\mathrm{i}} + (-5.60\ \mathrm{m/s})\hat{\mathrm{j}}.$ Therefore, the angle $\vec{v}$ makes with $+x$ is

$$\theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-5.60\ \mathrm{m/s}}{-3.88\ \mathrm{m/s}}\right) = 55.3° \text{ or } -125°.$$

We choose the latter ($-125°$) since $\vec{v}$ is in the third quadrant.

12. From the slope of the graph we find $a_x = 3.0$ m/s^2. Applying Newton's second law to the x axis (and taking θ to be the angle between F_1 and F_2), we have

$$F_1 + F_2 \cos\theta = m a_x \quad \Rightarrow \quad \theta = 56°.$$

13. (a) – (c) In all three cases the scale is not accelerating, which means that the two cords exert forces of equal magnitude on it. The scale reads the magnitude of either of these forces. In each case the tension force of the cord attached to the salami must be the same in magnitude as the weight of the salami because the salami is not accelerating. Thus the scale reading is mg, where m is the mass of the salami. Its value is (11.0 kg) (9.8 m/s^2) = 108 N.

14. Three vertical forces are acting on the block: the earth pulls down on the block with gravitational force 3.0 N; a spring pulls up on the block with elastic force 1.0 N; and, the surface pushes up on the block with normal force F_N. There is no acceleration, so

$$\sum F_y = 0 = F_N + (1.0\ \text{N}) + (-3.0\ \text{N})$$

yields $F_N = 2.0$ N.

(a) By Newton's third law, the force exerted by the block on the surface has that same magnitude but opposite direction: 2.0 N.

(b) The direction is down.

15. (a) From the fact that T_3 = 9.8 N, we conclude the mass of disk D is 1.0 kg. Both this and that of disk C cause the tension T_2 = 49 N, which allows us to conclude that disk C has a mass of 4.0 kg. The weights of these two disks plus that of disk B determine the tension T_1 = 58.8 N, which leads to the conclusion that m_B = 1.0 kg. The weights of all the disks must add to the 98 N force described in the problem; therefore, disk A has mass 4.0 kg.

(b) m_B = 1.0 kg, as found in part (a).

(c) m_C = 4.0 kg, as found in part (a).

(d) m_D = 1.0 kg, as found in part (a).

16. (a) There are six legs, and the vertical component of the tension force in each leg is $T\sin\theta$ where $\theta = 40°$. For vertical equilibrium (zero acceleration in the y direction) then Newton's second law leads to

$$6T\sin\theta = mg \Rightarrow T = \frac{mg}{6\sin\theta}$$

which (expressed as a multiple of the bug's weight mg) gives roughly $T/mg \approx 0.260$.

(b) The angle θ is measured from horizontal, so as the insect "straightens out the legs" θ will increase (getting closer to $90°$), which causes $\sin\theta$ to increase (getting closer to 1) and consequently (since $\sin\theta$ is in the denominator) causes T to decrease.

17. (a) The coin undergoes free fall. Therefore, with respect to ground, its acceleration is

$$\vec{a}_{\text{coin}} = \vec{g} = (-9.8 \text{ m/s}^2)\hat{\text{j}}.$$

(b) Since the customer is being pulled down with an acceleration of $\vec{a}'_{\text{customer}} = 1.24\vec{g} = (-12.15 \text{ m/s}^2)\hat{\text{j}}$, the acceleration of the coin with respect to the customer is

$$\vec{a}_{\text{rel}} = \vec{a}_{\text{coin}} - \vec{a}'_{\text{customer}} = (-9.8 \text{ m/s}^2)\hat{\text{j}} - (-12.15 \text{ m/s}^2)\hat{\text{j}} = (+2.35 \text{ m/s}^2)\hat{\text{j}}.$$

(c) The time it takes for the coin to reach the ceiling is

$$t = \sqrt{\frac{2h}{a_{\text{rel}}}} = \sqrt{\frac{2(2.20 \text{ m})}{2.35 \text{ m/s}^2}} = 1.37 \text{ s}.$$

(d) Since gravity is the only force acting on the coin, the actual force on the coin is

$$\vec{F}_{\text{coin}} = m\vec{a}_{\text{coin}} = m\vec{g} = (0.567\times10^{-3} \text{ kg})(-9.8 \text{ m/s}^2)\hat{\text{j}} = (-5.56\times10^{-3} \text{ N})\hat{\text{j}}.$$

(e) In the customer's frame, the coin travels upward at a constant acceleration. Therefore, the apparent force on the coin is

$$\vec{F}_{\text{app}} = m\vec{a}_{\text{rel}} = (0.567\times10^{-3} \text{ kg})(+2.35 \text{ m/s}^2)\hat{\text{j}} = (+1.33\times10^{-3} \text{ N})\hat{\text{j}}.$$

18. We note that the rope is 22.0° from vertical – and therefore 68.0° from horizontal.

(a) With $T = 760$ N, then its components are

$$\vec{T} = T\cos 68.0°\,\hat{\mathrm{i}} + T\sin 68.0°\,\hat{\mathrm{j}} = (285\,\mathrm{N})\,\hat{\mathrm{i}} + (705\,\mathrm{N})\,\hat{\mathrm{j}}.$$

(b) No longer in contact with the cliff, the only other force on Tarzan is due to earth's gravity (his weight). Thus,

$$\vec{F}_{\mathrm{net}} = \vec{T} + \vec{W} = (285\ \mathrm{N})\,\hat{\mathrm{i}} + (705\ \mathrm{N})\,\hat{\mathrm{j}} - (820\ \mathrm{N})\,\hat{\mathrm{j}} = (285\mathrm{N})\,\hat{\mathrm{i}} - (115\ \mathrm{N})\,\hat{\mathrm{j}}.$$

(c) In a manner that is efficiently implemented on a vector-capable calculator, we convert from rectangular (x, y) components to magnitude-angle notation:

$$\vec{F}_{\mathrm{net}} = (285,\ -115) \rightarrow (307\angle -22.0°)$$

so that the net force has a magnitude of 307 N.

(d) The angle (see part (c)) has been found to be –22.0°, or 22.0° below horizontal (away from cliff).

(e) Since $\vec{a} = \vec{F}_{\mathrm{net}}/m$ where $m = W/g = 83.7$ kg, we obtain $\vec{a} = 3.67\ \mathrm{m/s^2}$.

(f) Eq. 5-1 requires that $\vec{a} \parallel \vec{F}_{\mathrm{net}}$ so that the angle is also –22.0°, or 22.0° below horizontal (away from cliff).

19. (a) Since the acceleration of the block is zero, the components of the Newton's second law equation yield

$$T - mg \sin \theta = 0$$
$$F_N - mg \cos \theta = 0.$$

Solving the first equation for the tension in the string, we find

$$T = mg \sin \theta = (8.5 \text{ kg})(9.8 \text{ m/s}^2) \sin 30° = 42 \text{ N} .$$

(b) We solve the second equation in part (a) for the normal force F_N:

$$F_N = mg \cos \theta = (8.5 \text{ kg})(9.8 \text{ m/s}^2) \cos 30° = 72 \text{ N} .$$

(c) When the string is cut, it no longer exerts a force on the block and the block accelerates. The x component of the second law becomes $-mg\sin\theta = ma$, so the acceleration becomes

$$a = -g \sin \theta = -(9.8 \text{ m/s}^2) \sin 30° = -4.9 \text{ m/s}^2.$$

The negative sign indicates the acceleration is down the plane. The magnitude of the acceleration is 4.9 m/s^2.

20. We take rightwards as the $+x$ direction. Thus, $\vec{F}_1 = (20\text{ N})\hat{\text{i}}$. In each case, we use Newton's second law $\vec{F}_1 + \vec{F}_2 = m\vec{a}$ where $m = 2.0$ kg.

(a) If $\vec{a} = (+10\text{ m/s}^2)\,\hat{\text{i}}$, then the equation above gives $\vec{F}_2 = 0$.

(b) If , $\vec{a} = (+20\text{m/s}^2)\,\hat{\text{i}}$, then that equation gives $\vec{F}_2 = (20\text{N})\hat{\text{i}}$.

(c) If $\vec{a} = 0$, then the equation gives $\vec{F}_2 = (-20\text{N})\hat{\text{i}}$.

(d) If $\vec{a} = (-10\text{ m/s}^2)\,\hat{\text{i}}$, the equation gives $\vec{F}_2 = (-40\text{N})\hat{\text{i}}$.

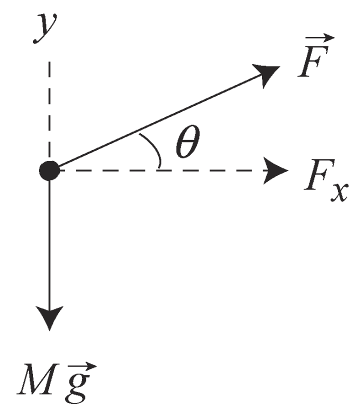
(e) If $\vec{a} = (-20\text{ m/s}^2)\,\hat{\text{i}}$, the equation gives $\vec{F}_2 = (-60\text{N})\hat{\text{i}}$.

21. (a) The slope of each graph gives the corresponding component of acceleration. Thus, we find a_x = 3.00 m/s^2 and a_y = –5.00 m/s^2. The magnitude of the acceleration vector is therefore $a = \sqrt{(3.00\ \mathrm{m/s^2})^2 + (-5.00\ \mathrm{m/s^2})^2} = 5.83\ \mathrm{m/s^2}$, and the force is obtained from this by multiplying with the mass (m= 2.00 kg). The result is $F = ma$ =11.7 N.

(b) The direction of the force is the same as that of the acceleration:

$$\theta = \tan^{-1}\,[(-5.00\ \mathrm{m/s^2})/(3.00\ \mathrm{m/s^2})] = -59.0°.$$

22. The free-body diagram of the cars is shown on the right. The force exerted by John Massis is



$$F = 2.5mg = 2.5(80\text{ kg})(9.8\text{ m/s}^2) = 1960\text{ N}.$$

Since the motion is along the horizontal x-axis, using Newton's second law, we have $Fx = F\cos\theta = Ma_x$, where M is the total mass of the railroad cars. Thus, the acceleration of the cars is

$$a_x = \frac{F\cos\theta}{M} = \frac{(1960\text{ N})\cos 30°}{(7.0\times10^5\text{ N}/9.8\text{ m/s}^2)} = 0.024\text{ m/s}^2.$$

Using Eq. 2-16, the speed of the car at the end of the pull is

$$v_x = \sqrt{2a_x\Delta x} = \sqrt{2(0.024\text{ m/s}^2)(1.0\text{ m})} = 0.22\text{ m/s}.$$

23. (a) The acceleration is

$$a = \frac{F}{m} = \frac{20\ \text{N}}{900\,\text{kg}} = 0.022\ \text{m/s}^2\ .$$

(b) The distance traveled in 1 day (= 86400 s) is

$$s = \frac{1}{2}at^2 = \frac{1}{2}\left(0.0222\,\text{m/s}^2\right)\left(86400\,\text{s}\right)^2 = 8.3 \times 10^7\ \text{m}\ .$$

(c) The speed it will be traveling is given by

$$v = at = \left(0.0222\ \text{m/s}^2\right)\left(86400\ \text{s}\right) = 1.9 \times 10^3\ \text{m/s}\ .$$

24. Some assumptions (not so much for realism but rather in the interest of using the given information efficiently) are needed in this calculation: we assume the fishing line and the path of the salmon are horizontal. Thus, the weight of the fish contributes only (via Eq. 5-12) to information about its mass ($m = W/g = 8.7$ kg). Our $+x$ axis is in the direction of the salmon's velocity (away from the fisherman), so that its acceleration (''deceleration'') is negative-valued and the force of tension is in the $-x$ direction: $\vec{T} = -T$. We use Eq. 2-16 and SI units (noting that $v = 0$).

$$v^2 = v_0^2 + 2a\Delta x \quad \Rightarrow \quad a = -\frac{v_0^2}{2\Delta x} = -\frac{(2.8\ \text{m/s})^2}{2(0.11\ \text{m})} = -36\ \text{m/s}^2 .$$

Assuming there are no significant horizontal forces other than the tension, Eq. 5-1 leads to

$$\vec{T} = m\vec{a} \quad \Rightarrow \quad -T = (8.7\,\text{kg})\left(-36\,\text{m/s}^2\right)$$

which results in $T = 3.1 \times 10^2$ N.

25. In terms of magnitudes, Newton's second law is $F = ma$, where $F = \left|\vec{F}_{\text{net}}\right|$, $a = |\vec{a}|$, and m is the (always positive) mass. The magnitude of the acceleration can be found using constant acceleration kinematics (Table 2-1). Solving $v = v_0 + at$ for the case where it starts from rest, we have $a = v/t$ (which we interpret in terms of magnitudes, making specification of coordinate directions unnecessary). The velocity is

$$v = (1600 \text{ km/h})\ (1000 \text{ m/km})/(3600 \text{ s/h}) = 444 \text{ m/s},$$

so

$$F = ma = m\frac{v}{t} = (500\,\text{kg})\ \frac{444\,\text{m/s}}{1.8\,\text{s}} = 1.2 \times 10^5 \text{ N}.$$

26. The stopping force $\vec{F}$ and the path of the passenger are horizontal. Our $+x$ axis is in the direction of the passenger's motion, so that the passenger's acceleration (''deceleration'') is negative-valued and the stopping force is in the $-x$ direction: $\vec{F} = -F\,\hat{\mathrm{i}}$. Using Eq. 2-16 with

$$v_0 = (53 \text{ km/h})(1000 \text{ m/km})/(3600 \text{ s/h}) = 14.7 \text{ m/s}$$

and $v = 0$, the acceleration is found to be

$$v^2 = v_0^2 + 2a\Delta x \quad \Rightarrow \quad a = -\frac{v_0^2}{2\Delta x} = -\frac{(14.7 \text{ m/s})^2}{2(0.65 \text{ m})} = -167 \text{ m/s}^2 .$$

Assuming there are no significant horizontal forces other than the stopping force, Eq. 5-1 leads to

$$\vec{F} = m\vec{a} \quad \Rightarrow \quad -F = (41 \text{ kg})\left(-167 \text{ m/s}^2\right)$$

which results in $F = 6.8 \times 10^3$ N.

27. We choose up as the $+y$ direction, so $\vec{a} = (-3.00\,\text{m/s}^2)\hat{\text{j}}$ (which, without the unit-vector, we denote as a since this is a 1-dimensional problem in which Table 2-1 applies). From Eq. 5-12, we obtain the firefighter's mass: $m = W/g = 72.7$ kg.

(a) We denote the force exerted by the pole on the firefighter $\vec{F}_{\text{fp}} = F_{\text{fp}}\,\hat{\text{j}}$ and apply Eq. 5-1. Since $\vec{F}_{\text{net}} = m\vec{a}$, we have

$$F_{\text{fp}} - F_g = ma \quad \Rightarrow \quad F_{\text{fp}} - 712\text{ N} = (72.7\text{ kg})(-3.00\text{ m/s}^2)$$

which yields $F_{\text{fp}} = 494$ N.

(b) The fact that the result is positive means $\vec{F}_{\text{fp}}$ points up.

(c) Newton's third law indicates $\vec{F}_{\text{fp}} = -\vec{F}_{\text{pf}}$, which leads to the conclusion that $|\vec{F}_{\text{pf}}| = 494\text{ N}$.

(d) The direction of $\vec{F}_{\text{pf}}$ is down.

28. The stopping force $\vec{F}$ and the path of the toothpick are horizontal. Our $+x$ axis is in the direction of the toothpick's motion, so that the toothpick's acceleration ("deceleration") is negative-valued and the stopping force is in the $-x$ direction: $\vec{F} = -F\,\hat{\mathrm{i}}$. Using Eq. 2-16 with $v_0 = 220$ m/s and $v = 0$, the acceleration is found to be

$$v^2 = v_0^2 + 2a\Delta x \quad \Rightarrow \quad a = -\frac{v_0^2}{2\Delta x} = -\frac{(220\ \text{m/s})^2}{2(0.015\ \text{m})} = -1.61\times10^6\ \text{m/s}^2.$$

Thus, the magnitude of the force exerted by the branch on the toothpick is

$$F = m\,|\,a\,| = (1.3\times10^{-4}\ \text{kg})(1.61\times10^6\ \text{m/s}^2) = 2.1\times10^2\ \text{N}.$$

29. The acceleration of the electron is vertical and for all practical purposes the only force acting on it is the electric force. The force of gravity is negligible. We take the $+x$ axis to be in the direction of the initial velocity and the $+y$ axis to be in the direction of the electrical force, and place the origin at the initial position of the electron. Since the force and acceleration are constant, we use the equations from Table 2-1: $x = v_0 t$ and

$$y = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{F}{m}\right)t^2 .$$

The time taken by the electron to travel a distance x (= 30 mm) horizontally is $t = x/v_0$ and its deflection in the direction of the force is

$$y = \frac{1}{2}\frac{F}{m}\left(\frac{x}{v_0}\right)^2 = \frac{1}{2}\left(\frac{4.5\times10^{-16}\ \text{N}}{9.11\times10^{-31}\ \text{kg}}\right)\left(\frac{30\times10^{-3}\ \text{m}}{1.2\times10^{7}\ \text{m/s}}\right)^2 = 1.5\times10^{-3}\ \text{m} .$$

30. The stopping force $\vec{F}$ and the path of the car are horizontal. Thus, the weight of the car contributes only (via Eq. 5-12) to information about its mass ($m = W/g = 1327$ kg). Our $+x$ axis is in the direction of the car's velocity, so that its acceleration ("deceleration") is negative-valued and the stopping force is in the $-x$ direction: $\vec{F} = -F\,\hat{\text{i}}$.

(a) We use Eq. 2-16 and SI units (noting that $v = 0$ and $v_0 = 40(1000/3600) = 11.1$ m/s).

$$v^2 = v_0^2 + 2a\Delta x \quad \Rightarrow \quad a = -\frac{v_0^2}{2\Delta x} = -\frac{(11.1\text{ m/s})^2}{2(15\text{ m})}$$

which yields $a = -\,4.12$ m/s^2. Assuming there are no significant horizontal forces other than the stopping force, Eq. 5-1 leads to

$$\vec{F} = m\vec{a} \quad \Rightarrow \quad -F = (1327\text{ kg})\left(-4.12\text{ m/s}^2\right)$$

which results in $F = 5.5 \times 10^3$ N.

(b) Eq. 2-11 readily yields $t = -v_0/a = 2.7$ s.

(c) Keeping F the same means keeping a the same, in which case (since $v = 0$) Eq. 2-16 expresses a direct proportionality between Δx and v_0^2. Therefore, doubling v_0 means quadrupling Δx. That is, the new over the old stopping distances is a factor of 4.0.

(d) Eq. 2-11 illustrates a direct proportionality between t and v_0 so that doubling one means doubling the other. That is, the new time of stopping is a factor of 2.0 greater than the one found in part (b).

31. The acceleration vector as a function of time is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(8.00t\,\hat{\mathrm{i}} + 3.00t^2\,\hat{\mathrm{j}}\right)\ \mathrm{m/s} = (8.00\,\hat{\mathrm{i}} + 6.00t\,\hat{\mathrm{j}})\ \mathrm{m/s^2}.$$

(a) The magnitude of the force acting on the particle is

$$F = ma = m\,|\vec{a}\,| = (3.00)\sqrt{(8.00)^2 + (6.00t)^2} = (3.00)\sqrt{64.0 + 36.0\,t^2}\ \mathrm{N}.$$

Thus, $F = 35.0\ \mathrm{N}$ corresponds to $t = 1.415\ \mathrm{s}$, and the acceleration vector at this instant is

$$\vec{a} = [8.00\,\hat{\mathrm{i}} + 6.00(1.415)\,\hat{\mathrm{j}}]\ \mathrm{m/s^2} = (8.00\ \mathrm{m/s^2})\,\hat{\mathrm{i}} + (8.49\ \mathrm{m/s^2})\hat{\mathrm{j}}.$$

The angle $\vec{a}$ makes with $+x$ is

$$\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{8.49\ \mathrm{m/s^2}}{8.00\ \mathrm{m/s^2}}\right) = 46.7°.$$

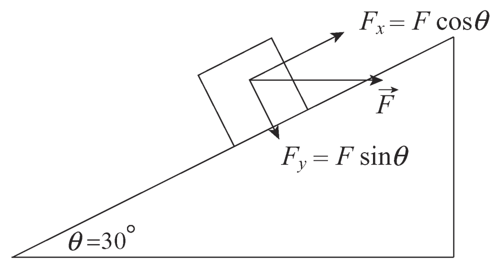
(b) The velocity vector at $t = 1.415\ \mathrm{s}$ is

$$\vec{v} = \left[8.00(1.415)\,\hat{\mathrm{i}} + 3.00(1.415)^2\,\hat{\mathrm{j}}\right]\mathrm{m/s} = (11.3\ \mathrm{m/s})\,\hat{\mathrm{i}} + (6.01\ \mathrm{m/s})\hat{\mathrm{j}}.$$

Therefore, the angle $\vec{v}$ makes with $+x$ is

$$\theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{6.01\ \mathrm{m/s}}{11.3\ \mathrm{m/s}}\right) = 28.0°.$$

32. We resolve this horizontal force into appropriate components.



(a) Newton's second law applied to the x-axis produces

$$F\cos\theta - mg\sin\theta = ma.$$

For $a = 0$, this yields $F = 566$ N.

(b) Applying Newton's second law to the y axis (where there is no acceleration), we have

$$F_N - F\sin\theta - mg\cos\theta = 0$$

which yields the normal force $F_N = 1.13 \times 10^3$ N.

33. (a) Since friction is negligible the force of the girl is the only horizontal force on the sled. The vertical forces (the force of gravity and the normal force of the ice) sum to zero. The acceleration of the sled is

$$a_s = \frac{F}{m_s} = \frac{5.2\,\text{N}}{8.4\,\text{kg}} = 0.62\ \text{m/s}^2\ .$$

(b) According to Newton's third law, the force of the sled on the girl is also 5.2 N. Her acceleration is

$$a_g = \frac{F}{m_g} = \frac{5.2\,\text{N}}{40\,\text{kg}} = 0.13\ \text{m/s}^2\ .$$

(c) The accelerations of the sled and girl are in opposite directions. Assuming the girl starts at the origin and moves in the $+x$ direction, her coordinate is given by $x_g = \frac{1}{2} a_g t^2$. The sled starts at x_0 = 15 m and moves in the $-x$ direction. Its coordinate is given by $x_s = x_0 - \frac{1}{2} a_s t^2$. They meet when $x_g = x_s$, or

$$\frac{1}{2} a_g t^2 = x_0 - \frac{1}{2} a_s t^2.$$

This occurs at time

$$t = \sqrt{\frac{2x_0}{a_g + a_s}}.$$

By then, the girl has gone the distance

$$x_g = \frac{1}{2} a_g t^2 = \frac{x_0 a_g}{a_g + a_s} = \frac{(15\text{ m})(0.13\text{ m/s}^2)}{0.13\text{ m/s}^2 + 0.62\text{ m/s}^2} = 2.6\text{ m}.$$

34. (a) Using notation suitable to a vector capable calculator, the $\vec{F}_{\text{net}} = 0$ condition becomes

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (6.00 \angle 150^\circ) + (7.00 \angle -60.0^\circ) + \vec{F}_3 = 0 .$$

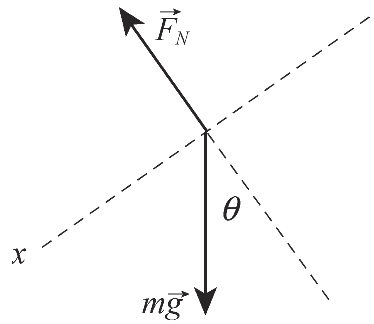
Thus, $\vec{F}_3 = (1.70 \text{ N})\,\hat{\text{i}} + (3.06 \text{ N})\hat{\text{j}}$.

(b) A constant velocity condition requires zero acceleration, so the answer is the same.

(c) Now, the acceleration is $\vec{a} = (13.0 \text{ m/s}^2)\,\hat{\text{i}} - (14.0 \text{ m/s}^2)\,\hat{\text{j}}$. Using $\vec{F}_{\text{net}} = m\,\vec{a}$ (with m = 0.025 kg) we now obtain

$$\vec{F}_3 = (2.02 \text{ N})\,\hat{\text{i}} + (2.71 \text{ N})\,\hat{\text{j}}.$$

35. The free-body diagram is shown next. $\vec{F}_N$ is the normal force of the plane on the block and $m\vec{g}$ is the force of gravity on the block. We take the $+x$ direction to be down the incline, in the direction of the acceleration, and the $+y$ direction to be in the direction of the normal force exerted by the incline on the block. The x component of Newton's second law is then $mg \sin \theta = ma$; thus, the acceleration is $a = g \sin \theta$.



(a) Placing the origin at the bottom of the plane, the kinematic equations (Table 2-1) for motion along the x axis which we will use are $v^2 = v_0^2 + 2ax$ and $v = v_0 + at$. The block momentarily stops at its highest point, where $v = 0$; according to the second equation, this occurs at time $t = -v_0/a$. The position where it stops is

$$x = -\frac{1}{2}\frac{v_0^2}{a} = -\frac{1}{2}\left(\frac{(-3.50\text{ m/s})^2}{\left(9.8\text{ m/s}^2\right)\sin 32.0°}\right) = -1.18\text{ m},$$

or $|x| = 1.18$ m.

(b) The time is

$$t = \frac{v_0}{a} = -\frac{v_0}{g\sin\theta} = -\frac{-3.50\,\text{m/s}}{\left(9.8\,\text{m/s}^2\right)\sin 32.0°} = 0.674\,\text{s}.$$

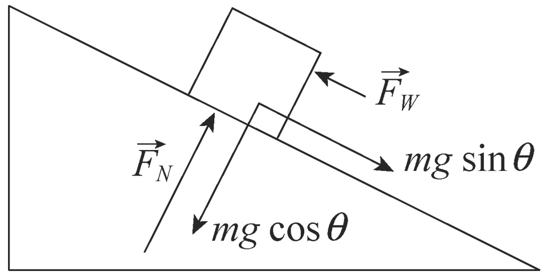
(c) That the return-speed is identical to the initial speed is to be expected since there are no dissipative forces in this problem. In order to prove this, one approach is to set $x = 0$ and solve $x = v_0 t + \frac{1}{2}at^2$ for the total time (up and back down) t. The result is

$$t = -\frac{2v_0}{a} = -\frac{2v_0}{g\sin\theta} = -\frac{2(-3.50\text{ m/s})}{\left(9.8\text{ m/s}^2\right)\sin 32.0°} = 1.35\text{ s}.$$

The velocity when it returns is therefore

$$v = v_0 + at = v_0 + gt\sin\theta = -3.50\text{ m/s} + (9.8\text{ m/s}^2)(1.35\text{ s})\sin 32° = 3.50\text{ m/s}.$$

36. We label the 40 kg skier "m" which is represented as a block in the figure shown. The force of the wind is denoted $\vec{F}_w$ and might be either "uphill" or "downhill" (it is shown uphill in our sketch). The incline angle θ is 10°. The $-x$ direction is downhill.



(a) Constant velocity implies zero acceleration; thus, application of Newton's second law along the x axis leads to

$$mg \sin\theta - F_w = 0 .$$

This yields $F_w = 68$ N (uphill).

(b) Given our coordinate choice, we have $a = |a| = 1.0$ m/s^2. Newton's second law

$$mg \sin\theta - F_w = ma$$

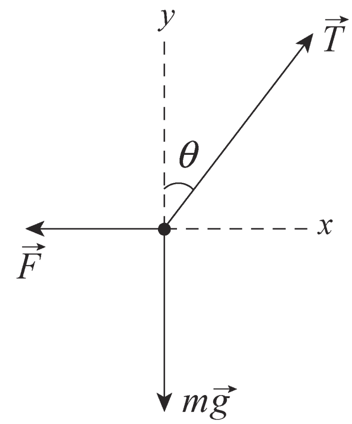
now leads to $F_w = 28$ N (uphill).

(c) Continuing with the forces as shown in our figure, the equation

$$mg \sin\theta - F_w = ma$$

will lead to $F_w = -12$ N when $|a| = 2.0$ m/s^2. This simply tells us that the wind is opposite to the direction shown in our sketch; in other words, $\vec{F}_w = 12\text{ N}$ *downhill*.

37. The solutions to parts (a) and (b) have been combined here. The free-body diagram is shown below, with the tension of the string $\vec{T}$, the force of gravity $m\vec{g}$, and the force of the air $\vec{F}$. Our coordinate system is shown. Since the sphere is motionless the net force on it is zero, and the x and the y components of the equations are:



$$T \sin \theta - F = 0$$
$$T \cos \theta - mg = 0,$$

where $\theta = 37°$. We answer the questions in the reverse order. Solving $T \cos \theta - mg = 0$ for the tension, we obtain

$$T = mg/ \cos \theta = (3.0 \times 10^{-4} \text{ kg}) (9.8 \text{ m/s}^2) / \cos 37° = 3.7 \times 10^{-3} \text{ N}.$$

Solving $T \sin \theta - F = 0$ for the force of the air:

$$F = T \sin \theta = (3.7 \times 10^{-3} \text{ N}) \sin 37° = 2.2 \times 10^{-3} \text{ N}.$$

38. The acceleration of an object (neither pushed nor pulled by any force other than gravity) on a smooth inclined plane of angle θ is $a = -\, g\sin\theta$. The slope of the graph shown with the problem statement indicates a = –2.50 m/s^2. Therefore, we find $\theta = 14.8°$. Examining the forces perpendicular to the incline (which must sum to zero since there is no component of acceleration in this direction) we find $F_N = mg\cos\theta$, where m = 5.00 kg. Thus, the normal (perpendicular) force exerted at the box/ramp interface is 47.4 N.

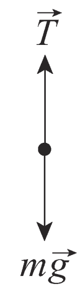
39. The free-body diagram is shown below. Let $\vec{T}$ be the tension of the cable and $m\vec{g}$ be the force of gravity. If the upward direction is positive, then Newton's second law is $T - mg = ma$, where a is the acceleration.

Thus, the tension is $T = m(g + a)$. We use constant acceleration kinematics (Table 2-1) to find the acceleration (where $v = 0$ is the final velocity, $v_0 = -12$ m/s is the initial velocity, and $y = -42$ m is the coordinate at the stopping point). Consequently, $v^2 = v_0^2 + 2ay$ leads to

$$a = -\frac{v_0^2}{2y} = -\frac{(-12\ \text{m/s})^2}{2(-42\ \text{m})} = 1.71\ \text{m/s}^2 .$$

We now return to calculate the tension:

$$\begin{aligned} T &= m(g + a) \\ &= (1600\ \text{kg})\,(9.8\ \text{m/s}^2 + 1.71\ \text{m/s}^2) \\ &= 1.8 \times 10^4\ \text{N} . \end{aligned}$$



40. (a) Constant velocity implies zero acceleration, so the "uphill" force must equal (in magnitude) the "downhill" force: $T = mg \sin\theta$. Thus, with m = 50 kg and $\theta = 8.0°$, the tension in the rope equals 68 N.

(b) With an uphill acceleration of 0.10 m/s^2, Newton's second law (applied to the x axis) yields

$$T - mg\sin\theta = ma \quad\Rightarrow\quad T - (50\text{ kg})\left(9.8\text{ m/s}^2\right)\sin 8.0° = (50\text{ kg})\left(0.10\text{ m/s}^2\right)$$

which leads to T = 73 N.

41. (a) The mass of the elevator is $m = (27800/9.80) = 2837$ kg and (with $+y$ upward) the acceleration is $a = +1.22$ m/s^2. Newton's second law leads to

$$T - mg = ma \;\Rightarrow\; T = m(g + a)$$

which yields $T = 3.13 \times 10^4$ N for the tension.

(b) The term "deceleration" means the acceleration vector is in the direction opposite to the velocity vector (which the problem tells us is upward). Thus (with $+y$ upward) the acceleration is now $a = -1.22$ m/s^2, so that the tension is

$$T = m\,(g + a) = 2.43 \times 10^4 \text{ N}.$$

42. (a) The term “deceleration” means the acceleration vector is in the direction opposite to the velocity vector (which the problem tells us is downward). Thus (with $+y$ upward) the acceleration is $a = +2.4$ m/s^2. Newton’s second law leads to

$$T - mg = ma \;\Rightarrow\; m = \frac{T}{g + a}$$

which yields $m = 7.3$ kg for the mass.

(b) Repeating the above computation (now to solve for the tension) with $a = +2.4$ m/s^2 will, of course, lead us right back to $T = 89$ N. Since the direction of the velocity did not enter our computation, this is to be expected.

43. The mass of the bundle is m = (449 N)/(9.80 m/s^2) = 45.8 kg and we choose $+y$ upward.

(a) Newton's second law, applied to the bundle, leads to

$$T - mg = ma \;\Rightarrow\; a = \frac{387\text{ N} - 449\text{ N}}{45.8\text{ kg}}$$

which yields a = –1.4 m/s^2 (or $|a|$ = 1.4 m/s^2) for the acceleration. The minus sign in the result indicates the acceleration vector points down. Any downward acceleration of magnitude greater than this is also acceptable (since that would lead to even smaller values of tension).

(b) We use Eq. 2-16 (with Δx replaced by Δy = –6.1 m). We assume v_0 = 0.

$$|v| = \sqrt{2a\Delta y} = \sqrt{2\left(-1.35\text{ m/s}^2\right)\left(-6.1\text{ m}\right)} = 4.1\text{ m/s}.$$

For downward accelerations greater than 1.4 m/s^2, the speeds at impact will be larger than 4.1 m/s.

44. With a_{ce} meaning "the acceleration of the coin relative to the elevator" and a_{eg} meaning "the acceleration of the elevator relative to the ground", we have

$$a_{ce} + a_{eg} = a_{cg} \quad \Rightarrow \quad -8.00 \text{ m/s}^2 + a_{eg} = -9.80 \text{ m/s}^2$$

which leads to $a_{eg} = -1.80$ m/s^2. We have chosen upward as the positive y direction. Then Newton's second law (in the "ground" reference frame) yields $T - m\,g = m\,a_{eg}$, or

$$T = m\,g + m\,a_{eg} = m(g + a_{eg}) = (2000 \text{ kg})(8.00 \text{ m/s}^2) = 16.0 \text{ kN}.$$

45. (a) The links are numbered from bottom to top. The forces on the bottom link are the force of gravity $m\vec{g}$, downward, and the force $\vec{F}_{2on1}$ of link 2, upward. Take the positive direction to be upward. Then Newton's second law for this link is $F_{2on1} - mg = ma$. Thus,

$$F_{2on1} = m(a + g) = (0.100 \text{ kg})\ (2.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) = 1.23 \text{ N}.$$

(b) The forces on the second link are the force of gravity $m\vec{g}$, downward, the force $\vec{F}_{1on2}$ of link 1, downward, and the force $\vec{F}_{3on2}$ of link 3, upward. According to Newton's third law $\vec{F}_{1on2}$ has the same magnitude as $\vec{F}_{2on1}$. Newton's second law for the second link is $F_{3on2} - F_{1on2} - mg = ma$, so

$$F_{3on2} = m(a + g) + F_{1on2} = (0.100 \text{ kg})\ (2.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) + 1.23 \text{ N} = 2.46 \text{ N}.$$

(c) Newton's second for link 3 is $F_{4on3} - F_{2on3} - mg = ma$, so

$$F_{4on3} = m(a + g) + F_{2on3} = (0.100 \text{ N})\ (2.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) + 2.46 \text{ N} = 3.69 \text{ N},$$

where Newton's third law implies $F_{2on3} = F_{3on2}$ (since these are magnitudes of the force vectors).

(d) Newton's second law for link 4 is $F_{5on4} - F_{3on4} - mg = ma$, so

$$F_{5on4} = m(a + g) + F_{3on4} = (0.100 \text{ kg})\ (2.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) + 3.69 \text{ N} = 4.92 \text{ N},$$

where Newton's third law implies $F_{3on4} = F_{4on3}$.

(e) Newton's second law for the top link is $F - F_{4on5} - mg = ma$, so

$$F = m(a + g) + F_{4on5} = (0.100 \text{ kg})\ (2.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) + 4.92 \text{ N} = 6.15 \text{ N},$$

where $F_{4on5} = F_{5on4}$ by Newton's third law.

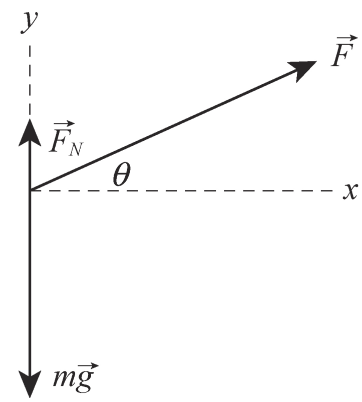
(f) Each link has the same mass and the same acceleration, so the same net force acts on each of them:

$$F_{net} = ma = (0.100 \text{ kg})\ (2.50 \text{ m/s}^2) = 0.250 \text{ N}.$$

46. Applying Newton's second law to cab B (of mass m) we have $a = \frac{T}{m} - g = 4.89 \text{ m/s}^2$. Next, we apply it to the box (of mass m_b) to find the normal force:

$$F_N = m_b(g + a) = 176 \text{ N}.$$

47. The free-body diagram (not to scale) for the block is shown below. $\vec{F}_N$ is the normal force exerted by the floor and $m\vec{g}$ is the force of gravity.



(a) The x component of Newton's second law is $F\cos\theta = ma$, where m is the mass of the block and a is the x component of its acceleration. We obtain

$$a = \frac{F\cos\theta}{m} = \frac{(12.0\,\text{N})\cos 25.0°}{5.00\,\text{kg}} = 2.18\ \text{m/s}^2.$$

This is its acceleration provided it remains in contact with the floor. Assuming it does, we find the value of F_N (and if F_N is positive, then the assumption is true but if F_N is negative then the block leaves the floor). The y component of Newton's second law becomes

$$F_N + F\sin\theta - mg = 0,$$

so

$$F_N = mg - F\sin\theta = (5.00\ \text{kg})(9.80\ \text{m/s}^2) - (12.0\ \text{N})\sin 25.0° = 43.9\ \text{N}.$$

Hence the block remains on the floor and its acceleration is $a = 2.18\ \text{m/s}^2$.

(b) If F is the minimum force for which the block leaves the floor, then $F_N = 0$ and the y component of the acceleration vanishes. The y component of the second law becomes

$$F\sin\theta - mg = 0 \quad \Rightarrow \quad F = \frac{mg}{\sin\theta} = \frac{(5.00\ \text{kg})(9.80\ \text{m/s}^2)}{\sin 25.0°} = 116\,\text{N}.$$

(c) The acceleration is still in the x direction and is still given by the equation developed in part (a):

$$a = \frac{F\cos\theta}{m} = \frac{(116\ \text{N})\cos 25.0°}{5.00\ \text{kg}} = 21.0\,\text{m/s}^2.$$

48. The direction of motion (the direction of the barge's acceleration) is $+\hat{\mathrm{i}}$, and $+\vec{\mathrm{j}}$ is chosen so that the pull $\vec{F}_\mathrm{h}$ from the horse is in the first quadrant. The components of the unknown force of the water are denoted simply F_x and F_y.

(a) Newton's second law applied to the barge, in the x and y directions, leads to

$$(7900\,\mathrm{N})\cos 18° + F_x = ma$$
$$(7900\,\mathrm{N})\sin 18° + F_y = 0$$

respectively. Plugging in a = 0.12 m/s^2 and m = 9500 kg, we obtain $F_x = -6.4 \times 10^3$ N and $F_y = -2.4 \times 10^3$ N. The magnitude of the force of the water is therefore

$$F_\text{water} = \sqrt{F_x^2 + F_y^2} = 6.8 \times 10^3\ \mathrm{N}.$$

(b) Its angle measured from $+\hat{\mathrm{i}}$ is either

$$\tan^{-1}\left(\frac{F_y}{F_x}\right) = +21°\,\text{or}\,201°.$$

The signs of the components indicate the latter is correct, so $\vec{F}_\text{water}$ is at 201° measured counterclockwise from the line of motion (+x axis).

49. Using Eq. 4-26, the launch speed of the projectile is

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta}} = \sqrt{\frac{(9.8\ \text{m/s}^2)(69\ \text{m})}{\sin 2(53°)}} = 26.52\ \text{m/s}\,.$$

The horizontal and vertical components of the speed are

$$\begin{aligned} v_x &= v_0 \cos\theta = (26.52\ \text{m/s})\cos 53° = 15.96\ \text{m/s} \\ v_y &= v_0 \sin\theta = (26.52\ \text{m/s})\sin 53° = 21.18\ \text{m/s}. \end{aligned}$$

Since the acceleration is constant, we can use Eq. 2-16 to analyze the motion. The component of the acceleration in the horizontal direction is

$$a_x = \frac{v_x^2}{2x} = \frac{(15.96\ \text{m/s})^2}{2(5.2\ \text{m})\cos 53°} = 40.7\ \text{m/s}^2,$$

and the force component is $F_x = ma_x = (85\ \text{kg})(40.7\ \text{m/s}^2) = 3460\ \text{N}$. Similarly, in the vertical direction, we have

$$a_y = \frac{v_y^2}{2y} = \frac{(21.18\ \text{m/s})^2}{2(5.2\ \text{m})\sin 53°} = 54.0\ \text{m/s}^2.$$

and the force component is

$$F_y = ma_y + mg = (85\ \text{kg})(54.0\ \text{m/s}^2 + 9.80\ \text{m/s}^2) = 5424\ \text{N}.$$

Thus, the magnitude of the force is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(3460\ \text{N})^2 + (5424\ \text{N})^2} = 6434\ \text{N} \approx 6.4\times 10^3\ \text{N},$$

to two significant figures.

50. First, we consider all the penguins (1 through 4, counting left to right) as one system, to which we apply Newton's second law:

$$T_4 = \left(m_1 + m_2 + m_3 + m_4\right)a \quad \Rightarrow \quad 222\,\text{N} = \left(12\,\text{kg} + m_2 + 15\,\text{kg} + 20\,\text{kg}\right)a.$$

Second, we consider penguins 3 and 4 as one system, for which we have

$$\begin{aligned} T_4 - T_2 &= \left(m_3 + m_4\right)a \\ 111\,\text{N} &= \left(15\text{ kg} + 20\,\text{kg}\right)a \quad \Rightarrow \quad a = 3.2\text{ m/s}^2. \end{aligned}$$

Substituting the value, we obtain $m_2 = 23$ kg.

51. We apply Newton's second law first to the three blocks as a single system and then to the individual blocks. The $+x$ direction is to the right in Fig. 5-49.

(a) With $m_{sys} = m_1 + m_2 + m_3 = 67.0$ kg, we apply Eq. 5-2 to the x motion of the system – in which case, there is only one force $\vec{T}_3 = +\vec{T}_3\ \hat{\mathrm{i}}$. Therefore,

$$T_3 = m_{\text{sys}} a \quad \Rightarrow \quad 65.0\,\text{N} = (67.0\,\text{kg})a$$

which yields $a = 0.970$ m/s^2 for the system (and for each of the blocks individually).

(b) Applying Eq. 5-2 to block 1, we find

$$T_1 = m_1 a = (12.0\,\text{kg})\left(0.970\,\text{m/s}^2\right) = 11.6\,\text{N}.$$

(c) In order to find T_2, we can either analyze the forces on block 3 or we can treat blocks 1 and 2 as a system and examine its forces. We choose the latter.

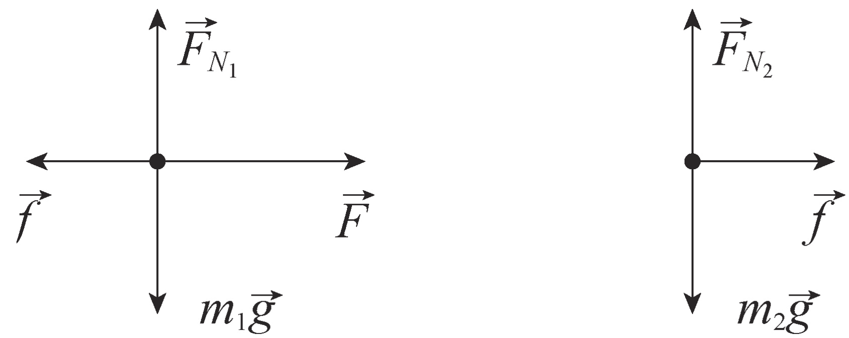
$$T_2 = \left(m_1 + m_2\right)a = \left(12.0\text{ kg} + 24.0\text{ kg}\right)\left(0.970\text{ m/s}^2\right) = 34.9\text{ N}.$$

52. Both situations involve the same applied force and the same total mass, so the accelerations must be the same in both figures.

(a) The (direct) force causing *B* to have this acceleration in the first figure is twice as big as the (direct) force causing *A* to have that acceleration. Therefore, *B* has the twice the mass of *A*. Since their total is given as 12.0 kg then *B* has a mass of m_B = 8.00 kg and *A* has mass m_A = 4.00 kg. Considering the first figure, (20.0 N)/(8.00 kg) = 2.50 m/s^2. Of course, the same result comes from considering the second figure ((10.0 N)/(4.00 kg) = 2.50 m/s^2).

(b) F_a = (12.0 kg)(2.50 m/s^2) = 30.0 N

53. The free-body diagrams for part (a) are shown below. $\vec{F}$ is the applied force and $\vec{f}$ is the force exerted by block 1 on block 2. We note that $\vec{F}$ is applied directly to block 1 and that block 2 exerts the force $-\vec{f}$ on block 1 (taking Newton's third law into account).



(a) Newton's second law for block 1 is $F - f = m_1 a$, where a is the acceleration. The second law for block 2 is $f = m_2 a$. Since the blocks move together they have the same acceleration and the same symbol is used in both equations. From the second equation we obtain the expression $a = f/m_2$, which we substitute into the first equation to get $F - f = m_1 f/m_2$. Therefore,

$$f = \frac{Fm_2}{m_1 + m_2} = \frac{(3.2\text{ N})(1.2\text{ kg})}{2.3\text{ kg} + 1.2\text{ kg}} = 1.1\text{ N}.$$

(b) If $\vec{F}$ is applied to block 2 instead of block 1 (and in the opposite direction), the force of contact between the blocks is

$$f = \frac{Fm_1}{m_1 + m_2} = \frac{(3.2\text{ N})(2.3\text{ kg})}{2.3\text{ kg} + 1.2\text{ kg}} = 2.1\text{ N}.$$

(c) We note that the acceleration of the blocks is the same in the two cases. In part (a), the force f is the only horizontal force on the block of mass m_2 and in part (b) f is the only horizontal force on the block with $m_1 > m_2$. Since $f = m_2 a$ in part (a) and $f = m_1 a$ in part (b), then for the accelerations to be the same, f must be larger in part (b).

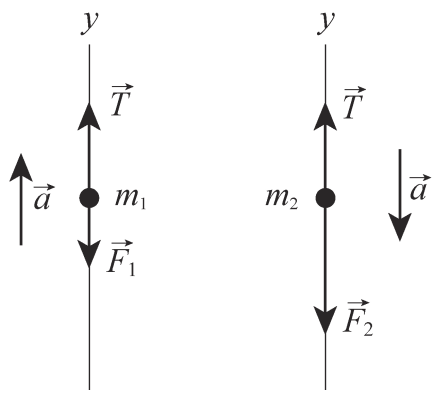
54. (a) The net force on the *system* (of total mass M = 80.0 kg) is the force of gravity acting on the total overhanging mass (m_{BC} = 50.0 kg). The magnitude of the acceleration is therefore $a = (m_{BC}\, g)/M = 6.125$ m/s^2. Next we apply Newton's second law to block C itself (choosing *down* as the $+y$ direction) and obtain

$$m_C g - T_{BC} = m_C a.$$

This leads to T_{BC} = 36.8 N.

(b) We use Eq. 2-15 (choosing *rightward* as the $+x$ direction): $\Delta x = 0 + \frac{1}{2}at^2 = 0.191$ m.

55. The free-body diagrams for m_1 and m_2 are shown in the figures below. The only forces on the blocks are the upward tension $\vec{T}$ and the downward gravitational forces $\vec{F}_1 = m_1 g$ and $\vec{F}_2 = m_2 g$. Applying Newton's second law, we obtain:



$$T - m_1 g = m_1 a$$

$$m_2 g - T = m_2 a$$

which can be solved to yield

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g$$

Substituting the result back, we have

$$T = \left(\frac{2 m_1 m_2}{m_1 + m_2}\right) g$$

(a) With $m_1 = 1.3$ kg and $m_2 = 2.8$ kg, the acceleration becomes

$$a = \left(\frac{2.80\ \text{kg} - 1.30\ \text{kg}}{2.80\ \text{kg} + 1.30\ \text{kg}}\right)(9.80\ \text{m/s}^2) = 3.59\ \text{m/s}^2.$$

(b) Similarly, the tension in the cord is

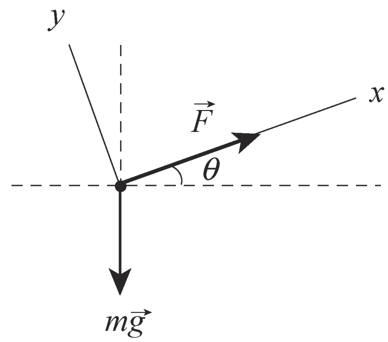
$$T = \frac{2(1.30\ \text{kg})(2.80\ \text{kg})}{1.30\ \text{kg} + 2.80\ \text{kg}}(9.80\ \text{m/s}^2) = 17.4\ \text{N}.$$

56. To solve the problem, we note that the acceleration along the slanted path depends on only the force components along the path, not the components perpendicular to the path.
(a) From the free-body diagram shown, we see that the net force on the putting shot along the $+x$-axis is

$$F_{\text{net},x} = F - mg\sin\theta = 380.0\ \text{N} - (7.260\ \text{kg})(9.80\ \text{m/s}^2)\sin 30° = 344.4\ \text{N},$$

which in turn gives

$$a_x = F_{\text{net},x}/m = (344.4\ \text{N})/(7.260\ \text{kg}) = 47.44\ \text{m/s}^2.$$



Using Eq. 2-16 for constant-acceleration motion, the speed of the shot at the end of the acceleration phase is

$$v = \sqrt{v_0^2 + 2a_x\Delta x} = \sqrt{(2.500\ \text{m/s})^2 + 2(47.44\ \text{m/s}^2)(1.650\ \text{m})}$$
$$= 12.76\ \text{m/s}.$$

(b) If $\theta = 42°$, then

$$a_x = \frac{F_{\text{net},x}}{m} = \frac{F - mg\sin\theta}{m} = \frac{380.0\ \text{N} - (7.260\ \text{kg})(9.80\ \text{m/s}^2)\sin 42.00°}{7.260\ \text{kg}} = 45.78\ \text{m/s}^2,$$

and the final (launch) speed is

$$v = \sqrt{v_0^2 + 2a_x\Delta x} = \sqrt{(2.500\ \text{m/s})^2 + 2(45.78\ \text{m/s}^2)(1.650\ \text{m})} = 12.54\ \text{m/s}.$$

(c) The decrease in launch speed when changing the angle from $30.00°$ to $42.00°$ is

$$\frac{12.76\ \text{m/s} - 12.54\ \text{m/s}}{12.76\ \text{m/s}} = 0.0169 = 16.9\%.$$

57. We take $+y$ to be up for both the monkey and the package.

(a) The force the monkey pulls downward on the rope has magnitude F. According to Newton's third law, the rope pulls upward on the monkey with a force of the same magnitude, so Newton's second law for forces acting on the monkey leads to

$$F - m_m g = m_m a_m,$$

where m_m is the mass of the monkey and a_m is its acceleration. Since the rope is massless $F = T$ is the tension in the rope. The rope pulls upward on the package with a force of magnitude F, so Newton's second law for the package is

$$F + F_N - m_p g = m_p a_p,$$

where m_p is the mass of the package, a_p is its acceleration, and F_N is the normal force exerted by the ground on it. Now, if F is the minimum force required to lift the package, then $F_N = 0$ and $a_p = 0$. According to the second law equation for the package, this means $F = m_p g$. Substituting $m_p g$ for F in the equation for the monkey, we solve for a_m:

$$a_m = \frac{F - m_m g}{m_m} = \frac{(m_p - m_m)g}{m_m} = \frac{(15\text{ kg} - 10\text{ kg})(9.8\text{ m/s}^2)}{10\text{ kg}} = 4.9\text{ m/s}^2.$$

(b) As discussed, Newton's second law leads to $F - m_p g = m_p a_p$ for the package and $F - m_m g = m_m a_m$ for the monkey. If the acceleration of the package is downward, then the acceleration of the monkey is upward, so $a_m = -a_p$. Solving the first equation for F

$$F = m_p(g + a_p) = m_p(g - a_m)$$

and substituting this result into the second equation, we solve for a_m:

$$a_m = \frac{(m_p - m_m)g}{m_p + m_m} = \frac{(15\text{ kg} - 10\text{ kg})(9.8\text{ m/s}^2)}{15\text{ kg} + 10\text{ kg}} = 2.0\text{ m/s}^2.$$

(c) The result is positive, indicating that the acceleration of the monkey is upward.

(d) Solving the second law equation for the package, we obtain

$$F = m_p(g - a_m) = (15\text{ kg})(9.8\text{ m/s}^2 - 2.0\text{ m/s}^2) = 120\text{N}.$$

58. Referring to Fig. 5-10(c) is helpful. In this case, viewing the man-rope-sandbag as a system means that we should be careful to choose a consistent positive direction of motion (though there are other ways to proceed – say, starting with individual application of Newton's law to each mass). We take *down* as positive for the man's motion and *up* as positive for the sandbag's motion and, without ambiguity, denote their acceleration as a. The net force on the system is the different between the weight of the man and that of the sandbag. The system mass is m_{sys} = 85 kg + 65 kg = 150 kg. Thus, Eq. 5-1 leads to

$$(85\ \text{kg})(9.8\ \text{m/s}^2) - (65\ \text{kg})(9.8\ \text{m/s}^2) = m_{\text{sys}}\, a$$

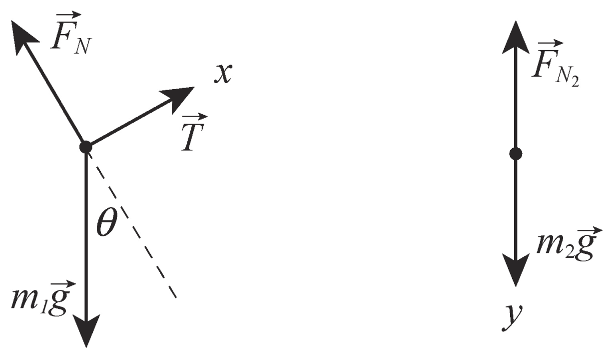
which yields a = 1.3 m/s^2. Since the system starts from rest, Eq. 2-16 determines the speed (after traveling Δy = 10 m) as follows:

$$v = \sqrt{2a\Delta y} = \sqrt{2(1.3\ \text{m/s}^2)(10\ \text{m})} = 5.1\ \text{m/s}.$$

59. The free-body diagram for each block is shown below. T is the tension in the cord and $\theta = 30°$ is the angle of the incline. For block 1, we take the $+x$ direction to be up the incline and the $+y$ direction to be in the direction of the normal force $\vec{F}_N$ that the plane exerts on the block. For block 2, we take the $+y$ direction to be down. In this way, the accelerations of the two blocks can be represented by the same symbol a, without ambiguity. Applying Newton's second law to the x and y axes for block 1 and to the y axis of block 2, we obtain

$$\begin{aligned} T - m_1 g \sin\theta &= m_1 a \\ F_N - m_1 g \cos\theta &= 0 \\ m_2 g - T &= m_2 a \end{aligned}$$

respectively. The first and third of these equations provide a simultaneous set for obtaining values of a and T. The second equation is not needed in this problem, since the normal force is neither asked for nor is it needed as part of some further computation (such as can occur in formulas for friction).



(a) We add the first and third equations above:

$$m_2 g - m_1 g \sin\theta = m_1 a + m_2 a.$$

Consequently, we find

$$a = \frac{(m_2 - m_1 \sin\theta)g}{m_1 + m_2} = \frac{[2.30\text{ kg} - (3.70\text{ kg})\sin 30.0°](9.80\text{ m/s}^2)}{3.70\text{ kg} + 2.30\text{ kg}} = 0.735\,\text{m/s}^2.$$

(b) The result for a is positive, indicating that the acceleration of block 1 is indeed up the incline and that the acceleration of block 2 is vertically down.

(c) The tension in the cord is

$$T = m_1 a + m_1 g \sin\theta = (3.70\text{ kg})(0.735\text{ m/s}^2) + (3.70\text{ kg})(9.80\text{ m/s}^2)\sin 30.0° = 20.8\,\text{N}.$$

60. The motion of the man-and-chair is positive if upward.

(a) When the man is grasping the rope, pulling with a force equal to the tension T in the rope, the total upward force on the man-and-chair due its two contact points with the rope is $2T$. Thus, Newton's second law leads to

$$2T - mg = ma$$

so that when $a = 0$, the tension is $T = 466$ N.

(b) When $a = +1.30$ m/s^2 the equation in part (a) predicts that the tension will be $T = 527\ \mathrm{N}$.

(c) When the man is not holding the rope (instead, the co-worker attached to the ground is pulling on the rope with a force equal to the tension T in it), there is only one contact point between the rope and the man-and-chair, and Newton's second law now leads to

$$T - mg = ma$$

so that when $a = 0$, the tension is $T = 931$ N.

(d) When $a = +1.30$ m/s^2, the equation in (c) yields $T = 1.05 \times 10^3$ N.

(e) The rope comes into contact (pulling down in each case) at the left edge and the right edge of the pulley, producing a total downward force of magnitude $2T$ on the ceiling. Thus, in part (a) this gives $2T = 931$ N.

(f) In part (b) the downward force on the ceiling has magnitude $2T = 1.05 \times 10^3$ N.

(g) In part (c) the downward force on the ceiling has magnitude $2T = 1.86 \times 10^3$ N.

(h) In part (d) the downward force on the ceiling has magnitude $2T = 2.11 \times 10^3$ N.

61. The forces on the balloon are the force of gravity $m\vec{g}$ (down) and the force of the air $\vec{F}_a$ (up). We take the $+y$ to be up, and use a to mean the *magnitude* of the acceleration (which is not its usual use in this chapter). When the mass is M (before the ballast is thrown out) the acceleration is downward and Newton's second law is

$$F_a - Mg = -Ma.$$

After the ballast is thrown out, the mass is $M - m$ (where m is the mass of the ballast) and the acceleration is upward. Newton's second law leads to

$$F_a - (M - m)g = (M - m)a.$$

The previous equation gives $F_a = M(g - a)$, and this plugs into the new equation to give

$$M(g - a) - (M - m)g = (M - m)a \;\Rightarrow\; m = \frac{2Ma}{g + a}.$$

62. The horizontal component of the acceleration is determined by the net horizontal force.

(a) If the rate of change of the angle is

$$\frac{d\theta}{dt} = (2.00\times10^{-2})°/\text{s} = (2.00\times10^{-2})°/\text{s}\cdot\left(\frac{\pi\ \text{rad}}{180°}\right) = 3.49\times10^{-4}\,\text{rad/s},$$

then, using $F_x = F\cos\theta$, we find the rate of change of acceleration to be

$$\frac{da_x}{dt} = \frac{d}{dt}\left(\frac{F\cos\theta}{m}\right) = -\frac{F\sin\theta}{m}\frac{d\theta}{dt} = -\frac{(20.0\ \text{N})\sin 25.0°}{5.00\ \text{kg}}\left(3.49\times10^{-4}\,\text{rad/s}\right)$$
$$= -5.90\times10^{-4}\ \text{m/s}^3.$$

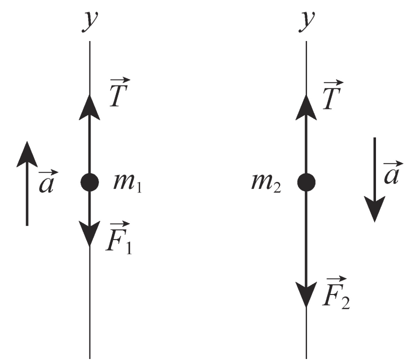
(b) If the rate of change of the angle is

$$\frac{d\theta}{dt} = -(2.00\times10^{-2})°/\text{s} = -(2.00\times10^{-2})°/\text{s}\cdot\left(\frac{\pi\ \text{rad}}{180°}\right) = -3.49\times10^{-4}\,\text{rad/s},$$

then the rate of change of acceleration would be

$$\frac{da_x}{dt} = \frac{d}{dt}\left(\frac{F\cos\theta}{m}\right) = -\frac{F\sin\theta}{m}\frac{d\theta}{dt} = -\frac{(20.0\ \text{N})\sin 25.0°}{5.00\ \text{kg}}\left(-3.49\times10^{-4}\,\text{rad/s}\right)$$
$$= +5.90\times10^{-4}\ \text{m/s}^3.$$

63. The free-body diagrams for m_1 and m_2 are shown in the figures below. The only forces on the blocks are the upward tension $\vec{T}$ and the downward gravitational forces $\vec{F}_1 = m_1 g$ and $\vec{F}_2 = m_2 g$. Applying Newton's second law, we obtain:



$$
\begin{aligned}
T - m_1 g &= m_1 a \\
m_2 g - T &= m_2 a
\end{aligned}
$$

which can be solved to give

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g$$

(a) At $t = 0$, $m_{10} = 1.30\ \text{kg}$. With $dm_1/dt = -0.200\ \text{kg/s}$, we find the rate of change of acceleration to be

$$\frac{da}{dt} = \frac{da}{dm_1}\frac{dm_1}{dt} = -\frac{2m_2 g}{(m_2 + m_{10})^2}\frac{dm_1}{dt} = -\frac{2(2.80\ \text{kg})(9.80\ \text{m/s}^2)}{(2.80\ \text{kg} + 1.30\ \text{kg})^2}(-0.200\ \text{kg/s}) = 0.653\ \text{m/s}^3.$$

(b) At $t = 3.00\ \text{s}$, $m_1 = m_{10} + (dm_1/dt)t = 1.30\ \text{kg} + (-0.200\ \text{kg/s})(3.00\ \text{s}) = 0.700\ \text{kg}$, and the rate of change of acceleration is

$$\frac{da}{dt} = \frac{da}{dm_1}\frac{dm_1}{dt} = -\frac{2m_2 g}{(m_2 + m_1)^2}\frac{dm_1}{dt} = -\frac{2(2.80\ \text{kg})(9.80\ \text{m/s}^2)}{(2.80\ \text{kg} + 0.700\ \text{kg})^2}(-0.200\ \text{kg/s}) = 0.896\ \text{m/s}^3.$$

(c) The acceleration reaches its maximum value when

$$0 = m_1 = m_{10} + (dm_1/dt)t = 1.30\ \text{kg} + (-0.200\ \text{kg/s})t,$$

or $t = 6.50\ \text{s}$.

64. We first use Eq. 4-26 to solve for the launch speed of the shot:

$$y - y_0 = (\tan\theta)x - \frac{gx^2}{2(v'\cos\theta)^2}.$$

With $\theta = 34.10°$, $y_0 = 2.11$ m and $(x, y) = (15.90 \text{ m}, 0)$, we find the launch speed to be $v' = 11.85$ m/s. During this phase, the acceleration is

$$a = \frac{v'^2 - v_0^2}{2L} = \frac{(11.85 \text{ m/s})^2 - (2.50 \text{ m/s})^2}{2(1.65 \text{ m})} = 40.63 \text{ m/s}^2.$$

Since the acceleration along the slanted path depends on only the force components along the path, not the components perpendicular to the path, the average force on the shot during the acceleration phase is

$$F = m(a + g\sin\theta) = (7.260 \text{ kg})\left[40.63 \text{ m/s}^2 + (9.80 \text{ m/s}^2)\sin 34.10°\right] = 334.8 \text{ N}.$$

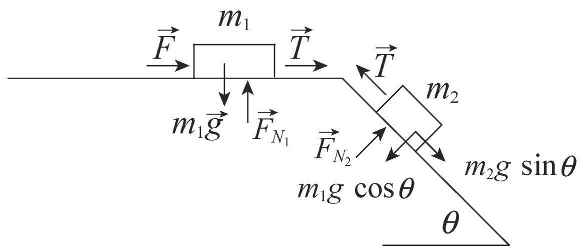
65. First we analyze the entire *system* with "clockwise" motion considered positive (that is, downward is positive for block C, rightward is positive for block B, and upward is positive for block A): $m_C g - m_A g = Ma$ (where M = mass of the *system* = 24.0 kg). This yields an acceleration of

$$a = g(m_C - m_A)/M = 1.63 \text{ m/s}^2.$$

Next we analyze the forces just on block C: $m_C g - T = m_C a$. Thus the tension is

$$T = m_C g(2m_A + m_B)/M = 81.7 \text{ N}.$$

66. The $+x$ direction for m_2=1.0 kg is “downhill” and the $+x$ direction for m_1=3.0 kg is rightward; thus, they accelerate with the same sign.



(a) We apply Newton’s second law to the x axis of each box:

$$m_2 g\ \sin\theta - T = m_2 a$$
$$F + T = m_1 a$$

Adding the two equations allows us to solve for the acceleration:

$$a = \frac{m_2 g \sin\theta + F}{m_1 + m_2}$$

With F = 2.3 N and $\theta = 30°$, we have a = 1.8 m/s^2. We plug back and find T = 3.1 N.

(b) We consider the “critical” case where the F has reached the *max* value, causing the tension to vanish. The first of the equations in part (a) shows that $a = g\sin 30°$ in this case; thus, a = 4.9 m/s^2. This implies (along with T = 0 in the second equation in part (a)) that

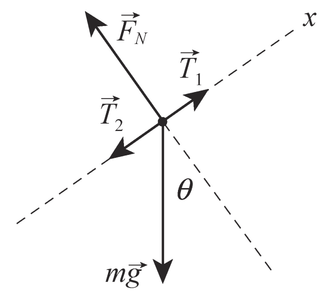
$$F = (3.0\ \text{kg})(4.9\ \text{m/s}^2) = 14.7\ \text{N} \approx 15\ \text{N}$$

in the critical case.

67. (a) The acceleration (which equals F/m in this problem) is the derivative of the velocity. Thus, the velocity is the integral of F/m, so we find the “area” in the graph (15 units) and divide by the mass (3) to obtain $v - v_o = 15/3 = 5$. Since $v_o = 3.0$ m/s, then $v = 8.0\,\text{m/s}$.

(b) Our positive answer in part (a) implies $\vec{v}$ points in the $+x$ direction.

68. The free-body diagram is shown on the right. Newton's second law for the mass m for the x direction leads to



$$T_1 - T_2 - mg\sin\theta = ma$$

which gives the difference in the tension in the pull cable:

$$\begin{aligned}T_1 - T_2 &= m\left(g\sin\theta + a\right) = \left(2800\ \text{kg}\right)\left[(9.8\ \text{m/s}^2)\sin 35° + 0.81\ \text{m/s}^2\right] \\ &= 1.8\times10^4\ \text{N}.\end{aligned}$$

69. (a) We quote our answers to many figures – probably more than are truly "significant." Here (7682 L)("1.77 kg/L") = 13597 kg. The quotation marks around the 1.77 are due to the fact that this was believed (by the flight crew) to be a legitimate conversion factor (it is not).

(b) The amount they felt should be added was 22300 kg – 13597 kg = 87083 kg, which they believed to be equivalent to (87083 kg)/("1.77 kg/L") = 4917 L.

(c) Rounding to 4 figures as instructed, the conversion factor is 1.77 lb/L $\rightarrow$ 0.8034 kg/L, so the amount on board was (7682 L)(0.8034 kg/L) = 6172 kg.

(d) The implication is that what as needed was 22300 kg – 6172 kg = 16128 kg, so the request should have been for (16128 kg)/(0.8034 kg/L) = 20075 L.

(e) The percentage of the required fuel was

$$\frac{7682\text{ L (on board)} + 4917\text{ L (added)}}{(22300\text{ kg required})\,/(0.8034\text{ kg/L})} = 45\%.$$

70. We are only concerned with horizontal forces in this problem (gravity plays no direct role). Without loss of generality, we take one of the forces along the $+x$ direction and the other at 80° (measured counterclockwise from the x axis). This calculation is efficiently implemented on a vector capable calculator in polar mode, as follows (using magnitude-angle notation, with angles understood to be in degrees):

$$\vec{F}_{\text{net}} = (20 \angle 0) + (35 \angle 80) = (43 \angle 53) \Rightarrow |\vec{F}_{\text{net}}| = 43 \text{ N} .$$

Therefore, the mass is $m = (43 \text{ N})/(20 \text{ m/s}^2) = 2.2$ kg.

71. The goal is to arrive at the least magnitude of $\vec{F}_{\text{net}}$, and as long as the magnitudes of $\vec{F}_2$ and $\vec{F}_3$ are (in total) less than or equal to $|\vec{F}_1|$ then we should orient them opposite to the direction of $\vec{F}_1$ (which is the +*x* direction).

(a) We orient both $\vec{F}_2$ and $\vec{F}_3$ in the –*x* direction. Then, the magnitude of the net force is 50 – 30 – 20 = 0, resulting in zero acceleration for the tire.

(b) We again orient $\vec{F}_2$ and $\vec{F}_3$ in the negative *x* direction. We obtain an acceleration along the +*x* axis with magnitude

$$a = \frac{F_1 - F_2 - F_3}{m} = \frac{50\,\text{N} - 30\,\text{N} - 10\,\text{N}}{12\,\text{kg}} = 0.83\ \text{m/s}^2 .$$

(c) In this case, the forces $\vec{F}_2$ and $\vec{F}_3$ are collectively strong enough to have *y* components (one positive and one negative) which cancel each other and still have enough *x* contributions (in the –*x* direction) to cancel $\vec{F}_1$. Since $|\vec{F}_2| = |\vec{F}_3|$, we see that the angle above the –*x* axis to one of them should equal the angle below the –*x* axis to the other one (we denote this angle θ). We require

$$-50\ \text{N} = F_{2x} + F_{3x} = -(30\text{N})\cos\theta - (30\text{N})\cos\theta$$

which leads to

$$\theta = \cos^{-1}\left(\frac{50\,\text{N}}{60\,\text{N}}\right) = 34^\circ .$$

72. (a) A small segment of the rope has mass and is pulled down by the gravitational force of the Earth. Equilibrium is reached because neighboring portions of the rope pull up sufficiently on it. Since tension is a force *along* the rope, at least one of the neighboring portions must slope up away from the segment we are considering. Then, the tension has an upward component which means the rope sags.

(b) The only force acting with a horizontal component is the applied force $\vec{F}$. Treating the block and rope as a single object, we write Newton's second law for it: $F = (M + m)a$, where a is the acceleration and the positive direction is taken to be to the right. The acceleration is given by $a = F/(M + m)$.

(c) The force of the rope F_r is the only force with a horizontal component acting on the block. Then Newton's second law for the block gives

$$F_r = Ma = \frac{MF}{M + m}$$

where the expression found above for a has been used.

(d) Treating the block and half the rope as a single object, with mass $M + \frac{1}{2}m$, where the horizontal force on it is the tension T_m at the midpoint of the rope, we use Newton's second law:

$$T_m = \left(M + \frac{1}{2}m\right)a = \frac{(M + m/2)F}{(M + m)} = \frac{(2M + m)F}{2(M + m)}.$$

73. Although the full specification of $\vec{F}_{\text{net}} = m\vec{a}$ in this situation involves both x and y axes, only the x-application is needed to find what this particular problem asks for. We note that $a_y = 0$ so that there is no ambiguity denoting a_x simply as a. We choose $+x$ to the right and $+y$ up. We also note that the x component of the rope's tension (acting on the crate) is

$$F_x = F\cos\theta = (450 \text{ N}) \cos 38° = 355 \text{ N},$$

and the resistive force (pointing in the $-x$ direction) has magnitude $f = 125$ N.

(a) Newton's second law leads to

$$F_x - f = ma \Rightarrow a = \frac{355 \text{ N} - 125 \text{ N}}{310 \text{ kg}} = 0.74 \text{ m/s}^2 .$$

(b) In this case, we use Eq. 5-12 to find the mass: $m = W/g = 31.6$ kg. Now, Newton's second law leads to

$$T_x - f = ma \quad \Rightarrow \quad a = \frac{355 \text{ N} - 125 \text{ N}}{31.6 \text{ kg}} = 7.3 \text{ m/s}^2 .$$

74. Since the velocity of the particle does not change, it undergoes no acceleration and must therefore be subject to zero net force. Therefore,

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0 .$$

Thus, the third force $\vec{F}_3$ is given by

$$\vec{F}_3 = -\vec{F}_1 - \vec{F}_2 = -\left(2\hat{\text{i}} + 3\hat{\text{j}} - 2\hat{\text{k}}\right)\text{N} - \left(-5\hat{\text{i}} + 8\hat{\text{j}} - 2\hat{\text{k}}\right)\text{N} = \left(3\hat{\text{i}} - 11\hat{\text{j}} + 4\hat{\text{k}}\right)\text{N}.$$

The specific value of the velocity is not used in the computation.

75. (a) Since the performer's weight is (52 kg)(9.8 m/s^2) = 510 N, the rope breaks.

(b) Setting $T = 425$ N in Newton's second law (with $+y$ upward) leads to

$$T - mg = ma \;\Rightarrow\; a = \frac{T}{m} - g$$

which yields $|a| = 1.6$ m/s^2.

76. (a) For the 0.50 meter drop in "free-fall", Eq. 2-16 yields a speed of 3.13 m/s. Using this as the "initial speed" for the final motion (over 0.02 meter) during which his motion slows at rate "a", we find the magnitude of his average acceleration from when his feet first touch the patio until the moment his body stops moving is $a = 245 \text{ m/s}^2$.

(b) We apply Newton's second law: $F_{\text{stop}} - mg = ma \Rightarrow F_{\text{stop}} = 20.4 \text{ kN}$.

77. We begin by examining a slightly different problem: similar to this figure but without the string. The motivation is that if (without the string) block A is found to accelerate faster (or exactly as fast) as block B then (returning to the original problem) the tension in the string is trivially zero. In the absence of the string,

$$a_A = F_A / m_A = 3.0 \text{ m/s}^2$$

$$a_B = F_B / m_B = 4.0 \text{ m/s}^2$$

so the trivial case does not occur. We now (with the string) consider the net force on the *system*: $Ma = F_A + F_B = 36$ N. Since $M = 10$ kg (the total mass of the system) we obtain $a = 3.6 \text{ m/s}^2$. The two forces on block A are F_A and T (in the same direction), so we have

$$m_A a = F_A + T \quad \Rightarrow \quad T = 2.4 \text{ N}.$$

78. With SI units understood, the net force on the box is

$$\vec{F}_{\text{net}} = \left(3.0 + 14\cos 30° - 11\right)\hat{\text{i}} + \left(14\sin 30° + 5.0 - 17\right)\hat{\text{j}}$$

which yields $\vec{F}_{\text{net}} = (4.1\text{ N})\,\hat{\text{i}} - (5.0\text{ N})\hat{\text{j}}$.

(a) Newton's second law applied to the m = 4.0 kg box leads to

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = (1.0\,\text{m/s}^2)\hat{\text{i}} - (1.3\,\text{m/s}^2)\hat{\text{j}}\,.$$

(b) The magnitude of $\vec{a}$ is $a = \sqrt{(1.0\text{ m/s}^2)^2 + \left(-1.3\text{ m/s}^2\right)^2} = 1.6\ \text{m/s}^2$.

(c) Its angle is $\tan^{-1}$ [(–1.3 m/s^2)/(1.0 m/s^2)] = –50° (that is, 50° measured clockwise from the rightward axis).

79. The "certain force" denoted F is assumed to be the net force on the object when it gives m_1 an acceleration $a_1 = 12$ m/s^2 and when it gives m_2 an acceleration $a_2 = 3.3$ m/s^2. Thus, we substitute $m_1 = F/a_1$ and $m_2 = F/a_2$ in appropriate places during the following manipulations.

(a) Now we seek the acceleration a of an object of mass $m_2 - m_1$ when F is the net force on it. Thus,

$$a = \frac{F}{m_2 - m_1} = \frac{F}{(F/a_2) - (F/a_1)} = \frac{a_1 a_2}{a_1 - a_2}$$

which yields $a = 4.6$ m/s^2.

(b) Similarly for an object of mass $m_2 + m_1$:

$$a = \frac{F}{m_2 + m_1} = \frac{F}{(F/a_2) + (F/a_1)} = \frac{a_1 a_2}{a_1 + a_2}$$

which yields $a = 2.6$ m/s^2.

80. We use the notation g as the acceleration due to gravity near the surface of Callisto, m as the mass of the landing craft, a as the acceleration of the landing craft, and F as the rocket thrust. We take down to be the positive direction. Thus, Newton's second law takes the form $mg - F = ma$. If the thrust is F_1 (= 3260 N), then the acceleration is zero, so $mg - F_1 = 0$. If the thrust is F_2 (= 2200 N), then the acceleration is a_2 (= 0.39 m/s^2), so $mg - F_2 = ma_2$.

(a) The first equation gives the weight of the landing craft: $mg = F_1 = 3260$ N.

(b) The second equation gives the mass:

$$m = \frac{mg - F_2}{a_2} = \frac{3260 \text{ N} - 2200 \text{ N}}{0.39 \text{ m/s}^2} = 2.7 \times 10^3 \text{ kg} .$$

(c) The weight divided by the mass gives the acceleration due to gravity:

$$g = (3260 \text{ N})/(2.7 \times 10^3 \text{ kg}) = 1.2 \text{ m/s}^2.$$

81. From the reading when the elevator was at rest, we know the mass of the object is $m = (65 \text{ N})/(9.8 \text{ m/s}^2) = 6.6$ kg. We choose $+y$ upward and note there are two forces on the object: mg downward and T upward (in the cord that connects it to the balance; T is the reading on the scale by Newton's third law).

(a) "Upward at constant speed" means constant velocity, which means no acceleration. Thus, the situation is just as it was at rest: $T = 65$ N.

(b) The term "deceleration" is used when the acceleration vector points in the direction opposite to the velocity vector. We're told the velocity is upward, so the acceleration vector points downward ($a = -2.4 \text{ m/s}^2$). Newton's second law gives

$$T - mg = ma \;\Rightarrow\; T = (6.6 \text{ kg})(9.8 \text{ m/s}^2 - 2.4 \text{ m/s}^2) = 49 \text{ N}.$$

82. We take $+x$ uphill for the m_2 = 1.0 kg box and $+x$ rightward for the m_1= 3.0 kg box (so the accelerations of the two boxes have the same magnitude and the same sign). The uphill force on m_2 is F and the downhill forces on it are T and $m_2g \sin \theta$, where θ = 37°. The only horizontal force on m_1 is the rightward-pointed tension. Applying Newton's second law to each box, we find

$$\begin{aligned} F - T - m_2 g \sin \theta &= m_2 a \\ T &= m_1 a \end{aligned}$$

which can be added to obtain $F - m_2g \sin \theta = (m_1 + m_2)a$. This yields the acceleration

$$a = \frac{12\ \text{N} - (1.0\ \text{kg})(9.8\ \text{m/s}^2)\sin 37°}{1.0\ \text{kg} + 3.0\ \text{kg}} = 1.53\ \text{m/s}^2.$$

Thus, the tension is $T = m_1a$ = (3.0 kg)(1.53 m/s^2) = 4.6 N.

83. We apply Eq. 5-12.

(a) The mass is $m = W/g = (22 \text{ N})/(9.8 \text{ m/s}^2) = 2.2$ kg. At a place where $g = 4.9 \text{ m/s}^2$, the mass is still 2.2 kg but the gravitational force is $F_g = mg = (2.2 \text{ kg})(4.0 \text{ m/s}^2) = 11$ N.

(b) As noted, $m = 2.2$ kg.

(c) At a place where $g = 0$ the gravitational force is zero.

(d) The mass is still 2.2 kg.

84. We use $W_p = mg_p$, where W_p is the weight of an object of mass m on the surface of a certain planet p, and g_p is the acceleration of gravity on that planet.

(a) The weight of the space ranger on Earth is

$$W_e = mg_e = (75 \text{ kg})\,(9.8 \text{ m/s}^2) = 7.4 \times 10^2 \text{ N}.$$

(b) The weight of the space ranger on Mars is

$$W_m = mg_m = (75 \text{ kg})\,(3.7 \text{ m/s}^2) = 2.8 \times 10^2 \text{ N}.$$

(c) The weight of the space ranger in interplanetary space is zero, where the effects of gravity are negligible.

(d) The mass of the space ranger remains the same, m=75 kg, at all the locations.

85. (a) When $\vec{F}_{\text{net}} = 3F - mg = 0$, we have

$$F = \frac{1}{3}mg = \frac{1}{3}(1400 \text{ kg})\left(9.8 \text{ m/s}^2\right) = 4.6 \times 10^3 \text{ N}$$

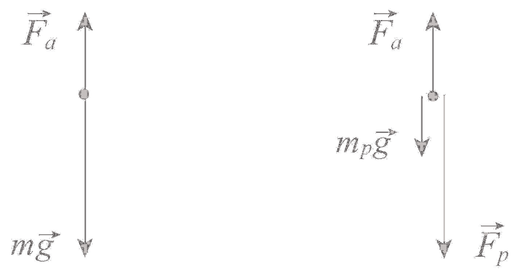
for the force exerted by each bolt on the engine.

(b) The force on each bolt now satisfies $3F - mg = ma$, which yields

$$F = \frac{1}{3}m(g + a) = \frac{1}{3}(1400 \text{ kg})\left(9.8 \text{ m/s}^2 + 2.6 \text{ m/s}^2\right) = 5.8 \times 10^3 \text{ N}.$$

86. We take the down to be the $+y$ direction.

(a) The first diagram (shown below left) is the free-body diagram for the person and parachute, considered as a single object with a mass of 80 kg + 5.0 kg = 85 kg.



$\vec{F}_a$ is the force of the air on the parachute and $m\vec{g}$ is the force of gravity. Application of Newton's second law produces $mg - F_a = ma$, where a is the acceleration. Solving for F_a we find

$$F_a = m(g - a) = (85 \text{ kg})(9.8 \text{ m/s}^2 - 2.5 \text{ m/s}^2) = 620 \text{ N}.$$

(b) The second diagram (above right) is the free-body diagram for the parachute alone. $\vec{F}_a$ is the force of the air, $m_p\vec{g}$ is the force of gravity, and $\vec{F}_p$ is the force of the person. Now, Newton's second law leads to

$$m_p g + F_p - F_a = m_p a.$$

Solving for F_p, we obtain

$$F_p = m_p(a - g) + F_a = (5.0 \text{ kg})(2.5 \text{ m/s}^2 - 9.8 \text{ m/s}^2) + 620 \text{ N} = 580 \text{ N}.$$

87. (a) Intuition readily leads to the conclusion that the heavier block should be the hanging one, for largest acceleration. The force that "drives" the system into motion is the weight of the hanging block (gravity acting on the block on the table has no effect on the dynamics, so long as we ignore friction). Thus, m = 4.0 kg.

The acceleration of the system and the tension in the cord can be readily obtained by solving

$$mg - T = ma$$
$$T = Ma.$$

(b) The acceleration is given by

$$a = \left(\frac{m}{m + M}\right) g = 6.5 \text{ m/s}^2.$$

(c) The tension is

$$T = Ma = \left(\frac{Mm}{m + M}\right) g = 13 \text{ N}.$$

88. We assume the direction of motion is $+x$ and assume the refrigerator starts from rest (so that the speed being discussed is the velocity $\vec{v}$ which results from the process). The only force along the x axis is the x component of the applied force $\vec{F}$.

(a) Since $v_0 = 0$, the combination of Eq. 2-11 and Eq. 5-2 leads simply to

$$F_x = m\left(\frac{v}{t}\right) \Rightarrow v_i = \left(\frac{F\cos\theta_i}{m}\right)t$$

for $i = 1$ or 2 (where we denote $\theta_1 = 0$ and $\theta_2 = \theta$ for the two cases). Hence, we see that the ratio v_2 over v_1 is equal to $\cos\theta$.

(b) Since $v_0 = 0$, the combination of Eq. 2-16 and Eq. 5-2 leads to

$$F_x = m\left(\frac{v^2}{2\Delta x}\right) \Rightarrow v_i = \sqrt{2\left(\frac{F\cos\theta_i}{m}\right)\Delta x}$$

for $i = 1$ or 2 (again, $\theta_1 = 0$ and $\theta_2 = \theta$ is used for the two cases). In this scenario, we see that the ratio v_2 over v_1 is equal to $\sqrt{\cos\theta}$.

89. The mass of the pilot is $m = 735/9.8 = 75$ kg. Denoting the upward force exerted by the spaceship (his seat, presumably) on the pilot as $\vec{F}$ and choosing upward the $+y$ direction, then Newton's second law leads to

$$F - mg_{\text{moon}} = ma \quad \Rightarrow \quad F = (75\ \text{kg})\left(1.6\ \text{m/s}^2 + 1.0\ \text{m/s}^2\right) = 195\ \text{N}.$$

90. We denote the thrust as T and choose $+y$ upward. Newton's second law leads to

$$T - Mg = Ma \quad \Rightarrow \quad a = \frac{2.6\times10^5 \text{ N}}{1.3\times10^4 \text{ kg}} - 9.8 \text{ m/s}^2 = 10 \text{ m/s}^2.$$

91. (a) The bottom cord is only supporting $m_2 = 4.5$ kg against gravity, so its tension is

$$T_2 = m_2 g = (4.5 \text{ kg})(9.8 \text{ m/s}^2) = 44 \text{ N}.$$

(b) The top cord is supporting a total mass of $m_1 + m_2 = (3.5 \text{ kg} + 4.5 \text{ kg}) = 8.0$ kg against gravity, so the tension there is

$$T_1 = (m_1 + m_2)g = (8.0 \text{ kg})(9.8 \text{ m/s}^2) = 78 \text{ N}.$$

(c) In the second picture, the lowest cord supports a mass of $m_5 = 5.5$ kg against gravity and consequently has a tension of $T_5 = (5.5 \text{ kg})(9.8 \text{ m/s}^2) = 54$ N.

(d) The top cord, we are told, has tension $T_3 = 199$ N which supports a total of (199 N)/(9.80 m/s^2) = 20.3 kg, 10.3 kg of which is already accounted for in the figure. Thus, the unknown mass in the middle must be $m_4 = 20.3 \text{ kg} - 10.3 \text{ kg} = 10.0$ kg, and the tension in the cord above it must be enough to support

$$m_4 + m_5 = (10.0 \text{ kg} + 5.50 \text{ kg}) = 15.5 \text{ kg},$$

so $T_4 = (15.5 \text{ kg})(9.80 \text{ m/s}^2) = 152$ N. Another way to analyze this is to examine the forces on m_3; one of the downward forces on it is T_4.

92. (a) With SI units understood, the net force is

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = \left(3.0\text{ N} + \left(-2.0\text{ N}\right)\right)\hat{\text{i}} + \left(4.0\text{ N} + \left(-6.0\text{ N}\right)\right)\hat{\text{j}}$$

which yields $\vec{F}_{\text{net}} = (1.0\text{ N})\,\hat{\text{i}} - (2.0\text{ N})\,\hat{\text{j}}$.

(b) The magnitude of $\vec{F}_{\text{net}}$ is $F_{\text{net}} = \sqrt{(1.0\text{ N})^2 + (-2.0\text{ N})^2} = 2.2\text{ N}$.

(c) The angle of $\vec{F}_{\text{net}}$ is

$$\theta = \tan^{-1}\left(\frac{-2.0\text{ N}}{1.0\text{ N}}\right) = -63°.$$

(d) The magnitude of $\vec{a}$ is

$$a = F_{\text{net}}/m = (2.2\text{ N})/(1.0\text{ kg}) = 2.2\text{ m/s}^2.$$

(e) Since $\vec{F}_{\text{net}}$ is equal to $\vec{a}$ multiplied by mass m, which is a positive scalar that cannot affect the direction of the vector it multiplies, $\vec{a}$ has the same angle as the net force, i.e, $\theta = -63°$. In magnitude-angle notation, we may write $\vec{a} = \left(2.2\text{ m/s}^2 \angle -63°\right)$.

93. According to Newton's second law, the magnitude of the force is given by $F = ma$, where a is the magnitude of the acceleration of the neutron. We use kinematics (Table 2-1) to find the acceleration that brings the neutron to rest in a distance d. Assuming the acceleration is constant, then $v^2 = v_0^2 + 2ad$ produces the value of a:

$$a = \frac{\left(v^2 - v_0^2\right)}{2d} = \frac{-\left(1.4 \times 10^7 \text{ m/s}\right)^2}{2\left(1.0 \times 10^{-14} \text{ m}\right)} = -9.8 \times 10^{27} \text{ m/s}^2.$$

The magnitude of the force is consequently

$$F = ma = \left(1.67 \times 10^{-27} \text{ kg}\right)\left(9.8 \times 10^{27} \text{ m/s}^2\right) = 16 \text{ N}.$$

94. Making separate free-body diagrams for the helicopter and the truck, one finds there are two forces on the truck ($\vec{T}$ upward, caused by the tension, which we'll think of as that of a single cable, and $m\vec{g}$ downward, where m = 4500 kg) and three forces on the helicopter ($\vec{T}$ downward, $\vec{F}_{\text{lift}}$ upward, and $M\vec{g}$ downward, where M = 15000 kg). With $+y$ upward, then $a = +1.4$ m/s^2 for both the helicopter and the truck.

(a) Newton's law applied to the helicopter and truck separately gives

$$F_{\text{lift}} - T - Mg = Ma$$
$$T - mg = ma$$

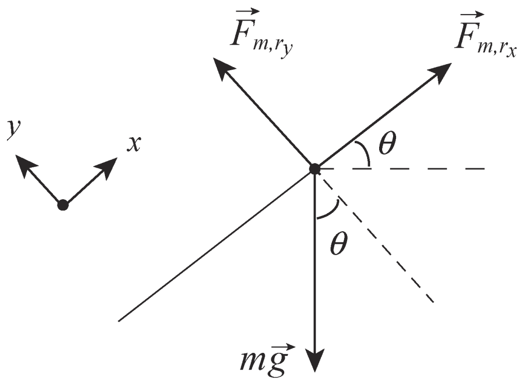
which we add together to obtain

$$F_{\text{lift}} - (M + m)g = (M + m)a.$$

From this equation, we find $F_{\text{lift}} = 2.2 \times 10^5$ N.

(b) From the truck equation $T - mg = ma$ we obtain $T = 5.0 \times 10^4$ N.

95. The free-body diagrams is shown on the right. Note that $F_{\mathrm{m,\,r}_y}$ and $F_{\mathrm{m,\,r}_x}$, respectively, and thought of as the y and x components of the force $\vec{F}_{\mathrm{m,\,r}}$ exerted by the motorcycle on the rider.



(a) Since the net force equals ma, then the magnitude of the net force on the rider is (60.0 kg) (3.0 m/s^2) = 1.8×10^2 N.

(b) We apply Newton's second law to the x axis:

$$F_{\mathrm{m,r}_x} - mg \sin\theta = ma$$

where m = 60.0 kg, a = 3.0 m/s^2, and θ = 10°. Thus, $F_{\mathrm{m,\,}r_x} = 282$ N Applying it to the y axis (where there is no acceleration), we have

$$F_{\mathrm{m,r}_y} - mg\cos\theta = 0$$

which produces $F_{\mathrm{m,\,r}_y} = 579$ N . Using the Pythagorean theorem, we find

$$\sqrt{F^2_{m,r_x} + F^2_{m,r_y}} = 644 \text{ N}.$$

Now, the magnitude of the force exerted on the rider by the motorcycle is the same magnitude of force exerted by the rider on the motorcycle, so the answer is 6.4×10^2 N, to two significant figures.

96. We write the length unit light-month, the distance traveled by light in one month, as c·month in this solution.

(a) The magnitude of the required acceleration is given by

$$a = \frac{\Delta v}{\Delta t} = \frac{(0.10)(3.0 \times 10^8 \text{ m/s})}{(3.0 \text{ days})(86400 \text{ s/day})} = 1.2 \times 10^2 \text{ m/s}^2 .$$

(b) The acceleration in terms of g is

$$a = \left(\frac{a}{g}\right) g = \left(\frac{1.2 \times 10^2 \text{ m/s}^2}{9.8 \text{ m/s}^2}\right) g = 12g .$$

(c) The force needed is

$$F = ma = (1.20 \times 10^6 \text{ kg})(1.2 \times 10^2 \text{ m/s}^2) = 1.4 \times 10^8 \text{ N} .$$

(d) The spaceship will travel a distance d = 0.1 c·month during one month. The time it takes for the spaceship to travel at constant speed for 5.0 light-months is

$$t = \frac{d}{v} = \frac{5.0 \text{ c} \cdot \text{months}}{0.1c} = 50 \text{ months} \approx 4.2 \text{ years.}$$

97. The coordinate choices are made in the problem statement.

(a) We write the velocity of the armadillo as $\vec{v} = v_x\,\hat{\mathrm{i}} + v_y\,\hat{\mathrm{j}}$. Since there is no net force exerted on it in the x direction, the x component of the velocity of the armadillo is a constant: v_x = 5.0 m/s. In the y direction at t = 3.0 s, we have (using Eq. 2-11 with $v_{0y} = 0$)

$$v_y = v_{0\,y} + a_y t = v_{0\,y} + \left(\frac{F_y}{m}\right)t = \left(\frac{17\text{ N}}{12\text{ kg}}\right)(3.0\text{ s}) = 4.3\text{ m/s}.$$

Thus, $\vec{v} = (5.0\,\text{m/s})\,\hat{\mathrm{i}} + (4.3\,\text{m/s})\,\hat{\mathrm{j}}$.

(b) We write the position vector of the armadillo as $\vec{r} = r_x\,\hat{\mathrm{i}} + r_y\,\hat{\mathrm{j}}$. At t = 3.0 s we have r_x = (5.0 m/s) (3.0 s) = 15 m and (using Eq. 2-15 with $v_{0\,y}$ = 0)

$$r_y = v_{0\,y}\,t + \frac{1}{2}a_y\,t^2 = \frac{1}{2}\left(\frac{F_y}{m}\right)t^2 = \frac{1}{2}\left(\frac{17\text{ N}}{12\text{ kg}}\right)(3.0\text{ s})^2 = 6.4\text{ m}.$$

The position vector at t = 3.0 s is therefore

$$\vec{r} = (15\text{ m})\hat{\mathrm{i}} + (6.4\text{ m})\hat{\mathrm{j}}.$$

98. (a) From Newton's second law, the magnitude of the maximum force on the passenger from the floor is given by

$$F_{\max} - mg = ma \quad \text{where} \quad a = a_{\max} = 2.0 \text{ m/s}^2$$

we obtain F_N = 590 N for m = 50 kg.

(b) The direction is upward.

(c) Again, we use Newton's second law, the magnitude of the minimum force on the passenger from the floor is given by

$$F_{\min} - mg = ma \quad \text{where} \quad a = a_{\min} = -3.0 \text{ m/s}^2.$$

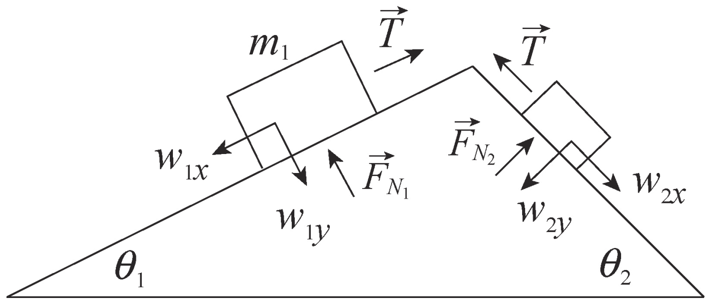
Now, we obtain F_N= 340 N.

(d) The direction is upward.

(e) Returning to part (a), we use Newton's third law, and conclude that the force exerted by the passenger on the floor is $|\vec{F}_{PF}| = 590$ N.

(f) The direction is downward.

99. The $+x$ axis is "uphill" for $m_1 = 3.0$ kg and "downhill" for $m_2 = 2.0$ kg (so they both accelerate with the same sign). The x components of the two masses along the x axis are given by $w_{1x} = m_1 g \sin\theta_1$ and $w_{2x} = m_2 g \sin\theta_2$, respectively.



Applying Newton's second law, we obtain

$$T - m_1 g \sin\theta_1 = m_1 a$$
$$m_2 g \sin\,\theta_2 - T = m_2 a$$

Adding the two equations allows us to solve for the acceleration:

$$a = \left(\frac{m_2 \sin\theta_2 - m_1 \sin\theta_1}{m_2 + m_1}\right) g$$

With $\theta_1 = 30°$ and $\theta_2 = 60°$, we have a = 0.45 m/s^2. This value is plugged back into either of the two equations to yield the tension T = 16 N.

100. (a) In unit vector notation,

$$m\vec{a} = (-\,3.76\text{ N})\,\hat{\text{i}} + (1.37\text{ N})\,\hat{\text{j}}.$$

Thus, Newton's second law leads to

$$\vec{F}_2 = m\vec{a} - \vec{F}_1 = (-\,6.26\text{ N})\,\hat{\text{i}} - (3.23\text{ N})\,\hat{\text{j}}.$$

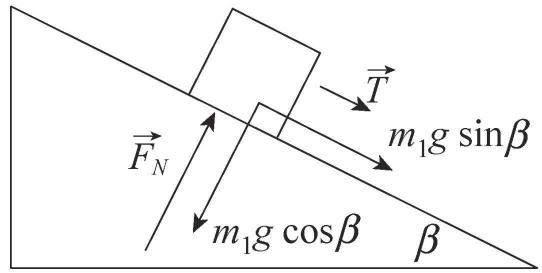
(b) The magnitude of $\vec{F}_2$ is $F_2 = \sqrt{(-6.26\text{ N})^2 + (-3.23\text{ N})^2} = 7.04\text{ N}.$

(c) Since $\vec{F}_2$ is in the third quadrant, the angle is

$$\theta = \tan^{-1}\left(\frac{-3.23\text{ N}}{-6.26\text{ N}}\right) = 207°.$$

counterclockwise from positive direction of x axis (or 153° *clockwise* from $+x$).

101. We first analyze the forces on m_1=1.0 kg.



The +x direction is "downhill" (parallel to $\vec{T}$).

With the acceleration (5.5 m/s^2) in the positive x direction for m_1, then Newton's second law, applied to the x axis, becomes

$$T + m_1 g \sin\beta = m_1\left(5.5\,\text{m/s}^2\right)$$

But for m_2=2.0 kg, using the more familiar vertical y axis (with *up* as the positive direction), we have the acceleration in the negative direction:

$$F + T - m_2 g = m_2\left(-5.5\,\text{m/s}^2\right)$$

where the tension comes in as an upward force (the cord can pull, not push).

(a) From the equation for m_2, with F = 6.0 N, we find the tension T = 2.6 N.

(b) From the equation for m, using the result from part (a), we obtain the angle $\beta = 17°$.

102. (a) The word "hovering" is taken to imply that the upward (thrust) force is equal in magnitude to the downward (gravitational) force: $mg = 4.9 \times 10^5$ N.

(b) Now the thrust must exceed the answer of part (a) by $ma = 10 \times 10^5$ N, so the thrust must be 1.5×10^6 N.

103. (a) Choosing the direction of motion as $+x$, Eq. 2-11 gives

$$a = \frac{88.5 \text{ km/h} - 0}{6.0 \text{ s}} = 15 \text{ km/h/s}.$$

Converting to SI, this is $a = 4.1 \text{ m/s}^2$.

(b) With mass $m = 2000/9.8 = 204$ kg, Newton's second law gives $\vec{F} = m\vec{a} = 836$ N in the $+x$ direction.

104. (a) With $v_0 = 0$, Eq. 2-16 leads to

$$a = \frac{v^2}{2\Delta x} = \frac{(6.0\times10^6\ \text{m/s})^2}{2(0.015\ \text{m})} = 1.2\times10^{15}\ \text{m/s}^2.$$

The force responsible for producing this acceleration is

$$F = ma = (9.11\times\ 10^{-31}\ \text{kg})\ (1.2\times10^{15}\ \text{m/s}^2) = 1.1\times10^{-15}\ \text{N}.$$

(b) The weight is $mg = 8.9 \times 10^{-30}$ N, many orders of magnitude smaller than the result of part (a). As a result, gravity plays a negligible role in most atomic and subatomic processes.

Chapter 6

1. We do not consider the possibility that the bureau might tip, and treat this as a purely horizontal motion problem (with the person's push $\vec{F}$ in the $+x$ direction). Applying Newton's second law to the x and y axes, we obtain

$$F - f_{s,\,\max} = ma$$
$$F_N - mg = 0$$

respectively. The second equation yields the normal force $F_N = mg$, whereupon the maximum static friction is found to be (from Eq. 6-1) $f_{s,\max} = \mu_s mg$. Thus, the first equation becomes

$$F - \mu_s mg = ma = 0$$

where we have set $a = 0$ to be consistent with the fact that the static friction is still (just barely) able to prevent the bureau from moving.

(a) With $\mu_s = 0.45$ and $m = 45$ kg, the equation above leads to $F = 198$ N. To bring the bureau into a state of motion, the person should push with any force greater than this value. Rounding to two significant figures, we can therefore say the minimum required push is $F = 2.0 \times 10^2$ N.

(b) Replacing $m = 45$ kg with $m = 28$ kg, the reasoning above leads to roughly $F = 1.2\times10^2$ N.

2. To maintain the stone's motion, a horizontal force (in the $+x$ direction) is needed that cancels the retarding effect due to kinetic friction. Applying Newton's second to the x and y axes, we obtain

$$F - f_k = ma$$
$$F_N - mg = 0$$

respectively. The second equation yields the normal force $F_N = mg$, so that (using Eq. 6-2) the kinetic friction becomes $f_k = \mu_k mg$. Thus, the first equation becomes

$$F - \mu_k mg = ma = 0$$

where we have set $a = 0$ to be consistent with the idea that the horizontal velocity of the stone should remain constant. With $m = 20$ kg and $\mu_k = 0.80$, we find $F = 1.6 \times 10^2$ N.

3. We denote $\vec{F}$ as the horizontal force of the person exerted on the crate (in the $+x$ direction), $\vec{f}_k$ is the force of kinetic friction (in the $-x$ direction), F_N is the vertical normal force exerted by the floor (in the $+y$ direction), and $m\vec{g}$ is the force of gravity. The magnitude of the force of friction is given by $f_k = \mu_k F_N$ (Eq. 6-2). Applying Newton's second law to the x and y axes, we obtain

$$F - f_k = ma$$
$$F_N - mg = 0$$

respectively.

(a) The second equation yields the normal force $F_N = mg$, so that the friction is

$$f_k = \mu_k mg = (0.35)(55 \text{ kg})(9.8 \text{ m/s}^2) = 1.9 \times 10^2 \text{ N} .$$

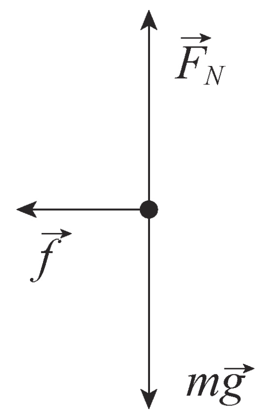
(b) The first equation becomes

$$F - \mu_k mg = ma$$

which (with $F = 220$ N) we solve to find

$$a = \frac{F}{m} - \mu_k g = 0.56 \text{ m/s}^2 .$$

4. The free-body diagram for the player is shown next. $\vec{F}_N$ is the normal force of the ground on the player, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of friction. The force of friction is related to the normal force by $f = \mu_k F_N$. We use Newton's second law applied to the vertical axis to find the normal force. The vertical component of the acceleration is zero, so we obtain $F_N - mg = 0$; thus, $F_N = mg$. Consequently,



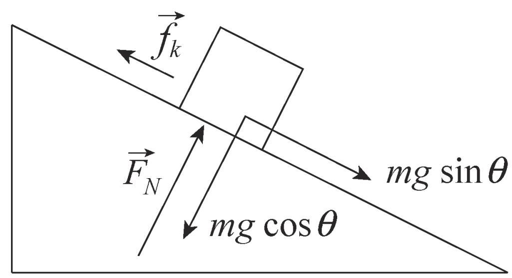
$$\mu_k = \frac{f}{F_N} = \frac{470\ \text{N}}{(79\ \text{kg})\,(9.8\ \text{m/s}^2)} = 0.61 .$$

5. The greatest deceleration (of magnitude a) is provided by the maximum friction force (Eq. 6-1, with $F_N = mg$ in this case). Using Newton's second law, we find

$$a = f_{s,\max}/m = \mu_s g.$$

Eq. 2-16 then gives the shortest distance to stop: $|\Delta x| = v^2/2a = 36$ m. In this calculation, it is important to first convert v to 13 m/s.

6. We first analyze the forces on the pig of mass m. The incline angle is θ.



The $+x$ direction is "downhill.''

Application of Newton's second law to the x- and y-axes leads to

$$\begin{aligned} mg\sin\theta - f_k &= ma \\ F_N - mg\cos\theta &= 0. \end{aligned}$$

Solving these along with Eq. 6-2 ($f_k = \mu_k F_N$) produces the following result for the pig's downhill acceleration:

$$a = g(\sin\theta - \mu_k\cos\theta)\,.$$

To compute the time to slide from rest through a downhill distance ℓ, we use Eq. 2-15:

$$\ell = v_0 t + \frac{1}{2}at^2 \quad\Rightarrow\quad t = \sqrt{\frac{2\ell}{a}}\,.$$

We denote the frictionless ($\mu_k = 0$) case with a prime and set up a ratio:

$$\frac{t}{t'} = \frac{\sqrt{2\ell/a}}{\sqrt{2\ell/a'}} = \sqrt{\frac{a'}{a}}$$

which leads us to conclude that if $t/t' = 2$ then $a' = 4a$. Putting in what we found out above about the accelerations, we have

$$g\sin\theta = 4g\,(\sin\theta - \mu_k\cos\theta)\,.$$

Using $\theta = 35°$, we obtain $\mu_k = 0.53$.

7. We choose $+x$ horizontally rightwards and $+y$ upwards and observe that the 15 N force has components $F_x = F\cos\theta$ and $F_y = -F\sin\theta$.

(a) We apply Newton's second law to the y axis:

$$F_N - F\sin\theta - mg = 0 \;\Rightarrow\; F_N = (15\text{ N})\sin 40° + (3.5\text{ kg})(9.8\text{ m/s}^2) = 44\text{ N}.$$

With $\mu_k = 0.25$, Eq. 6-2 leads to $f_k = 11$ N.

(b) We apply Newton's second law to the x axis:

$$F\cos\theta - f_k = ma \;\Rightarrow\; a = \frac{(15\text{ N})\cos 40° - 11\text{ N}}{3.5\text{ kg}} = 0.14\text{ m/s}^2.$$

Since the result is positive-valued, then the block is accelerating in the $+x$ (rightward) direction.

8. In addition to the forces already shown in Fig. 6-21, a free-body diagram would include an upward normal force $\vec{F}_N$ exerted by the floor on the block, a downward $m\vec{g}$ representing the gravitational pull exerted by Earth, and an assumed-leftward $\vec{f}$ for the kinetic or static friction. We choose +x rightwards and +y upwards. We apply Newton's second law to these axes:

$$F - f = ma$$
$$P + F_N - mg = 0$$

where F = 6.0 N and m = 2.5 kg is the mass of the block.

(a) In this case, P = 8.0 N leads to

$$F_N = (2.5 \text{ kg})(9.8 \text{ m/s}^2) - 8.0 \text{ N} = 16.5 \text{ N}.$$

Using Eq. 6-1, this implies $f_{s,\max} = \mu_s F_N = 6.6$ N , which is larger than the 6.0 N rightward force – so the block (which was initially at rest) does not move. Putting $a = 0$ into the first of our equations above yields a static friction force of $f = P = 6.0$ N.

(b) In this case, P = 10 N, the normal force is $F_N = (2.5 \text{ kg})(9.8 \text{ m/s}^2) - 10 \text{ N} = 14.5$ N. Using Eq. 6-1, this implies $f_{s,\max} = \mu_s F_N = 5.8$ N, which is less than the 6.0 N rightward force – so the block does move. Hence, we are dealing not with static but with kinetic friction, which Eq. 6-2 reveals to be $f_k = \mu_k F_N = 3.6$ N .

(c) In this last case, P = 12 N leads to F_N = 12.5 N and thus to $f_{s,\max} = \mu_s F_N = 5.0$ N, which (as expected) is less than the 6.0 N rightward force – so the block moves. The kinetic friction force, then, is $f_k = \mu_k F_N = 3.1$ N .

9. Applying Newton's second law to the horizontal motion, we have $F - \mu_k\, m\, g = ma$, where we have used Eq. 6-2, assuming that $F_N = mg$ (which is equivalent to assuming that the vertical force from the broom is negligible). Eq. 2-16 relates the distance traveled and the final speed to the acceleration: $v^2 = 2a\Delta x$. This gives $a = 1.4$ m/s^2. Returning to the force equation, we find (with $F = 25$ N and $m = 3.5$ kg) that $\mu_k = 0.58$.

10. There is no acceleration, so the (upward) static friction forces (there are four of them, one for each thumb and one for each set of opposing fingers) equals the magnitude of the (downward) pull of gravity. Using Eq. 6-1, we have

$$4\mu_s F_N = mg = (79\ \text{kg})(9.8\ \text{m/s}^2)$$

which, with $\mu_s = 0.70$, yields $F_N = 2.8 \times 10^2$ N.

11. We denote the magnitude of 110 N force exerted by the worker on the crate as F. The magnitude of the static frictional force can vary between zero and $f_{s,\max} = \mu_s F_N$.

(a) In this case, application of Newton's second law in the vertical direction yields $F_N = mg$. Thus,

$$f_{s,\,\max} = \mu_s F_N = \mu_s mg = (0.37)(35\,\text{kg})(9.8\,\text{m/s}^2) = 1.3\times10^2\ \text{N}$$

which is greater than F.

(b) The block, which is initially at rest, stays at rest since $F < f_{s,\,\max}$. Thus, it does not move.

(c) By applying Newton's second law to the horizontal direction, that the magnitude of the frictional force exerted on the crate is $f_s = 1.1\times10^2\ \text{N}$.

(d) Denoting the upward force exerted by the second worker as F_2, then application of Newton's second law in the vertical direction yields $F_N = mg - F_2$, which leads to

$$f_{s,\max} = \mu_s F_N = \mu_s (mg - F_2)\,.$$

In order to move the crate, F must satisfy the condition $F > f_{s,\max} = \mu_s\ (mg - F_2)$

or

$$110\,\text{N} > (0.37)\left[(35\,\text{kg})(9.8\,\text{m/s}^2) - F_2\right].$$

The minimum value of F_2 that satisfies this inequality is a value slightly bigger than $45.7\ \text{N}$, so we express our answer as $F_{2,\,\min} = 46$ N.

(e) In this final case, moving the crate requires a greater horizontal push from the worker than static friction (as computed in part (a)) can resist. Thus, Newton's law in the horizontal direction leads to

$$F + F_2 > f_{s,\,\max} \quad\Rightarrow\quad 110\ \text{N} + F_2 > 126.9\ \text{N}$$

which leads (after appropriate rounding) to $F_{2,\,\min} = 17$ N.

12. (a) Using the result obtained in Sample Problem 6-2, the maximum angle for which static friction applies is

$$\theta_{\max} = \tan^{-1} \mu_s = \tan^{-1} 0.63 \approx 32° .$$

This is greater than the dip angle in the problem, so the block does not slide.

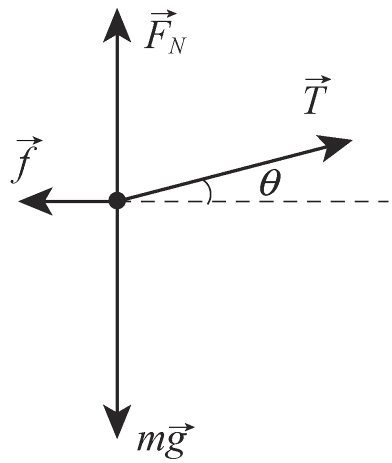
(b) We analyze forces in a manner similar to that shown in Sample Problem 6-3, but with the addition of a downhill force F.

$$F + mg \sin \theta - f_{s,\,\max} = ma = 0$$
$$F_N - mg \cos \theta = 0.$$

Along with Eq. 6-1 ($f_{s,\,\max} = \mu_s F_N$) we have enough information to solve for F. With $\theta = 24°$ and $m = 1.8 \times 10^7$ kg, we find

$$F = mg\left(\mu_s \cos\theta - \sin\theta\right) = 3.0\times10^7 \text{ N}.$$

13. (a) The free-body diagram for the crate is shown on the right. $\vec{T}$ is the tension force of the rope on the crate, $\vec{F}_N$ is the normal force of the floor on the crate, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of friction. We take the $+x$ direction to be horizontal to the right and the $+y$ direction to be up. We assume the crate is motionless. The equations for the x and the y components of the force according to Newton's second law are:



$$T\cos\theta - f = 0$$
$$T\sin\theta + F_N - mg = 0$$

where $\theta = 15°$ is the angle between the rope and the horizontal. The first equation gives $f = T\cos\theta$ and the second gives $F_N = mg - T\sin\theta$. If the crate is to remain at rest, f must be less than $\mu_s F_N$, or $T\cos\theta < \mu_s(mg - T\sin\theta)$. When the tension force is sufficient to just start the crate moving, we must have

$$T\cos\theta = \mu_s(mg - T\sin\theta).$$

We solve for the tension:

$$T = \frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta} = \frac{(0.50)\,(68\text{ kg})\,\left(9.8\text{ m/s}^2\right)}{\cos 15° + 0.50\sin 15°} = 304\text{ N} \approx 3.0\times10^2\text{ N}.$$

(b) The second law equations for the moving crate are

$$T\cos\theta - f = ma$$
$$F_N + T\sin\theta - mg = 0.$$

Now $f = \mu_k F_N$, and the second equation gives $F_N = mg - T\sin\theta$, which yields $f = \mu_k(mg - T\sin\theta)$. This expression is substituted for f in the first equation to obtain

$$T\cos\theta - \mu_k(mg - T\sin\theta) = ma,$$

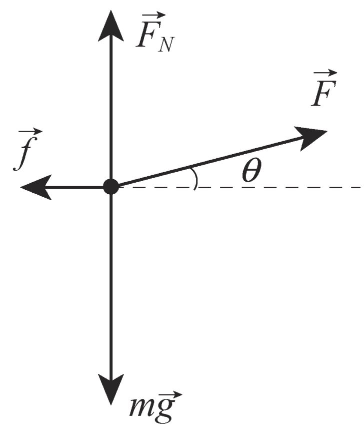
so the acceleration is

$$a = \frac{T\left(\cos\theta + \mu_k\sin\theta\right)}{m} - \mu_k g\,.$$

Numerically, it is given by

$$a = \frac{(304\text{ N})\left(\cos 15° + 0.35\sin 15°\right)}{68\text{ kg}} - (0.35)\left(9.8\text{ m/s}^2\right) = 1.3\text{ m/s}^2\,.$$

14. (a) The free-body diagram for the block is shown on the right, with $\vec{F}$ being the force applied to the block, $\vec{F}_N$ the normal force of the floor on the block, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction. We take the $+x$ direction to be horizontal to the right and the $+y$ direction to be up. The equations for the x and the y components of the force according to Newton's second law are:



$$F_x = F\cos\theta - f = ma$$
$$F_y = F\sin\theta + F_N - mg = 0$$

Now $f = \mu_k F_N$, and the second equation gives $F_N = mg - F\sin\theta$, which yields $f = \mu_k(mg - F\sin\theta)$. This expression is substituted for f in the first equation to obtain

$$F\cos\theta - \mu_k(mg - F\sin\theta) = ma,$$

so the acceleration is

$$a = \frac{F}{m}(\cos\theta + \mu_k\sin\theta) - \mu_k g\,.$$

(a) If $\mu_s = 0.600$ and $\mu_k = 0.500$, then the magnitude of $\vec{f}$ has a maximum value of

$$f_{s,\max} = \mu_s F_N = (0.600)(mg - 0.500mg\sin 20°) = 0.497mg.$$

On the other hand, $F\cos\theta = 0.500mg\cos 20° = 0.470mg$. Therefore, $F\cos\theta < f_{s,\max}$ and the block remains stationary with $a = 0$.

(b) If $\mu_s = 0.400$ and $\mu_k = 0.300$, then the magnitude of $\vec{f}$ has a maximum value of

$$f_{s,\max} = \mu_s F_N = (0.400)(mg - 0.500mg\sin 20°) = 0.332mg.$$

In this case, $F\cos\theta = 0.500mg\cos 20° = 0.470mg > f_{s,\max}$. Therefore, the acceleration of the block is

$$\begin{aligned}a &= \frac{F}{m}(\cos\theta + \mu_k\sin\theta) - \mu_k g\\ &= (0.500)(9.80\ \text{m/s}^2)\left[\cos 20° + (0.300)\sin 20°\right] - (0.300)(9.80\ \text{m/s}^2)\\ &= 2.17\ \text{m/s}^2.\end{aligned}$$

15. An excellent discussion and equation development related to this problem is given in Sample Problem 6-2. We merely quote (and apply) their main result:

$$\theta = \tan^{-1}\mu_s = \tan^{-1} 0.04 \approx 2^\circ .$$

16. (a) We apply Newton's second law to the "downhill" direction:

$$mg\sin\theta - f = ma,$$

where, using Eq. 6-11,

$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta .$$

Thus, with $\mu_k = 0.600$, we have

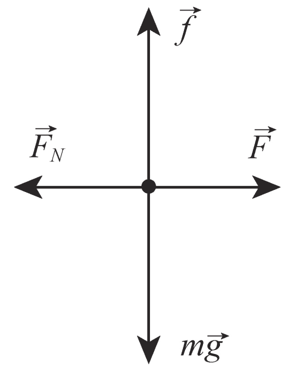
$$a = g\sin\theta - \mu_k\cos\theta = -3.72 \text{ m/s}^2$$

which means, since we have chosen the positive direction in the direction of motion (down the slope) then the acceleration vector points "uphill"; it is decelerating. With $v_0 = 18.0$ m/s and $\Delta x = d = 24.0$ m, Eq. 2-16 leads to

$$v = \sqrt{v_0^2 + 2ad} = 12.1 \text{ m/s}.$$

(b) In this case, we find $a = +1.1$ m/s^2, and the speed (when impact occurs) is 19.4 m/s.

17. (a) The free-body diagram for the block is shown below. $\vec{F}$ is the applied force, $\vec{F}_N$ is the normal force of the wall on the block, $\vec{f}$ is the force of friction, and $m\vec{g}$ is the force of gravity. To determine if the block falls, we find the magnitude f of the force of friction required to hold it without accelerating and also find the normal force of the wall on the block. We compare f and $\mu_s F_N$. If $f < \mu_s F_N$, the block does not slide on the wall but if $f > \mu_s F_N$, the block does slide. The horizontal component of Newton's second law is $F - F_N = 0$, so $F_N = F = 12$ N and



$$\mu_s F_N = (0.60)(12 \text{ N}) = 7.2 \text{ N}.$$

The vertical component is $f - mg = 0$, so $f = mg = 5.0$ N. Since $f < \mu_s F_N$ the block does not slide.

(b) Since the block does not move $f = 5.0$ N and $F_N = 12$ N. The force of the wall on the block is

$$\vec{F}_w = -F_N\,\hat{\text{i}} + f\,\hat{\text{j}} = -(12\text{N})\,\hat{\text{i}} + (5.0\text{N})\,\hat{\text{j}}$$

where the axes are as shown on Fig. 6-26 of the text.

18. We find the acceleration from the slope of the graph (recall Eq. 2-11): a = 4.5 m/s^2. Thus, Newton's second law leads to

$$F - \mu_k mg = ma,$$

where F = 40.0 N is the constant horizontal force applied. With m = 4.1 kg, we arrive at μ_k =0.54.

19. Fig. 6-4 in the textbook shows a similar situation (using ϕ for the unknown angle) along with a free-body diagram. We use the same coordinate system as in that figure.

(a) Thus, Newton's second law leads to

$$\begin{aligned} x:& \quad T\cos\phi - f = ma \\ y:& \quad T\sin\phi + F_N - mg = 0 \end{aligned}$$

Setting a = 0 and $f = f_{s,\max} = \mu_s F_N$, we solve for the mass of the box-and-sand (as a function of angle):

$$m = \frac{T}{g}\left(\sin\phi + \frac{\cos\phi}{\mu_s}\right)$$

which we will solve with calculus techniques (to find the angle ϕ_m corresponding to the maximum mass that can be pulled).

$$\frac{dm}{dt} = \frac{T}{g}\left(\cos\phi_m - \frac{\sin\phi_m}{\mu_s}\right) = 0$$

This leads to tan $\phi_m = \mu_s$ which (for $\mu_s = 0.35$) yields $\phi_m = 19°$.

(b) Plugging our value for ϕ_m into the equation we found for the mass of the box-and-sand yields m = 340 kg. This corresponds to a weight of $mg = 3.3 \times 10^3$ N.

20. (a) In this situation, we take $\vec{f}_s$ to point uphill and to be equal to its maximum value, in which case $f_{s,\,\max} = \mu_s F_N$ applies, where $\mu_s = 0.25$. Applying Newton's second law to the block of mass $m = W/g = 8.2$ kg, in the x and y directions, produces

$$\begin{aligned} F_{\min 1} - mg\sin\theta + f_{s,\,\max} &= ma = 0 \\ F_N - mg\cos\theta &= 0 \end{aligned}$$

which (with $\theta = 20°$) leads to

$$F_{\min 1} - mg\left(\sin\theta + \mu_s\cos\theta\right) = 8.6\ \text{N}.$$

(b) Now we take $\vec{f}_s$ to point downhill and to be equal to its maximum value, in which case $f_{s,\,\max} = \mu_s F_N$ applies, where $\mu_s = 0.25$. Applying Newton's second law to the block of mass $m = W/g = 8.2$ kg, in the x and y directions, produces

$$\begin{aligned} F_{\min 2} = mg\sin\theta - f_{s,\,\max} &= ma = 0 \\ F_N - mg\cos\theta &= 0 \end{aligned}$$

which (with $\theta = 20°$) leads to

$$F_{\min 2} = mg\left(\sin\theta + \mu_s\cos\theta\right) = 46\ \text{N}.$$

A value slightly larger than the "exact" result of this calculation is required to make it accelerate uphill, but since we quote our results here to two significant figures, 46 N is a "good enough" answer.

(c) Finally, we are dealing with kinetic friction (pointing downhill), so that

$$\begin{aligned} 0 &= F - mg\sin\theta - f_k = ma \\ 0 &= F_N - mg\cos\theta \end{aligned}$$

along with $f_k = \mu_k F_N$ (where $\mu_k = 0.15$) brings us to

$$F = mg\left(\sin\theta + \mu_k\cos\theta\right) = 39\ \text{N}.$$

21. If the block is sliding then we compute the kinetic friction from Eq. 6-2; if it is not sliding, then we determine the extent of static friction from applying Newton's law, with zero acceleration, to the *x* axis (which is parallel to the incline surface). The question of whether or not it is sliding is therefore crucial, and depends on the maximum static friction force, as calculated from Eq. 6-1. The forces are resolved in the incline plane coordinate system in Figure 6-5 in the textbook. The acceleration, if there is any, is along the *x* axis, and we are taking uphill as $+x$. The net force along the *y* axis, then, is certainly zero, which provides the following relationship:

$$\sum \vec{F}_y = 0 \;\Rightarrow\; F_N = W\cos\theta$$

where $W = mg = 45$ N is the weight of the block, and $\theta = 15°$ is the incline angle. Thus, $F_N = 43.5$ N, which implies that the maximum static friction force should be

$$f_{s,\max} = (0.50)\,(43.5\text{ N}) = 21.7\text{ N}.$$

(a) For $\vec{P} = (-5.0\text{ N})\hat{\text{i}}$, Newton's second law, applied to the *x* axis becomes

$$f - |P| - mg\sin\theta = ma\,.$$

Here we are assuming $\vec{f}$ is pointing uphill, as shown in Figure 6-5, and if it turns out that it points downhill (which *is* a possibility), then the result for f_s will be negative. If $f = f_s$ then $a = 0$, we obtain

$$f_s = |\,P\,| + mg\sin\theta = 5.0\text{ N} + (43.5\text{ N})\sin 15° = 17\text{ N},$$

or $\vec{f}_s = (17\text{ N})\hat{\text{i}}$. This is clearly allowed since f_s is less than $f_{s,\max}$.

(b) For $\vec{P} = (-8.0\text{ N})\hat{\text{i}}$, we obtain (from the same equation) $\vec{f}_s = (20\text{ N})\hat{\text{i}}$, which is still allowed since it is less than $f_{s,\max}$.

(c) But for $\vec{P} = (-15\text{ N})\hat{\text{i}}$, we obtain (from the same equation) $f_s = 27$ N, which is not allowed since it is larger than $f_{s,\max}$. Thus, we conclude that it is the kinetic friction instead of the static friction that is relevant in this case. The result is

$$\vec{f}_k = \mu_k F_N\hat{\text{i}} = (0.34)(43.5\text{ N})\,\hat{\text{i}} = (15\text{ N})\,\hat{\text{i}}\,.$$

22. Treating the two boxes as a single system of total mass $m_C + m_W = 1.0 + 3.0 = 4.0$ kg, subject to a total (leftward) friction of magnitude 2.0 N + 4.0 N = 6.0 N, we apply Newton's second law (with $+x$ rightward):

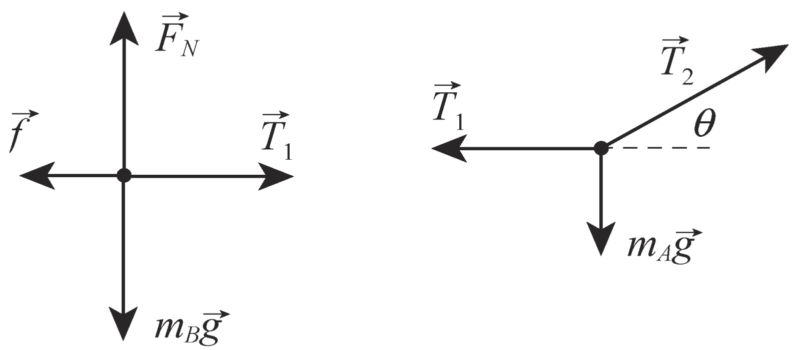
$$F - f_{\text{total}} = m_{\text{total}}\, a \quad \Rightarrow \quad 12.0\ \text{N} - 6.0\ \text{N} = (4.0\ \text{kg})a$$

which yields the acceleration $a = 1.5$ m/s^2. We have treated F as if it were known to the nearest tenth of a Newton so that our acceleration is "good" to two significant figures. Turning our attention to the larger box (the Wheaties box of mass $m_W = 3.0$ kg) we apply Newton's second law to find the contact force F' exerted by the Cheerios box on it.

$$F' - f_W = m_W a \quad \Rightarrow \quad F' - 4.0\ \text{N} = (3.0\ \text{kg})(1.5\ \text{m/s}^2)\,.$$

From the above equation, we find the contact force to be $F' = 8.5$ N.

23. The free-body diagrams for block B and for the knot just above block A are shown next. $\vec{T}_1$ is the tension force of the rope pulling on block B or pulling on the knot (as the case may be), $\vec{T}_2$ is the tension force exerted by the second rope (at angle $\theta = 30°$) on the knot, $\vec{f}$ is the force of static friction exerted by the horizontal surface on block B, $\vec{F}_N$ is normal force exerted by the surface on block B, W_A is the weight of block A (W_A is the magnitude of $m_A\vec{g}$), and W_B is the weight of block B (W_B = 711 N is the magnitude of $m_B\vec{g}$).



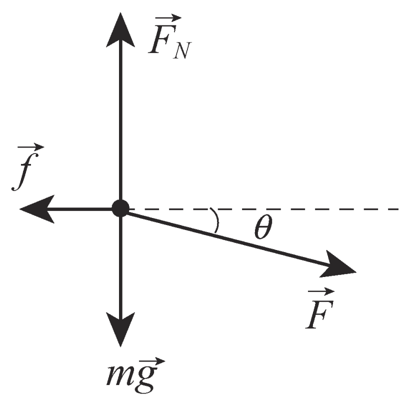
For each object we take $+x$ horizontally rightward and $+y$ upward. Applying Newton's second law in the x and y directions for block B and then doing the same for the knot results in four equations:

$$\begin{aligned} T_1 - f_{s,\max} &= 0 \\ F_N - W_B &= 0 \\ T_2 \cos\theta - T_1 &= 0 \\ T_2 \sin\theta - W_A &= 0 \end{aligned}$$

where we assume the static friction to be at its maximum value (permitting us to use Eq. 6-1). Solving these equations with $\mu_s = 0.25$, we obtain $W_A = 103 \text{ N} \approx 1.0\times10^2 \text{ N}$.

24. The free-body diagram for the block is shown below, with $\vec{F}$ being the force applied to the block, $\vec{F}_N$ the normal force of the floor on the block, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction. We take the $+x$ direction to be horizontal to the right and the $+y$ direction to be up. The equations for the x and the y components of the force according to Newton's second law are:

$$F_x = F\cos\theta - f = ma$$
$$F_y = F_N - F\sin\theta - mg = 0$$



Now $f = \mu_k F_N$, and the second equation gives $F_N = mg + F\sin\theta$, which yields

$$f = \mu_k(mg + F\sin\theta).$$

This expression is substituted for f in the first equation to obtain

$$F\cos\theta - \mu_k(mg + F\sin\theta) = ma,$$

so the acceleration is

$$a = \frac{F}{m}(\cos\theta - \mu_k\sin\theta) - \mu_k g.$$

From Fig. 6-32, we see that $a = 3.0\ \text{m/s}^2$ when $\mu_k = 0$. This implies

$$3.0\ \text{m/s}^2 = \frac{F}{m}\cos\theta.$$

We also find $a = 0$ when $\mu_k = 0.20$:

$$0 = \frac{F}{m}(\cos\theta - (0.20)\sin\theta) - (0.20)(9.8\ \text{m/s}^2) = 3.00\ \text{m/s}^2 - 0.20\frac{F}{m}\sin\theta - 1.96\ \text{m/s}^2$$
$$= 1.04\ \text{m/s}^2 - 0.20\frac{F}{m}\sin\theta$$

which yields $5.2\ \text{m/s}^2 = \dfrac{F}{m}\sin\theta$. Combining the two results, we get

$$\tan\theta = \left(\frac{5.2\ \text{m/s}^2}{3.0\ \text{m/s}^2}\right) = 1.73 \quad\Rightarrow\quad \theta = 60°.$$

25. Let the tensions on the strings connecting m_2 and m_3 be $T_{23,}$ and that connecting m_2 and m_1 be T_{12}, respectively. Applying Newton's second law (and Eq. 6-2, with $F_N = m_2 g$ in this case) to the *system* we have

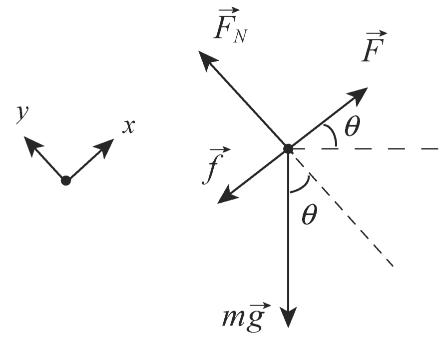
$$\begin{aligned} m_3 g - T_{23} &= m_3 a \\ T_{23} - \mu_k m_2 g - T_{12} &= m_2 a \\ T_{12} - m_1 g &= m_1 a \end{aligned}$$

Adding up the three equations and using $m_1 = M, m_2 = m_3 = 2M$, we obtain

$$2Mg - 2\mu_k Mg - Mg = 5Ma \ .$$

With a = 0.500 m/s^2 this yields μ_k = 0.372. Thus, the coefficient of kinetic friction is roughly μ_k = 0.37.

26. The free-body diagram for the sled is shown on the right, with $\vec{F}$ being the force applied to the sled, $\vec{F}_N$ the normal force of the inclined plane on the sled, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction. We take the $+x$ direction to be along the inclined plane and the $+y$ direction to be in its normal direction. The equations for the x and the y components of the force according to Newton's second law are:



$$F_x = F - f - mg\sin\theta = ma = 0$$
$$F_y = F_N - mg\cos\theta = 0$$

Now $f = \mu F_N$, and the second equation gives $F_N = mg\cos\theta$, which yields $f = \mu mg\cos\theta$. This expression is substituted for f in the first equation to obtain

$$F = mg(\sin\theta + \mu\cos\theta)$$

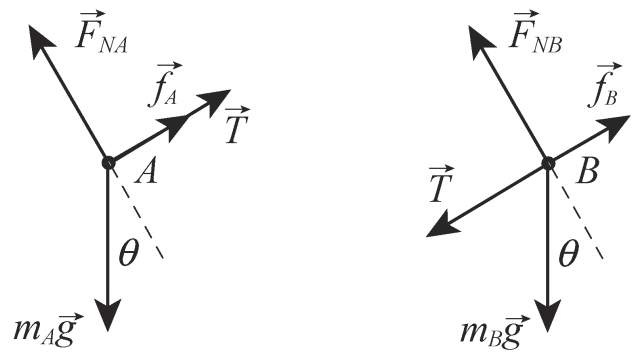
From Fig. 6-34, we see that $F = 2.0$ N when $\mu = 0$. This implies $mg\sin\theta = 2.0$ N. Similarly, we also find $F = 5.0$ N when $\mu = 0.5$:

$$5.0\text{ N} = mg(\sin\theta + 0.50\cos\theta) = 2.0\text{ N} + 0.50mg\cos\theta$$

which yields $mg\cos\theta = 6.0$ N. Combining the two results, we get

$$\tan\theta = \frac{2}{6} = \frac{1}{3} \quad\Rightarrow\quad \theta = 18°.$$

27. The free-body diagrams for the two blocks are shown next. T is the magnitude of the tension force of the string, $\vec{F}_{NA}$ is the normal force on block A (the leading block), $\vec{F}_{NB}$ is the normal force on block B, $\vec{f}_A$ is kinetic friction force on block A, $\vec{f}_B$ is kinetic friction force on block B. Also, m_A is the mass of block A (where $m_A = W_A/g$ and $W_A = 3.6$ N), and m_B is the mass of block B (where $m_B = W_B/g$ and $W_B = 7.2$ N). The angle of the incline is $\theta = 30°$.



For each block we take $+x$ downhill (which is toward the lower-left in these diagrams) and $+y$ in the direction of the normal force. Applying Newton's second law to the x and y directions of both blocks A and B, we arrive at four equations:

$$W_A \sin\theta - f_A - T = m_A\ a$$
$$F_{NA} - W_A \cos\theta = 0$$
$$W_B \sin\theta - f_B + T = m_B\ a$$
$$F_{NB} - W_B \cos\theta = 0$$

which, when combined with Eq. 6-2 ($f_A = \mu_{kA} F_{NA}$ where $\mu_{k\,A} = 0.10$ and $f_B = \mu_{kB} F_{NB}$$f_B$ where $\mu_{k\,B} = 0.20$), fully describe the dynamics of the system so long as the blocks have the same acceleration and $T > 0$.

(a) From these equations, we find the acceleration to be

$$a = g\left(\sin\theta - \left(\frac{\mu_{k\,A} W_A + \mu_{k\,B} W_B}{W_A + W_B}\right)\cos\theta\right) = 3.5\ \text{m/s}^2.$$

(b) We solve the above equations for the tension and obtain

$$T = \left(\frac{W_A W_B}{W_A + W_B}\right)\ (\mu_{k\,B} - \mu_{k\,A})\cos\theta = 0.21\ \text{N}.$$

Simply returning the value for a found in part (a) into one of the above equations is certainly fine, and probably easier than solving for T algebraically as we have done, but the algebraic form does illustrate the $\mu_{k\,B} - \mu_{k\,A}$ factor which aids in the understanding of the next part.

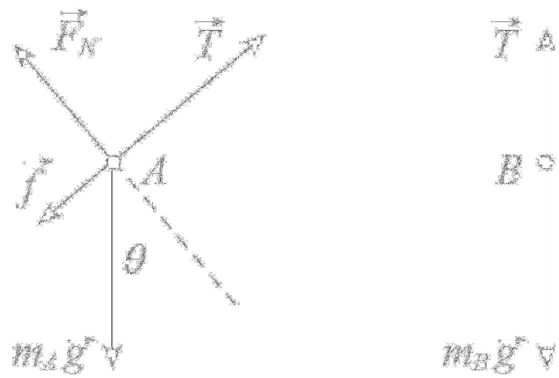
28. (a) Applying Newton's second law to the *system* (of total mass M = 60.0 kg) and using Eq. 6-2 (with $F_N = Mg$ in this case) we obtain

$$F - \mu_k Mg = Ma \;\Rightarrow\; a = 0.473 \text{ m/s}^2.$$

Next, we examine the forces just on m_3 and find $F_{32} = m_3(a + \mu_k g) =$ 147 N. If the algebra steps are done more systematically, one ends up with the interesting relationship: $F_{32} = (m_3 / M)F$ (which is independent of the friction!).

(b) As remarked at the end of our solution to part (a), the result does not depend on the frictional parameters. The answer here is the same as in part (a).

29. First, we check to see if the bodies start to move. We assume they remain at rest and compute the force of (static) friction which holds them there, and compare its magnitude with the maximum value $\mu_s F_N$. The free-body diagrams are shown below. T is the magnitude of the tension force of the string, f is the magnitude of the force of friction on body A, F_N is the magnitude of the normal force of the plane on body A, $m_A\vec{g}$ is the force of gravity on body A (with magnitude W_A = 102 N), and $m_B\vec{g}$ is the force of gravity on body B (with magnitude W_B = 32 N). θ = 40° is the angle of incline. We are told the direction of $\vec{f}$ but we assume it is downhill. If we obtain a negative result for f, then we know the force is actually up the plane.



(a) For A we take the $+x$ to be uphill and $+y$ to be in the direction of the normal force. The x and y components of Newton's second law become

$$T - f - W_A \sin\theta = 0$$
$$F_N - W_A \cos\theta = 0.$$

Taking the positive direction to be *downward* for body B, Newton's second law leads to $W_B - T = 0$. Solving these three equations leads to

$$f = W_B - W_A \sin\theta = 32\text{ N} - (102\text{ N})\sin 40° = -34\text{ N}$$

(indicating that the force of friction is *uphill*) and to

$$F_N = W_A \cos\theta = (102\text{ N})\cos 40° = 78\text{N}$$

which means that

$$f_{s,\max} = \mu_s F_N = (0.56)\,(78\text{ N}) = 44\text{ N}.$$

Since the magnitude f of the force of friction that holds the bodies motionless is less than $f_{s,\max}$ the bodies remain at rest. The acceleration is zero.

(b) Since A is moving up the incline, the force of friction is downhill with magnitude $f_k = \mu_k F_N$. Newton's second law, using the same coordinates as in part (a), leads to

$$\begin{aligned} T - f_k - W_A \sin\theta &= m_A a \\ F_N - W_A \cos\theta &= 0 \\ W_B - T &= m_B a \end{aligned}$$

for the two bodies. We solve for the acceleration:

$$\begin{aligned} a &= \frac{W_B - W_A \sin\theta - \mu_k W_A \cos\theta}{m_B + m_A} = \frac{32\text{N} - (102\text{N})\sin 40° - (0.25)(102\text{N})\cos 40°}{(32\text{N}+102\text{N}) \big/ \left(9.8\text{ m/s}^2\right)} \\ &= -3.9\text{ m/s}^2. \end{aligned}$$

The acceleration is down the plane, i.e., $\vec{a} = (-3.9\text{ m/s}^2)\hat{\mathrm{i}}$, which is to say (since the initial velocity was uphill) that the objects are slowing down. We note that $m = W/g$ has been used to calculate the masses in the calculation above.

(c) Now body A is initially moving down the plane, so the force of friction is uphill with magnitude $f_k = \mu_k F_N$. The force equations become

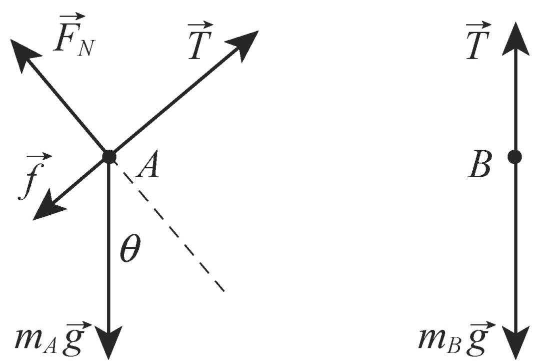
$$\begin{aligned} T + f_k - W_A \sin\theta &= m_A a \\ F_N - W_A \cos\theta &= 0 \\ W_B - T &= m_B a \end{aligned}$$

which we solve to obtain

$$\begin{aligned} a &= \frac{W_B - W_A \sin\theta + \mu_k W_A \cos\theta}{m_B + m_A} = \frac{32\text{N} - (102\text{N})\sin 40° + (0.25)(102\text{N})\cos 40°}{(32\text{N}+102\text{N}) \big/ \left(9.8\text{ m/s}^2\right)} \\ &= -1.0\text{ m/s}^2. \end{aligned}$$

The acceleration is again downhill the plane, i.e., $\vec{a} = (-1.0\text{ m/s}^2)\hat{\mathrm{i}}$. In this case, the objects are speeding up.

30. The free-body diagrams are shown below. T is the magnitude of the tension force of the string, f is the magnitude of the force of friction on block A, F_N is the magnitude of the normal force of the plane on block A, $m_A\vec{g}$ is the force of gravity on body A (where m_A = 10 kg), and $m_B\vec{g}$ is the force of gravity on block B. θ = 30° is the angle of incline. For A we take the $+x$ to be uphill and $+y$ to be in the direction of the normal force; the positive direction is chosen *downward* for block B.



Since A is moving down the incline, the force of friction is uphill with magnitude $f_k = \mu_k F_N$ (where μ_k = 0.20). Newton's second law leads to

$$\begin{aligned} T - f_k + m_A g \sin\theta &= m_A a = 0 \\ F_N - m_A g \cos\theta &= 0 \\ m_B g - T &= m_B a = 0 \end{aligned}$$

for the two bodies (where a = 0 is a consequence of the velocity being constant). We solve these for the mass of block B.

$$m_B = m_A\left(\sin\theta - \mu_k \cos\theta\right) = 3.3 \text{ kg}.$$

31. (a) Free-body diagrams for the blocks A and C, considered as a single object, and for the block B are shown below. T is the magnitude of the tension force of the rope, F_N is the magnitude of the normal force of the table on block A, f is the magnitude of the force of friction, W_{AC} is the combined weight of blocks A and C (the magnitude of force $\vec{F}_{g\,AC}$ shown in the figure), and W_B is the weight of block B (the magnitude of force $\vec{F}_{g\,B}$ shown). Assume the blocks are not moving. For the blocks on the table we take the x axis to be to the right and the y axis to be upward. From Newton's second law, we have

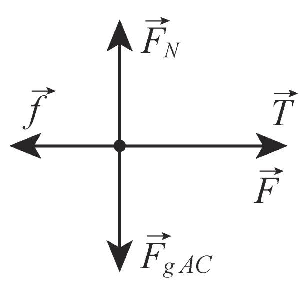
$$x \text{ component:} \qquad T - f = 0$$

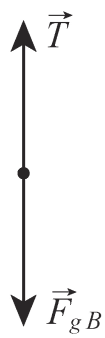
$$y \text{ component:} \qquad F_N - W_{AC} = 0.$$

For block B take the downward direction to be positive. Then Newton's second law for that block is $W_B - T = 0$. The third equation gives $T = W_B$ and the first gives $f = T = W_B$. The second equation gives $F_N = W_{AC}$. If sliding is not to occur, f must be less than $\mu_s F_N$, or $W_B < \mu_s W_{AC}$. The smallest that W_{AC} can be with the blocks still at rest is

$$W_{AC} = W_B/\mu_s = (22\text{ N})/(0.20) = 110\text{ N}.$$

Since the weight of block A is 44 N, the least weight for C is $(110 - 44)$ N = 66 N.





(b) The second law equations become

$$T - f = (W_A/g)a$$
$$F_N - W_A = 0$$
$$W_B - T = (W_B/g)a.$$

In addition, $f = \mu_k F_N$. The second equation gives $F_N = W_A$, so $f = \mu_k W_A$. The third gives $T = W_B - (W_B/g)a$. Substituting these two expressions into the first equation, we obtain

$$W_B - (W_B/g)a - \mu_k W_A = (W_A/g)a.$$

Therefore,

$$a = \frac{g\left(W_B - \mu_k W_A\right)}{W_A + W_B} = \frac{(9.8\text{ m/s}^2)\left(22\text{ N} - (0.15)(44\text{ N})\right)}{44\text{ N} + 22\text{ N}} = 2.3\text{ m/s}^2.$$

32. We use the familiar horizontal and vertical axes for x and y directions, with rightward and upward positive, respectively. The rope is assumed massless so that the force exerted by the child $\vec{F}$ is identical to the tension uniformly through the rope. The x and y components of $\vec{F}$ are $F\cos\theta$ and $F\sin\theta$, respectively. The static friction force points leftward.

(a) Newton's Law applied to the y-axis, where there is presumed to be no acceleration, leads to

$$F_N + F\sin\theta - mg = 0$$

which implies that the maximum static friction is $\mu_s(mg - F\sin\theta)$. If $f_s = f_{s,\,\max}$ is assumed, then Newton's second law applied to the x axis (which also has $a = 0$ even though it is "verging" on moving) yields

$$F\cos\theta - f_s = ma \quad \Rightarrow \quad F\cos\theta - \mu_s(mg - F\sin\theta) = 0$$

which we solve, for $\theta = 42°$ and $\mu_s = 0.42$, to obtain $F = 74$ N.

(b) Solving the above equation algebraically for F, with W denoting the weight, we obtain

$$F = \frac{\mu_s W}{\cos\theta + \mu_s \sin\theta} = \frac{(0.42)(180\text{ N})}{\cos\theta + (0.42)\sin\theta} = \frac{76\text{ N}}{\cos\theta + (0.42)\sin\theta}.$$

(c) We minimize the above expression for F by working through the condition:

$$\frac{dF}{d\theta} = \frac{\mu_s W(\sin\theta - \mu_s\cos\theta)}{(\cos\theta + \mu_s\sin\theta)^2} = 0$$

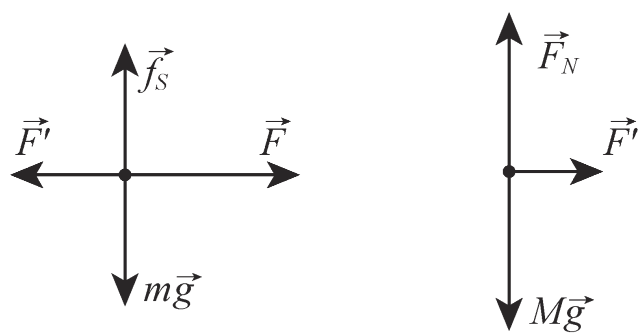
which leads to the result $\theta = \tan^{-1}\mu_s = 23°$.

(d) Plugging $\theta = 23°$ into the above result for F, with $\mu_s = 0.42$ and $W = 180$ N, yields $F = 70\text{ N}$.

33. The free-body diagrams for the two blocks, treated individually, are shown below (first m and then M). F' is the contact force between the two blocks, and the static friction force $\vec{f}_s$ is at its maximum value (so Eq. 6-1 leads to $f_s = f_{s,\max} = \mu_s F'$ where $\mu_s = 0.38$).

Treating the two blocks together as a single system (sliding across a frictionless floor), we apply Newton's second law (with $+x$ rightward) to find an expression for the acceleration:

$$F = m_{\text{total}}\, a \quad \Rightarrow a = \frac{F}{m+M}$$



This is equivalent to having analyzed the two blocks individually and then combined their equations. Now, when we analyze the small block individually, we apply Newton's second law to the x and y axes, substitute in the above expression for a, and use Eq. 6-1.

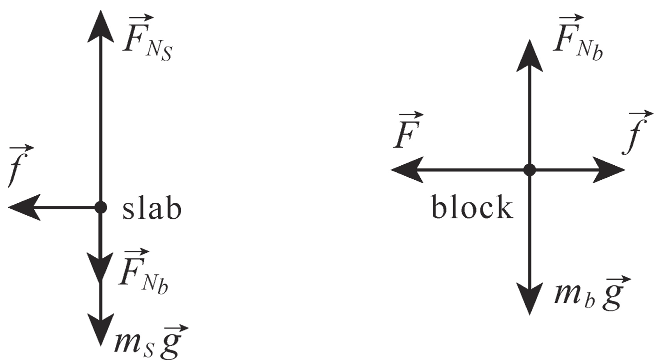
$$\begin{aligned} F - F' = ma \quad &\Rightarrow \quad F' = F - m\left(\frac{F}{m+M}\right) \\ f_s - mg = 0 \quad &\Rightarrow \quad \mu_s F' - mg = 0 \end{aligned}$$

These expressions are combined (to eliminate F') and we arrive at

$$F = \frac{mg}{\mu_s\left(1 - \dfrac{m}{m+M}\right)}$$

which we find to be $F = 4.9 \times 10^2$ N.

34. The free-body diagrams for the slab and block are shown below.



$\vec{F}$ is the 100 N force applied to the block, $\vec{F}_{Ns}$ is the normal force of the floor on the slab, F_{Nb} is the magnitude of the normal force between the slab and the block, $\vec{f}$ is the force of friction between the slab and the block, m_s is the mass of the slab, and m_b is the mass of the block. For both objects, we take the $+x$ direction to be to the right and the $+y$ direction to be up.

Applying Newton's second law for the x and y axes for (first) the slab and (second) the block results in four equations:

$$\begin{aligned} -f &= m_s a_s \\ F_{Ns} - F_{Ns} - m_s g &= 0 \\ f - F &= m_b a_b \\ F_{Nb} - m_b g &= 0 \end{aligned}$$

from which we note that the maximum possible static friction magnitude would be

$$\mu_s F_{Nb} = \mu_s m_b g = (0.60)(10\ \text{kg})(9.8\ \text{m/s}^2) = 59\ \text{N}\ .$$

We check to see if the block slides on the slab. Assuming it does not, then $a_s = a_b$ (which we denote simply as a) and we solve for f:

$$f = \frac{m_s F}{m_s + m_b} = \frac{(40\ \text{kg})(100\ \text{N})}{40\ \text{kg} + 10\ \text{kg}} = 80\ \text{N}$$

which is greater than $f_{s,\max}$ so that we conclude the block is sliding across the slab (their accelerations are different).

(a) Using $f = \mu_k F_{Nb}$ the above equations yield

$$a_b = \frac{\mu_k m_b g - F}{m_b} = \frac{(0.40)(10\ \text{kg})(9.8\ \text{m/s}^2) - 100\ \text{N}}{10\ \text{kg}} = -6.1\ \text{m/s}^2.$$

The negative sign means that the acceleration is leftward. That is, $\vec{a}_b = (-6.1\ \text{m/s}^2)\hat{\text{i}}$

(b) We also obtain

$$a_s = -\frac{\mu_k m_b g}{m_s} = -\frac{(0.40)(10\ \text{kg})(9.8\ \text{m/s}^2)}{40\ \text{kg}} = -0.98\ \text{m/s}^2.$$

As mentioned above, this means it accelerates to the left. That is, $\vec{a}_s = (-0.98\ \text{m/s}^2)\hat{\text{i}}$

35. We denote the magnitude of the frictional force αv, where $\alpha = 70\ \mathrm{N \cdot s/m}$. We take the direction of the boat's motion to be positive. Newton's second law gives

$$-\alpha v = m\frac{dv}{dt}.$$

Thus,

$$\int_{v_0}^{v} \frac{dv}{v} = -\frac{\alpha}{m}\int_0^t dt$$

where v_0 is the velocity at time zero and v is the velocity at time t. The integrals are evaluated with the result

$$\ln\left(\frac{v}{v_0}\right) = -\frac{\alpha t}{m}$$

We take $v = v_0/2$ and solve for time:

$$t = \frac{m}{\alpha}\ln 2 = \frac{1000\ \mathrm{kg}}{70\ \mathrm{N \cdot s/m}}\ln 2 = 9.9\ \mathrm{s}\ .$$

36. Using Eq. 6-16, we solve for the area

$$A\frac{2m\,g}{C\rho\, v_t^2}$$

which illustrates the inverse proportionality between the area and the speed-squared. Thus, when we set up a ratio of areas – of the slower case to the faster case – we obtain

$$\frac{A_{\text{slow}}}{A_{\text{fast}}}=\left(\frac{310\ \text{km/h}}{160\ \text{km/h}}\right)^2=3.75.$$

37. For the passenger jet $D_j = \frac{1}{2}C\rho_1 A v_j^2$, and for the prop-driven transport $D_t = \frac{1}{2}C\rho_2 A v_t^2$, where ρ_1 and ρ_2 represent the air density at 10 km and 5.0 km, respectively. Thus the ratio in question is

$$\frac{D_j}{D_t} = \frac{\rho_1 v_j^2}{\rho_2 v_t^2} = \frac{\left(0.38\ \text{kg/m}^3\right)\left(1000\ \text{km/h}\right)^2}{\left(0.67\ \text{kg/m}^3\right)\left(500\ \text{km/h}\right)^2} = 2.3.$$

38. This problem involves Newton's second law for motion along the slope.

(a) The force along the slope is given by

$$\begin{aligned} F_g &= mg\sin\theta - \mu F_N = mg\sin\theta - \mu mg\cos\theta = mg(\sin\theta - \mu\cos\theta) \\ &= (85.0\text{ kg})(9.80\text{ m/s}^2)\left[\sin 40.0° - (0.04000)\cos 40.0°\right] \\ &= 510\text{ N}. \end{aligned}$$

Thus, the terminal speed of the skier is

$$v_t = \sqrt{\frac{2F_g}{C\rho A}} = \sqrt{\frac{2(510\text{ N})}{(0.150)(1.20\text{ kg/m}^3)(1.30\text{ m}^2)}} = 66.0\text{ m/s}.$$

(b) Differentiating v_t with respect to C, we obtain

$$\begin{aligned} dv_t &= -\frac{1}{2}\sqrt{\frac{2F_g}{\rho A}}\,C^{-3/2}dC = -\frac{1}{2}\sqrt{\frac{2(510\text{ N})}{(1.20\text{ kg/m}^3)(1.30\text{ m}^2)}}(0.150)^{-3/2}\,dC \\ &= -(2.20\times10^2\text{ m/s})dC. \end{aligned}$$

39. In the solution to exercise 4, we found that the force provided by the wind needed to equal $F = 157$ N (where that last figure is not "significant'').

(a) Setting $F = D$ (for Drag force) we use Eq. 6-14 to find the wind speed V along the ground (which actually is relative to the moving stone, but we assume the stone is moving slowly enough that this does not invalidate the result):

$$V = \sqrt{\frac{2F}{C\rho A}} = \sqrt{\frac{2(157\ \text{N})}{(0.80)(1.21\,\text{kg/m}^3)(0.040\ \text{m}^2)}} = 90\ \text{m/s} = 3.2\times10^2\ \text{km/h}.$$

(b) Doubling our previous result, we find the reported speed to be 6.5×10^2 km/h.

(c) The result is not reasonable for a terrestrial storm. A category 5 hurricane has speeds on the order of 2.6×10^2 m/s.

40. (a) From Table 6-1 and Eq. 6-16, we have

$$v_t = \sqrt{\frac{2F_g}{C\rho A}} \Rightarrow C\rho A = 2\frac{mg}{v_t^2}$$

where v_t = 60 m/s. We estimate the pilot's mass at about m = 70 kg. Now, we convert v = 1300(1000/3600) ≈ 360 m/s and plug into Eq. 6-14:

$$D = \frac{1}{2}C\rho A v^2 = \frac{1}{2}\left(2\frac{mg}{v_t^2}\right)v^2 = mg\left(\frac{v}{v_t}\right)^2$$

which yields $D = (70 \text{ kg})(9.8 \text{ m/s}^2)(360/60)^2 \approx 2 \times 10^4$ N.

(b) We assume the mass of the ejection seat is roughly equal to the mass of the pilot. Thus, Newton's second law (in the horizontal direction) applied to this system of mass $2m$ gives the magnitude of acceleration:

$$|a| = \frac{D}{2m} = \frac{g}{2}\left(\frac{v}{v_t}\right)^2 = 18g\ .$$

41. The magnitude of the acceleration of the cyclist as it rounds the curve is given by v^2/R, where v is the speed of the cyclist and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newton's second law is $f = mv^2/R$. If F_N is the normal force of the road on the bicycle and m is the mass of the bicycle and rider, the vertical component of Newton's second law leads to $F_N = mg$. Thus, using Eq. 6-1, the maximum value of static friction is $f_{s,\max} = \mu_s F_N = \mu_s mg$. If the bicycle does not slip, $f \le \mu_s mg$. This means

$$\frac{v^2}{R} \le \mu_s g \;\Rightarrow\; R \ge \frac{v^2}{\mu_s g}.$$

Consequently, the minimum radius with which a cyclist moving at 29 km/h = 8.1 m/s can round the curve without slipping is

$$R_{\min} = \frac{v^2}{\mu_s g} = \frac{(8.1\text{ m/s})^2}{(0.32)(9.8\text{ m/s}^2)} = 21\text{ m}.$$

42. With v = 96.6 km/h = 26.8 m/s, Eq. 6-17 readily yields

$$a = \frac{v^2}{R} = \frac{(26.8\ \text{m/s})^2}{7.6\ \text{m}} = 94.7\ \text{m/s}^2$$

which we express as a multiple of g:

$$a = \left(\frac{a}{g}\right) g = \left(\frac{94.7\ \text{m/s}^2}{9.80\ \text{m/s}^2}\right) g = 9.7g.$$

43. Perhaps surprisingly, the equations pertaining to this situation are exactly those in Sample Problem 6-9, although the logic is a little different. In the Sample Problem, the car moves along a (stationary) road, whereas in this problem the cat is stationary relative to the merry-go-around platform. But the static friction plays the same role in both cases since the bottom-most point of the car tire is instantaneously at rest with respect to the race track, just as static friction applies to the contact surface between cat and platform. Using Eq. 6-23 with Eq. 4-35, we find

$$\mu_s = (2\pi R/T)^2/gR = 4\pi^2 R/gT^2.$$

With $T = 6.0$ s and $R = 5.4$ m, we obtain $\mu_s = 0.60$.

44. The magnitude of the acceleration of the car as it rounds the curve is given by v^2/R, where v is the speed of the car and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newton's second law is $f = mv^2/R$. If F_N is the normal force of the road on the car and m is the mass of the car, the vertical component of Newton's second law leads to $F_N = mg$. Thus, using Eq. 6-1, the maximum value of static friction is

$$f_{s,\max} = \mu_s F_N = \mu_s mg.$$

If the car does not slip, $f \le \mu_s mg$. This means

$$\frac{v^2}{R} \le \mu_s g \quad \Rightarrow \quad v \le \sqrt{\mu_s R g}.$$

Consequently, the maximum speed with which the car can round the curve without slipping is

$$v_{\max} = \sqrt{\mu_s R g} = \sqrt{(0.60)(30.5\text{ m})(9.8\text{ m/s}^2)} = 13\text{ m/s} \approx 48\text{ km/h}.$$

45. (a) Eq. 4-35 gives $T = 2\pi R/v = 2\pi(10\text{ m})/(6.1\text{ m/s}) = 10\text{ s}$.

(b) The situation is similar to that of Sample Problem 6-7 but with the normal force direction reversed. Adapting Eq. 6-19, we find

$$F_N = m(g - v^2/R) = 486\text{ N} \approx 4.9 \times 10^2\text{ N}.$$

(c) Now we reverse both the normal force direction and the acceleration direction (from what is shown in Sample Problem 6-7) and adapt Eq. 6-19 accordingly. Thus we obtain

$$F_N = m(g + v^2/R) = 1081\text{ N} \approx 1.1\text{ kN}.$$

46. We will start by assuming that the normal force (on the car from the rail) points up. Note that gravity points down, and the y axis is chosen positive upwards. Also, the direction to the center of the circle (the direction of centripetal acceleration) is down. Thus, Newton's second law leads to

$$F_N - mg = m\left(-\frac{v^2}{r}\right).$$

(a) When $v = 11$ m/s, we obtain $F_N = 3.7 \times 10^3$ N.

(b) $\vec{F}_N$ points upward.

(c) When $v = 14$ m/s, we obtain $F_N = -1.3 \times 10^3$ N, or $|F_N| = 1.3 \times 10^3$ N.

(d) The fact that this answer is negative means that $\vec{F}_N$ points opposite to what we had assumed. Thus, the magnitude of $\vec{F}_N$ is $|\vec{F}_N| = 1.3$ kN and its direction is *down*.

47. At the top of the hill, the situation is similar to that of Sample Problem 6-7 but with the normal force direction reversed. Adapting Eq. 6-19, we find

$$F_N = m(g - v^2/R).$$

Since $F_N = 0$ there (as stated in the problem) then $v^2 = gR$. Later, at the bottom of the valley, we reverse both the normal force direction and the acceleration direction (from what is shown in Sample Problem 6-7) and adapt Eq. 6-19 accordingly. Thus we obtain

$$F_N = m(g + v^2/R) = 2mg = 1372 \text{ N} \approx 1.37 \times 10^3 \text{ N}.$$

48. (a) We note that the speed 80.0 km/h in SI units is roughly 22.2 m/s. The horizontal force that keeps her from sliding must equal the centripetal force (Eq. 6-18), and the upward force on her must equal mg. Thus,

$$F_{\text{net}} = \sqrt{(mg)^2 + (m\text{v}^2/R)^2} \;= 547 \text{ N}.$$

(b) The angle is $\tan^{-1}[(mv^2/R)/(mg)] = \tan^{-1}(v^2/gR) = 9.53^\circ$ (as measured from a vertical axis).

49. (a) At the top (the highest point in the circular motion) the seat pushes up on the student with a force of magnitude F_N = 556 N. Earth pulls down with a force of magnitude W = 667 N. The seat is pushing up with a force that is smaller than the student's weight, and we say the student experiences a decrease in his "apparent weight" at the highest point. Thus, he feels "light."

(b) Now F_N is the magnitude of the upward force exerted by the seat when the student is at the lowest point. The net force toward the center of the circle is $F_b - W = mv^2/R$ (note that we are now choosing upward as the positive direction). The Ferris wheel is "steadily rotating" so the value mv^2/R is the same as in part (a). Thus,

$$F_N = \frac{mv^2}{R} + W = 111\text{ N} + 667\text{ N} = 778\text{ N}.$$

(c) If the speed is doubled, mv^2/R increases by a factor of 4, to 444 N. Therefore, at the highest point we have $W - F_N = mv^2/R$, which leads to

$$F_N = 667\text{ N} - 444\text{ N} = 223\text{ N}.$$

(d) Similarly, the normal force at the lowest point is now found to be

$$F_N = 667\text{ N} + 444\text{ N} \approx 1.11\text{ kN}.$$

50. The situation is somewhat similar to that shown in the "loop-the-loop" example done in the textbook (see Figure 6-10) except that, instead of a downward normal force, we are dealing with the force of the boom $\vec{F}_B$ on the car – which is capable of pointing any direction. We will assume it to be upward as we apply Newton's second law to the car (of total weight 5000 N): $F_B - W = ma$ where $m = W/g$ and $a = -v^2/r$. Note that the centripetal acceleration is downward (our choice for negative direction) for a body at the top of its circular trajectory.

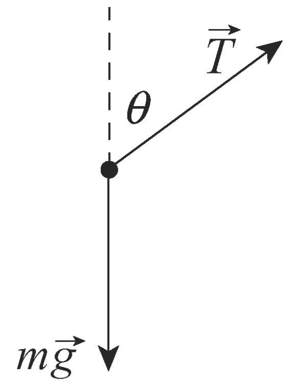
(a) If r = 10 m and v = 5.0 m/s, we obtain $F_B = 3.7 \times 10^3$ N = 3.7 kN.

(b) The direction of $\vec{F}_B$ is up.

(c) If r = 10 m and v = 12 m/s, we obtain $F_B = -2.3 \times 10^3$ N = – 2.3 kN, or $|F_B|$ = 2.3 kN.

(d) The minus sign indicates that $\vec{F}_B$ points downward.

51. The free-body diagram (for the hand straps of mass m) is the view that a passenger might see if she was looking forward and the streetcar was curving towards the right (so $\vec{a}$ points rightwards in the figure). We note that $|\vec{a}| = v^2 / R$ where v = 16 km/h = 4.4 m/s.



Applying Newton's law to the axes of the problem (+x is rightward and +y is upward) we obtain

$$T\sin\theta = m\frac{v^2}{R}$$
$$T\cos\theta = mg.$$

We solve these equations for the angle:

$$\theta = \tan^{-1}\left(\frac{v^2}{Rg}\right)$$

which yields $\theta = 12°$.

52. The centripetal force on the passenger is $F = mv^2 / r$.

(a) The variation of F with respect to r while holding v constant is

$$dF = -\frac{mv^2}{r^2} dr .$$

(b) The variation of F with respect to v while holding r constant is

$$dF = \frac{2mv}{r} dv .$$

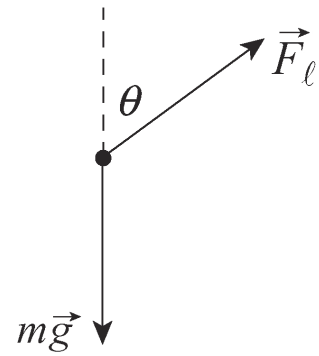
(c) The period of the circular ride is $T = 2\pi r / v$. Thus,

$$F = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 mr}{T^2},$$

and the variation of F with respect to T while holding r constant is

$$dF = -\frac{8\pi^2 mr}{T^3} dT = -8\pi^2 mr\left(\frac{v}{2\pi r}\right)^3 dT = -\left(\frac{mv^3}{\pi r^2}\right) dT.$$

53. The free-body diagram (for the airplane of mass m) is shown below. We note that $\vec{F}_\ell$ is the force of aerodynamic lift and $\vec{a}$ points rightwards in the figure. We also note that $|\vec{a}| = v^2/R$ where v = 480 km/h = 133 m/s.



Applying Newton's law to the axes of the problem (+x rightward and +y upward) we obtain

$$F_\ell \sin\theta = m\frac{v^2}{R}$$
$$F_\ell \cos\theta = mg$$

where θ = 40°. Eliminating mass from these equations leads to

$$\tan\theta = \frac{v^2}{gR}$$

which yields $R = v^2/g \tan\theta = 2.2 \times 10^3$ m.

54. The centripetal force on the passenger is $F = mv^2/r$.

(a) The slope of the plot at $v = 8.30$ m/s is

$$\left.\frac{dF}{dv}\right|_{v=8.30\text{ m/s}} = \left.\frac{2mv}{r}\right|_{v=8.30\text{ m/s}} = \frac{2(85.0\text{ kg})(8.30\text{ m/s})}{3.50\text{ m}} = 403\text{ N}\cdot\text{s/m}.$$

(b) The period of the circular ride is $T = 2\pi r/v$. Thus,

$$F = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 mr}{T^2},$$

and the variation of F with respect to T while holding r constant is

$$dF = -\frac{8\pi^2 mr}{T^3}dT.$$

The slope of the plot at $T = 2.50$ s is

$$\left.\frac{dF}{dT}\right|_{T=2.50\text{ s}} = -\left.\frac{8\pi^2 mr}{T^3}\right|_{T=2.50\text{ s}} = \frac{8\pi^2(85.0\text{ kg})(3.50\text{ m})}{(2.50\text{ s})^3} = -1.50\times 10^3\text{ N/s}.$$

55. For the puck to remain at rest the magnitude of the tension force T of the cord must equal the gravitational force Mg on the cylinder. The tension force supplies the centripetal force that keeps the puck in its circular orbit, so $T = mv^2/r$. Thus $Mg = mv^2/r$. We solve for the speed:

$$v = \sqrt{\frac{Mgr}{m}} = \sqrt{\frac{(2.50\,\text{kg})(9.80\,\text{m/s}^2)(0.200\ \text{m})}{1.50\,\text{kg}}} = 1.81\ \text{m/s}.$$

56. (a) Using the kinematic equation given in Table 2-1, the deceleration of the car is

$$v^2 = v_0^2 + 2ad \quad \Rightarrow \quad 0 = (35 \text{ m/s})^2 + 2a(107 \text{ m})$$

which gives $a = -5.72 \text{ m/s}^2$. Thus, the force of friction required to stop by car is

$$f = m\,|a| = (1400 \text{ kg})(5.72 \text{ m/s}^2) \approx 8.0 \times 10^3 \text{ N}.$$

(b) The maximum possible static friction is

$$f_{s,\max} = \mu_s mg = (0.50)(1400 \text{ kg})(9.80 \text{ m/s}^2) \approx 6.9 \times 10^3 \text{ N}.$$

(c) If $\mu_k = 0.40$, then $f_k = \mu_k mg$ and the deceleration is $a = -\mu_k g$. Therefore, the speed of the car when it hits the wall is

$$v = \sqrt{v_0^2 + 2ad} = \sqrt{(35 \text{ m/s})^2 - 2(0.40)(9.8 \text{ m/s}^2)(107 \text{ m})} \approx 20 \text{ m/s}.$$

(d) The force required to keep the motion circular is

$$F_r = \frac{mv_0^2}{r} = \frac{(1400 \text{ kg})(35.0 \text{ m/s})^2}{107 \text{ m}} = 1.6 \times 10^4 \text{ N}.$$

(e) Since $F_r > f_{s,\max}$, no circular path is possible.

57. We note that the period T is eight times the time between flashes ($\frac{1}{2000}$ s), so T = 0.0040 s. Combining Eq. 6-18 with Eq. 4-35 leads to

$$F = \frac{4m\pi^2 R}{T^2} = \frac{4(0.030\text{ kg})\pi^2(0.035\text{ m})}{(0.0040\text{ s})^2} = 2.6 \times 10^3\text{ N}.$$

58. We refer the reader to Sample Problem 6-10, and use the result Eq. 6-26:

$$\theta = \tan^{-1}\left(\frac{v^2}{gR}\right)$$

with v = 60(1000/3600) = 17 m/s and R = 200 m. The banking angle is therefore θ = 8.1°. Now we consider a vehicle taking this banked curve at v' = 40(1000/3600) = 11 m/s. Its (horizontal) acceleration is $a' = v'^2/R$, which has components parallel the incline and perpendicular to it:

$$a_{||} = a'\cos\theta = \frac{v'^2\cos\theta}{R}$$
$$a_{\perp} = a'\sin\theta = \frac{v'^2\sin\theta}{R}.$$

These enter Newton's second law as follows (choosing downhill as the $+x$ direction and away-from-incline as $+y$):

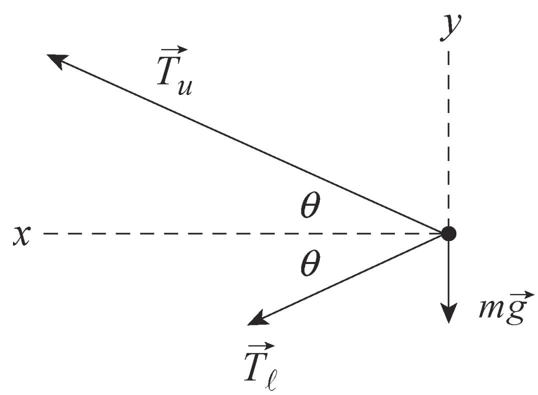
$$mg\sin\theta - f_s = ma_{||}$$
$$F_N - mg\cos\theta = ma_{\perp}$$

and we are led to

$$\frac{f_s}{F_N} = \frac{mg\sin\theta - mv'^2\cos\theta/R}{mg\cos\theta + mv'^2\sin\theta/R}.$$

We cancel the mass and plug in, obtaining f_s/F_N = 0.078. The problem implies we should set $f_s = f_{s,\max}$ so that, by Eq. 6-1, we have μ_s = 0.078.

59. The free-body diagram for the ball is shown below. $\vec{T}_u$ is the tension exerted by the upper string on the ball, $\vec{T}_\ell$ is the tension force of the lower string, and m is the mass of the ball. Note that the tension in the upper string is greater than the tension in the lower string. It must balance the downward pull of gravity and the force of the lower string.



(a) We take the $+x$ direction to be leftward (toward the center of the circular orbit) and $+y$ upward. Since the magnitude of the acceleration is $a = v^2/R$, the x component of Newton's second law is

$$T_u \cos\theta + T_\ell \cos\theta = \frac{mv^2}{R},$$

where v is the speed of the ball and R is the radius of its orbit. The y component is

$$T_u \sin\theta - T_\ell \sin\theta - mg = 0.$$

The second equation gives the tension in the lower string: $T_\ell = T_u - mg/\sin\theta$. Since the triangle is equilateral $\theta = 30.0°$. Thus

$$T_\ell = 35.0\text{ N} - \frac{(1.34\text{ kg})(9.80\text{ m/s}^2)}{\sin 30.0°} = 8.74\text{ N}.$$

(b) The net force has magnitude

$$F_{\text{net,str}} = \left(T_u + T_\ell\right)\cos\theta = (35.0\text{ N} + 8.74\text{ N})\cos 30.0° = 37.9\text{ N}.$$

(c) The radius of the path is

$$R = ((1.70\text{ m})/2)\tan 30.0° = 1.47\text{ m}.$$

Using $F_{\text{net,str}} = mv^2/R$, we find that the speed of the ball is

$$v = \sqrt{\frac{RF_{\text{net,str}}}{m}} = \sqrt{\frac{(1.47\text{ m})(37.9\text{ N})}{1.34\text{ kg}}} = 6.45\text{ m/s}.$$

(d) The direction of $\vec{F}_{\text{net,str}}$ is leftward ("radially inward").

60. (a) We note that R (the horizontal distance from the bob to the axis of rotation) is the circumference of the circular path divided by 2π; therefore, $R = 0.94/2\pi = 0.15$ m. The angle that the cord makes with the horizontal is now easily found:

$$\theta = \cos^{-1}(R/L) = \cos^{-1}(0.15 \text{ m}/0.90 \text{ m}) = 80^\circ.$$

The vertical component of the force of tension in the string is $T\sin\theta$ and must equal the downward pull of gravity (mg). Thus,

$$T = \frac{mg}{\sin\theta} = 0.40 \text{ N}.$$

Note that we are using T for tension (not for the period).

(b) The horizontal component of that tension must supply the centripetal force (Eq. 6-18), so we have $T\cos\theta = mv^2/R$. This gives speed $v = 0.49$ m/s. This divided into the circumference gives the time for one revolution: 0.94/0.49 = 1.9 s.

61. The layer of ice has a mass of

$$m_{\text{ice}} = \left(917 \text{ kg/m}^3\right) \ (400 \text{ m} \times 500 \text{ m} \times 0.0040 \text{ m}) = 7.34 \times 10^5 \text{ kg}.$$

This added to the mass of the hundred stones (at 20 kg each) comes to $m = 7.36 \times 10^5$ kg.

(a) Setting $F = D$ (for Drag force) we use Eq. 6-14 to find the wind speed v along the ground (which actually is relative to the moving stone, but we assume the stone is moving slowly enough that this does not invalidate the result):

$$v = \sqrt{\frac{\mu_k mg}{4C_{\text{ice}}\rho A_{\text{ice}}}} = \sqrt{\frac{(0.10)\left(7.36 \times 10^5 \text{ kg}\right)\left(9.8 \text{ m/s}^2\right)}{4(0.002)\left(1.21 \text{ kg/m}^3\right)\left(400 \times 500 \text{ m}^2\right)}} = 19 \text{ m/s} \ \approx 69 \text{ km/h}.$$

(b) Doubling our previous result, we find the reported speed to be 139 km/h.

(c) The result is reasonable for storm winds. (A category-5 hurricane has speeds on the order of 2.6×10^2 m/s.)

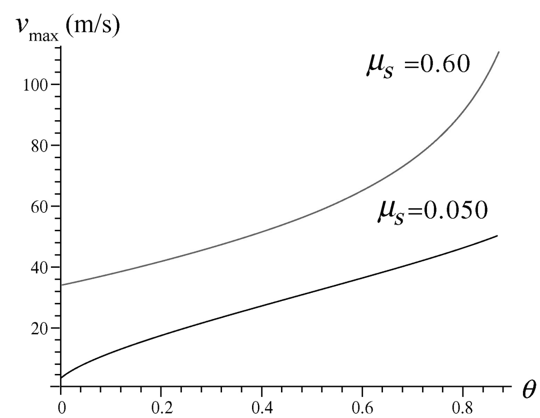
62. (a) To be on the verge of sliding out means that the force of static friction is acting "down the bank" (in the sense explained in the problem statement) with maximum possible magnitude. We first consider the vector sum $\vec{F}$ of the (maximum) static friction force and the normal force. Due to the facts that they are perpendicular and their magnitudes are simply proportional (Eq. 6-1), we find $\vec{F}$ is at angle (measured from the *vertical* axis) $\phi = \theta + \theta_s$, where $\tan\theta_s = \mu_s$ (compare with Eq. 6-13), and θ is the bank angle (as stated in the problem). Now, the vector sum of $\vec{F}$ and the vertically downward pull (mg) of gravity must be equal to the (horizontal) centripetal force (mv^2/R), which leads to a surprisingly simple relationship:

$$\tan\phi = \frac{mv^2/R}{mg} = \frac{v^2}{Rg} \quad .$$

Writing this as an expression for the maximum speed, we have

$$v_{\max} = \sqrt{Rg\tan(\theta + \tan^{-1}\mu_s)} = \sqrt{\frac{Rg(\tan\theta + \mu_s)}{1 - \mu_s\tan\theta}}$$

(b) The graph is shown below (with θ in radians):



(c) Either estimating from the graph (μ_s = 0.60, upper curve) or calculated it more carefully leads to v = 41.3 m/s = 149 km/h when θ = 10º = 0.175 radian.

(d) Similarly (for μ_s = 0.050, the lower curve) we find v = 21.2 m/s = 76.2 km/h when θ = 10º = 0.175 radian.

63. (a) With $\theta = 60°$, we apply Newton's second law to the "downhill" direction:

$$mg\sin\theta - f = ma$$
$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta.$$

Thus,

$$a = g(\sin\theta - \mu_k\cos\theta) = 7.5 \text{ m/s}^2.$$

(b) The direction of the acceleration $\vec{a}$ is down the slope.

(c) Now the friction force is in the "downhill" direction (which is our positive direction) so that we obtain

$$a = g(\sin\theta + \mu_k\cos\theta) = 9.5 \text{ m/s}^2.$$

(d) The direction is down the slope.

64. Note that since no static friction coefficient is mentioned, we assume f_s is not relevant to this computation. We apply Newton's second law to each block's x axis, which for m_1 is positive rightward and for m_2 is positive downhill:

$$\begin{aligned} T - f_k &= m_1 a \\ m_2 g \sin\theta - T &= m_2 a \end{aligned}$$

Adding the equations, we obtain the acceleration:

$$a = \frac{m_2 g \sin\theta - f_k}{m_1 + m_2}$$

For $f_k = \mu_k F_N = \mu_k m_1 g$, we obtain

$$a = \frac{(3.0 \text{ kg})(9.8 \text{ m/s}^2)\sin 30° - (0.25)(2.0 \text{ kg})(9.8 \text{ m/s}^2)}{3.0 \text{ kg} + 2.0 \text{ kg}} = 1.96 \text{ m/s}^2 .$$

Returning this value to either of the above two equations, we find $T = 8.8$ N.

65. (a) Using $F = \mu_s mg$, the coefficient of static friction for the surface between the two blocks is $\mu_s = (12\text{ N})/(39.2\text{ N}) = 0.31$, where $m_t\, g$ = (4.0 kg)(9.8 m/s^2)=39.2 N is the weight of the top block. Let $M = m_t + m_b = 9.0$ kg be the total *system* mass, then the maximum horizontal force has a magnitude $Ma = M\mu_s g$ = 27 N.

(b) The acceleration (in the maximal case) is $a = \mu_s g$ =3.0 m/s^2.

66. With $\theta = 40°$, we apply Newton's second law to the "downhill" direction:

$$mg\sin\theta - f = ma,$$
$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta$$

using Eq. 6-12. Thus,

$$a = 0.75 \text{ m/s}^2 = g(\sin\theta - \mu_k\cos\theta)$$

determines the coefficient of kinetic friction: $\mu_k = 0.74$.

67. (a) To be "on the verge of sliding" means the applied force is equal to the maximum possible force of static friction (Eq. 6-1, with $F_N = mg$ in this case):

$$f_{s,\max} = \mu_s mg = 35.3 \text{ N}.$$

(b) In this case, the applied force $\vec{F}$ indirectly decreases the maximum possible value of friction (since its y component causes a reduction in the normal force) as well as directly opposing the friction force itself (because of its x component). The normal force turns out to be

$$F_N = mg - F\sin\theta$$

where $\theta = 60°$, so that the horizontal equation (the x application of Newton's second law) becomes

$$F\cos\theta - f_{s,\max} = F\cos\theta - \mu_s(mg - F\sin\theta) = 0 \quad \Rightarrow \ F = 39.7 \text{ N}.$$

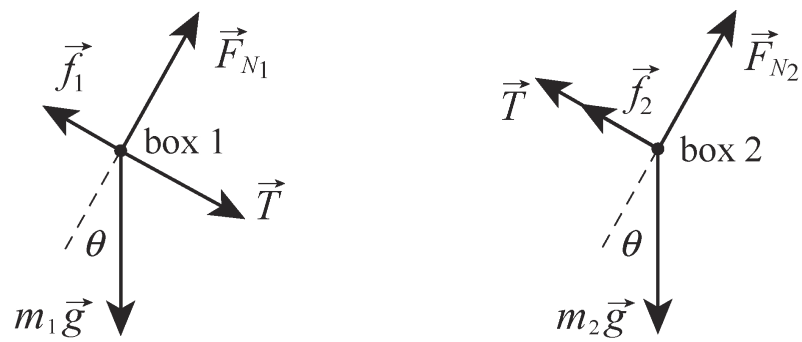
(c) Now, the applied force $\vec{F}$ indirectly increases the maximum possible value of friction (since its y component causes a reduction in the normal force) as well as directly opposing the friction force itself (because of its x component). The normal force in this case turns out to be

$$F_N = mg + F\sin\theta,$$

where $\theta = 60°$, so that the horizontal equation becomes

$$F\cos\theta - f_{s,\max} = F\cos\theta - \mu_s(mg + F\sin\theta) = 0 \quad \Rightarrow \ F = 320 \text{ N}.$$

68. The free-body diagrams for the two boxes are shown below. T is the magnitude of the force in the rod (when $T > 0$ the rod is said to be in tension and when $T < 0$ the rod is under compression), $\vec{F}_{N2}$ is the normal force on box 2 (the uncle box), $\vec{F}_{N1}$ is the the normal force on the aunt box (box 1), $\vec{f}_1$ is kinetic friction force on the aunt box, and $\vec{f}_2$ is kinetic friction force on the uncle box. Also, $m_1 = 1.65$ kg is the mass of the aunt box and $m_2 = 3.30$ kg is the mass of the uncle box (which is a lot of ants!).



For each block we take $+x$ downhill (which is toward the lower-right in these diagrams) and $+y$ in the direction of the normal force. Applying Newton's second law to the x and y directions of first box 2 and next box 1, we arrive at four equations:

$$\begin{aligned} m_2 g \sin\theta - f_2 - T &= m_2\, a \\ F_{N2} - m_2 g \cos\theta &= 0 \\ m_1 g \sin\theta - f_1 + T &= m_1 a \\ F_{N1} - m_1 g \cos\theta &= 0 \end{aligned}$$

which, when combined with Eq. 6-2 ($f_1 = \mu_1 F_{N1}$ where $\mu_1 = 0.226$ and $f_2 = \mu_2 F_{N2}$ where $\mu_2 = 0.113$), fully describe the dynamics of the system.

(a) We solve the above equations for the tension and obtain

$$T = \left(\frac{m_2 m_1 g}{m_2 + m_1}\right)(\mu_1 - \mu_2)\cos\theta = 1.05 \text{ N}.$$

(b) These equations lead to an acceleration equal to

$$a = g\left(\sin\theta - \left(\frac{\mu_2 m_2 + \mu_1 m_1}{m_2 + m_1}\right)\cos\theta\right) = 3.62 \text{ m/s}^2.$$

(c) Reversing the blocks is equivalent to switching the labels. We see from our algebraic result in part (a) that this gives a negative value for T (equal in magnitude to the result we got before). Thus, the situation is as it was before except that the rod is now in a state of compression.

69. Each side of the trough exerts a normal force on the crate. The first diagram shows the view looking in toward a cross section. The net force is along the dashed line. Since each of the normal forces makes an angle of 45° with the dashed line, the magnitude of the resultant normal force is given by

$$F_{Nr} = 2F_N \cos 45° = \sqrt{2}F_N .$$

The second diagram is the free-body diagram for the crate (from a "side" view, similar to that shown in the first picture in Fig. 6-53). The force of gravity has magnitude mg, where m is the mass of the crate, and the magnitude of the force of friction is denoted by f. We take the $+x$ direction to be down the incline and $+y$ to be in the direction of $\vec{F}_{Nr}$. Then the x and the y components of Newton's second law are

$$\begin{aligned} x&: \quad mg \sin \theta - f = ma \\ y&: \quad F_{Nr} - mg \cos \theta = 0. \end{aligned}$$

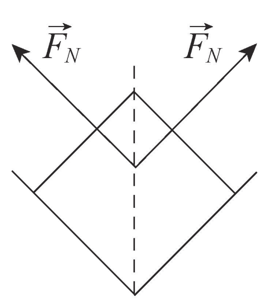
Since the crate is moving, each side of the trough exerts a force of kinetic friction, so the total frictional force has magnitude

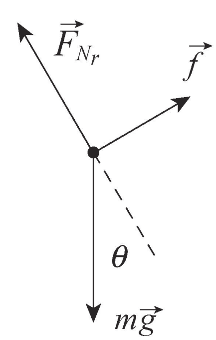
$$f = 2\mu_k F_N = 2\mu_k F_{Nr} / \sqrt{2} = \sqrt{2}\mu_k F_{Nr}$$

Combining this expression with $F_{Nr} = mg \cos \theta$ and substituting into the x component equation, we obtain

$$mg \sin\theta - \sqrt{2}mg \cos\theta = ma .$$

Therefore $a = g(\sin\theta - \sqrt{2}\mu_k \cos\theta)$.





70. (a) The coefficient of static friction is $\mu_s = \tan(\theta_{\text{slip}}) = 0.577 \approx 0.58$.

(b) Using

$$mg\sin\theta - f = ma$$

$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta$$

and $a = 2d/t^2$ (with $d = 2.5$ m and $t = 4.0$ s), we obtain $\mu_k = 0.54$.

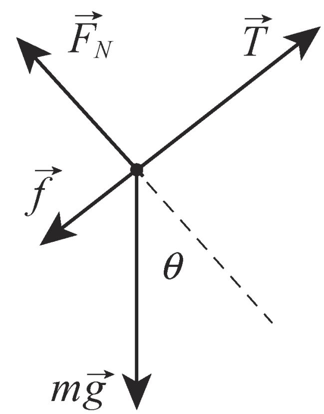
71. We may treat all 25 cars as a single object of mass $m = 25 \times 5.0 \times 10^4$ kg and (when the speed is 30 km/h = 8.3 m/s) subject to a friction force equal to

$$f = 25 \times 250 \times 8.3 = 5.2 \times 10^4 \text{ N}.$$

(a) Along the level track, this object experiences a "forward" force T exerted by the locomotive, so that Newton's second law leads to

$$T - f = ma \quad \Rightarrow \quad T = 5.2\times10^4 + (1.25\times10^6)(0.20) = 3.0\times10^5 \text{ N}.$$

(b) The free-body diagram is shown next, with θ as the angle of the incline. The $+x$ direction (which is the only direction to which we will be applying Newton's second law) is uphill (to the upper right in our sketch).



Thus, we obtain

$$T - f - mg\sin\theta = ma$$

where we set $a = 0$ (implied by the problem statement) and solve for the angle. We obtain $\theta = 1.2°$.

72. An excellent discussion and equation development related to this problem is given in Sample Problem 6-2. Using the result, we obtain

$$\theta = \tan^{-1} \mu_s = \tan^{-1} 0.50 = 27°$$

which implies that the angle through which the slope should be *reduced* is

$$\phi = 45° - 27° \approx 20°.$$

73. We make use of Eq. 6-16 which yields

$$\sqrt{\frac{2mg}{C\rho\pi R^2}} = \sqrt{\frac{2(6)(9.8)}{(1.6)(1.2)\pi(0.03)^2}} = 147 \text{ m/s}.$$

74. (a) The upward force exerted by the car on the passenger is equal to the downward force of gravity ($W = 500$ N) on the passenger. So the *net* force does not have a vertical contribution; it only has the contribution from the horizontal force (which is necessary for maintaining the circular motion). Thus $\left|\vec{F}_{\text{net}}\right| = F = 210\ \text{N}$.

(b) Using Eq. 6-18, we have

$$v = \sqrt{\frac{FR}{m}} = \sqrt{\frac{(210\ \text{N})(470\ \text{m})}{51.0\ \text{kg}}} = 44.0\ \text{m/s}.$$

75. (a) We note that $F_N = mg$ in this situation, so

$$f_{s,\max} = \mu_s mg = (0.52)(11 \text{ kg})(9.8 \text{ m/s}^2) = 56 \text{ N}.$$

Consequently, the horizontal force $\vec{F}$ needed to initiate motion must be (at minimum) slightly more than 56 N.

(b) Analyzing vertical forces when $\vec{F}$ is at nonzero θ yields

$$F\sin\,\theta + F_N = mg \;\Rightarrow\; f_{s,\max} = \mu_s(mg - F\ \sin\,\theta).$$

Now, the horizontal component of $\vec{F}$ needed to initiate motion must be (at minimum) slightly more than this, so

$$F\cos\theta = \mu_s(mg - F\sin\,\theta) \;\Rightarrow\; F = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\,\theta}$$

which yields $F = 59$ N when $\theta = 60°$.

(c) We now set $\theta = -60°$ and obtain

$$F = \frac{(0.52)(11\text{ kg})(9.8\text{ m/s}^2)}{\cos(-60°) + (0.52)\sin(-60°)} = 1.1\times10^3\ \text{N}.$$

76. We use Eq. 6-14, $D = \frac{1}{2} C\rho A v^2$, where ρ is the air density, A is the cross-sectional area of the missile, v is the speed of the missile, and C is the drag coefficient. The area is given by $A = \pi R^2$, where $R = 0.265$ m is the radius of the missile. Thus

$$D = \frac{1}{2}(0.75)\left(1.2 \text{ kg/m}^3\right)\pi\left(0.265 \text{ m}\right)^2\left(250 \text{ m/s}\right)^2 = 6.2 \times 10^3 \text{ N}.$$

77. The magnitude of the acceleration of the cyclist as it moves along the horizontal circular path is given by v^2/R, where v is the speed of the cyclist and R is the radius of the curve.

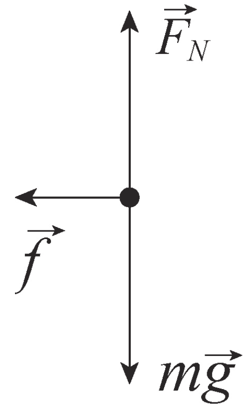
(a) The horizontal component of Newton's second law is $f = mv^2/R$, where f is the static friction exerted horizontally by the ground on the tires. Thus,

$$f = \frac{(85.0\text{ kg})(9.00\text{ m/s})^2}{25.0\text{ m}} = 275\ \text{N}.$$

(b) If F_N is the vertical force of the ground on the bicycle and m is the mass of the bicycle and rider, the vertical component of Newton's second law leads to $F_N = mg = 833$ N. The magnitude of the force exerted by the ground on the bicycle is therefore

$$\sqrt{f^2 + F_N^2} = \sqrt{(275\text{ N})^2 + (833\text{ N})^2} = 877\text{ N}.$$

78. The free-body diagram for the puck is shown below. $\vec{F}_N$ is the normal force of the ice on the puck, $\vec{f}$ is the force of friction (in the $-x$ direction), and $m\vec{g}$ is the force of gravity.



(a) The horizontal component of Newton's second law gives $-f = ma$, and constant acceleration kinematics (Table 2-1) can be used to find the acceleration.

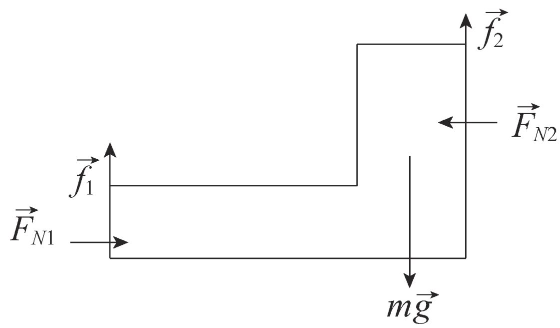
Since the final velocity is zero, $v^2 = v_0^2 + 2ax$ leads to $a = -v_0^2/2x$. This is substituted into the Newton's law equation to obtain

$$f = \frac{mv_0^2}{2x} = \frac{(0.110 \text{ kg})(6.0 \text{ m/s})^2}{2(15 \text{ m})} = 0.13 \text{ N}.$$

(b) The vertical component of Newton's second law gives $F_N - mg = 0$, so $F_N = mg$ which implies (using Eq. 6-2) $f = \mu_k\, mg$. We solve for the coefficient:

$$\mu_k = \frac{f}{mg} = \frac{0.13 \text{ N}}{(0.110 \text{ kg})(9.8 \text{ m/s}^2)} = 0.12 .$$

79. (a) The free-body diagram for the person (shown as an L-shaped block) is shown below. The force that she exerts on the rock slabs is not directly shown (since the diagram should only show forces exerted on her), but it is related by Newton's third law) to the normal forces $\vec{F}_{N1}$ and $\vec{F}_{N2}$ exerted horizontally by the slabs onto her shoes and back, respectively. We will show in part (b) that $F_{N1} = F_{N2}$ so that we there is no ambiguity in saying that the magnitude of her push is F_{N2}. The total upward force due to (maximum) static friction is $\vec{f} = \vec{f}_1 + \vec{f}_2$ where $f_1 = \mu_{s1}F_{N1}$ and $f_2 = \mu_{s2}F_{N2}$. The problem gives the values $\mu_{s1} = 1.2$ and $\mu_{s2} = 0.8$.



(b) We apply Newton's second law to the x and y axes (with $+x$ rightward and $+y$ upward and there is no acceleration in either direction).

$$F_{N1} - F_{N2} = 0$$
$$f_1 + f_2 - mg = 0$$

The first equation tells us that the normal forces are equal $F_{N1} = F_{N2} = F_N$. Consequently, from Eq. 6-1,

$$f_1 = \mu_{s\,1}F_N$$
$$f_2 = \mu_{s\,2}F_N$$

we conclude that

$$f_1 = \left(\frac{\mu_{s\,1}}{\mu_{s\,2}}\right) f_2 \; .$$

Therefore, $f_1 + f_2 - mg = 0$ leads to

$$\left(\frac{\mu_{s\,1}}{\mu_{s\,2}} + 1\right) f_2 = mg$$

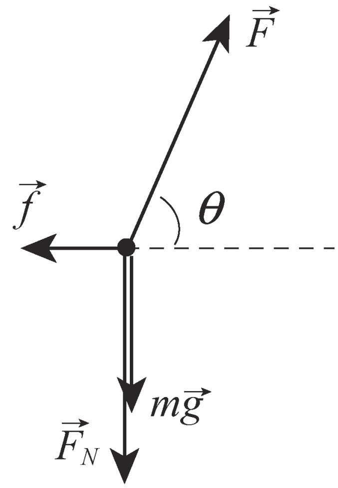
which (with m = 49 kg) yields f_2 = 192 N. From this we find $F_N = f_2 / \mu_{s2} = 240$ N. This is equal to the magnitude of the push exerted by the rock climber.

(c) From the above calculation, we find $f_1 = \mu_{s1}F_N = 288$ N which amounts to a fraction

$$\frac{f_1}{W} = \frac{288}{(49)\,(9.8)} = 0.60$$

or 60% of her weight.

80. The free-body diagram for the stone is shown on the right, with $\vec{F}$ being the force applied to the stone, $\vec{F}_N$ the *downward* normal force of the ceiling on the stone, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction. We take the +*x* direction to be horizontal to the right and the +*y* direction to be up. The equations for the *x* and the *y* components of the force according to Newton's second law are:



$$F_x = F\cos\theta - f = ma$$
$$F_y = F\sin\theta - F_N - mg = 0$$

Now $f = \mu_k F_N$, and the second equation gives $F_N = F\sin\theta - mg$, which yields $f = \mu_k(F\sin\theta - mg)$. This expression is substituted for *f* in the first equation to obtain

$$F\cos\theta - \mu_k(F\sin\theta - mg) = ma.$$

For $a = 0$, the force is

$$F = \frac{-\mu_k mg}{\cos\theta - \mu_k\sin\theta}.$$

With $\mu_k = 0.65$, $m = 5.0$ kg, and $\theta = 70^\circ$, we obtain $F = 118$ N.

81. (a) If we choose "downhill" positive, then Newton's law gives

$$m_A g \sin\theta - f_A - T = m_A a$$

for block A (where $\theta = 30°$). For block B we choose leftward as the positive direction and write $T - f_B = m_B a$. Now

$$f_A = \mu_{k,\text{incline}} F_{NA} = \mu' m_A g \cos\theta$$

using Eq. 6-12 applies to block A, and

$$f_B = \mu_k F_{NB} = \mu_k m_B g.$$

In this particular problem, we are asked to set $\mu' = 0$, and the resulting equations can be straightforwardly solved for the tension: $T = 13$ N.

(b) Similarly, finding the value of a is straightforward:

$$a = g(m_A \sin\theta - \mu_k m_B)/(m_A + m_B) = 1.6 \text{ m/s}^2.$$

82. (a) If the skier covers a distance L during time t with zero initial speed and a constant acceleration a, then $L = at^2/2$, which gives the acceleration a_1 for the first (old) pair of skis:

$$a_1 = \frac{2L}{t_1^2} = \frac{2(200\,\text{m})}{(61\text{ s})^2} = 0.11\ \text{m/s}^2 .$$

(b) The acceleration a_2 for the second (new) pair is

$$a_2 = \frac{2L}{t_2^2} = \frac{2(200\,\text{m})}{(42\,\text{s})^2} = 0.23\ \text{m/s}^2\ .$$

(c) The net force along the slope acting on the skier of mass m is

$$F_{\text{net}} = mg\sin\theta - f_k = mg(\sin\theta - \mu_k\cos\theta) = ma$$

which we solve for μ_{k1} for the first pair of skis:

$$\mu_{k1} = \tan\theta - \frac{a_1}{g\cos\theta} = \tan 3.0° - \frac{0.11\,\text{m/s}^2}{(9.8\ \text{m/s}^2)\cos 3.0°} = 0.041$$

(d) For the second pair, we have

$$\mu_{k2} = \tan\theta - \frac{a_2}{g\cos\theta} = \tan 3.0° - \frac{0.23\ \text{m/s}^2}{(9.8\ \text{m/s}^2)\cos 3.0°} = 0.029\ .$$

83. If we choose "downhill" positive, then Newton's law gives

$$m\,g \sin\theta - f_k = m\,a$$

for the sliding child. Now using Eq. 6-12

$$f_k = \mu_k F_N = \mu_k m\,g,$$

so we obtain $a = g(\sin\theta - \mu_k \cos\theta) = -\,0.5$ m/s^2 (note that the problem gives the direction of the acceleration vector as uphill, even though the child is sliding downhill, so it is a deceleration). With $\theta = 35°$, we solve for the coefficient and find $\mu_k = 0.76$.

84. At the top of the hill the vertical forces on the car are the upward normal force exerted by the ground and the downward pull of gravity. Designating $+y$ downward, we have

$$mg - F_N = \frac{mv^2}{R}$$

from Newton's second law. To find the greatest speed without leaving the hill, we set $F_N = 0$ and solve for v:

$$v = \sqrt{gR} = \sqrt{(9.8\ \text{m/s}^2)(250\ \text{m})} = 49.5\ \text{m/s} = 49.5(3600/1000)\ \text{km/h} = 178\ \text{km/h}.$$

85. The mass of the car is m = (10700/9.80) kg = 1.09×10^3 kg. We choose "inward" (horizontally towards the center of the circular path) as the positive direction.

(a) With v = 13.4 m/s and R = 61 m, Newton's second law (using Eq. 6-18) leads to

$$f_s = \frac{mv^2}{R} = 3.21 \times 10^3 \text{ N} .$$

(b) Noting that $F_N = mg$ in this situation, the maximum possible static friction is found to be

$$f_{s,\max} = \mu_s mg = (0.35)(10700 \text{ N}) = 3.75 \times 10^3 \text{ N}$$

using Eq. 6-1. We see that the static friction found in part (a) is less than this, so the car rolls (no skidding) and successfully negotiates the curve.

86. (a) Our $+x$ direction is horizontal and is chosen (as we also do with $+y$) so that the components of the 100 N force $\vec{F}$ are non-negative. Thus, $F_x = F \cos \theta = 100$ N, which the textbook denotes F_h in this problem.

(b) Since there is no vertical acceleration, application of Newton's second law in the y direction gives

$$F_N + F_y = mg \Rightarrow F_N = mg - F \sin\theta$$

where $m = 25.0$ kg. This yields $F_N = 245$ N in this case ($\theta = 0°$).

(c) Now, $F_x = F_h = F \cos \theta = 86.6$ N for $\theta = 30.0°$.

(d) And $F_N = mg - F \sin \theta = 195$ N.

(e) We find $F_x = F_h = F \cos \theta = 50.0$ N for $\theta = 60.0°$.

(f) And $F_N = mg - F \sin \theta = 158$ N.

(g) The condition for the chair to slide is

$$F_x > f_{s,\max} = \mu_s F_N \quad \text{where} \quad \mu_s = 0.42.$$

For $\theta = 0°$, we have

$$F_x = 100 \text{ N} < f_{s,\max} = (0.42)(245 \text{ N}) = 103 \text{ N}$$

so the crate remains at rest.

(h) For $\theta = 30.0°$, we find

$$F_x = 86.6 \text{ N} > f_{s,\max} = (0.42)(195 \text{ N}) = 81.9 \text{ N}$$

so the crate slides.

(i) For $\theta = 60°$, we get

$$F_x = 50.0 \text{ N} < f_{s,\max} = (0.42)(158 \text{ N}) = 66.4 \text{ N}$$

which means the crate must remain at rest.

87. For simplicity, we denote the 70° angle as θ and the magnitude of the push (80 N) as P. The vertical forces on the block are the downward normal force exerted on it by the ceiling, the downward pull of gravity (of magnitude mg) and the vertical component of $\vec{P}$ (which is upward with magnitude $P \sin \theta$). Since there is no acceleration in the vertical direction, we must have

$$F_N = P\sin\theta - mg$$

in which case the leftward-pointed kinetic friction has magnitude

$$f_k = \mu_k (P\sin\theta - mg).$$

Choosing $+x$ rightward, Newton's second law leads to

$$P\cos\ \theta - f_k = ma \ \Rightarrow\ a = \frac{P\cos\theta - u_k (P\sin\theta - mg)}{m}$$

which yields $a = 3.4\ \text{m/s}^2$ when $\mu_k = 0.40$ and $m = 5.0$ kg.

88. (a) The intuitive conclusion, that the tension is greatest at the bottom of the swing, is certainly supported by application of Newton's second law there:

$$T - mg = \frac{mv^2}{R} \Rightarrow T = m\left(g + \frac{v^2}{R}\right)$$

where Eq. 6-18 has been used. Increasing the speed eventually leads to the tension at the bottom of the circle reaching that breaking value of 40 N.

(b) Solving the above equation for the speed, we find

$$v = \sqrt{R\left(\frac{T}{m} - g\right)} = \sqrt{(0.91\text{ m})\left(\frac{40\text{ N}}{0.37\text{ kg}} - 9.8\text{ m/s}^2\right)}$$

which yields $v = 9.5$ m/s.

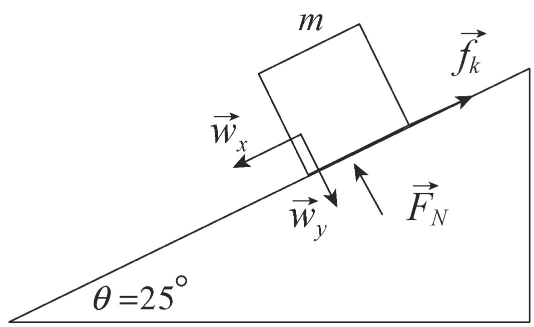
89. (a) The push (to get it moving) must be at least as big as $f_{s,\max} = \mu_s F_N$ (Eq. 6-1, with $F_N = mg$ in this case), which equals (0.51)(165 N) = 84.2 N.

(b) While in motion, constant velocity (zero acceleration) is maintained if the push is equal to the kinetic friction force $f_k = \mu_k F_N = \mu_k mg = 52.8$ N.

(c) We note that the mass of the crate is 165/9.8 = 16.8 kg. The acceleration, using the push from part (a), is

$$a = (84.2 \text{ N} - 52.8 \text{ N})/(16.8 \text{ kg}) \approx 1.87 \text{ m/s}^2.$$

90. In the figure below, $m = 140/9.8 = 14.3$ kg is the mass of the child. We use $\vec{w}_x$ and $\vec{w}_y$ as the components of the gravitational pull of Earth on the block; their magnitudes are $w_x = mg \sin\theta$ and $w_y = mg \cos\theta$.



(a) With the x axis directed up along the incline (so that $a = -0.86$ m/s^2), Newton's second law leads to

$$f_k - 140 \sin 25° = m(-0.86)$$

which yields $f_k = 47$ N. We also apply Newton's second law to the y axis (perpendicular to the incline surface), where the acceleration-component is zero:

$$F_N - 140 \cos 25° = 0 \quad \Rightarrow \quad F_N = 127 \text{ N.}$$

Therefore, $\mu_k = f_k/F_N = 0.37$.

(b) Returning to our first equation in part (a), we see that if the downhill component of the weight force were insufficient to overcome static friction, the child would not slide at all. Therefore, we require $140 \sin 25° > f_{s,\max} = \mu_s F_N$, which leads to $\tan 25° = 0.47 > \mu_s$. The minimum value of μ_s equals μ_k and is more subtle; reference to §6-1 is recommended. If μ_k exceeded μ_s then when static friction were overcome (as the incline is raised) then it should start to move – which is impossible if f_k is large enough to cause deceleration! The bounds on μ_s are therefore given by $0.47 > \mu_s > 0.37$.

91. We apply Newton's second law (as $F_{\text{push}} - f = ma$). If we find $F_{\text{push}} < f_{\max}$, we conclude "no, the cabinet does not move" (which means a is actually 0 and $f = F_{\text{push}}$), and if we obtain $a > 0$ then it is moves (so $f = f_k$). For $f_{\max}$ and f_k we use Eq. 6-1 and Eq. 6-2 (respectively), and in those formulas we set the magnitude of the normal force equal to 556 N. Thus, $f_{\max} = 378$ N and $f_k = 311$ N.

(a) Here we find $F_{\text{push}} < f_{\max}$ which leads to $f = F_{\text{push}} = 222$ N.

(b) Again we find $F_{\text{push}} < f_{\max}$ which leads to $f = F_{\text{push}} = 334$ N.

(c) Now we have $F_{\text{push}} > f_{\max}$ which means it moves and $f = f_k = 311$ N.

(d) Again we have $F_{\text{push}} > f_{\max}$ which means it moves and $f = f_k = 311$ N.

(e) The cabinet moves in (c) and (d).

92. (a) The tension will be the greatest at the lowest point of the swing. Note that there is no substantive difference between the tension T in this problem and the normal force F_N in Sample Problem 6-7. Eq. 6-19 of that Sample Problem examines the situation at the top of the circular path (where F_N is the least), and rewriting that for the bottom of the path leads to

$$T = mg + mv^2/r$$

where F_N is at its greatest value.

(b) At the breaking point $T = 33$ N $= m(g + v^2/r)$ where $m = 0.26$ kg and $r = 0.65$ m. Solving for the speed, we find that the cord should break when the speed (at the lowest point) reaches 8.73 m/s.

93. (a) The component of the weight along the incline (with downhill understood as the positive direction) is $mg\sin\theta$ where $m = 630$ kg and $\theta = 10.2°$. With $f = 62.0$ N, Newton's second law leads to

$$mg\sin\theta - f = ma$$

which yields $a = 1.64$ m/s^2. Using Eq. 2-15, we have

$$80.0 \text{ m} = \left(6.20 \frac{\text{m}}{\text{s}}\right) t + \frac{1}{2}\left(1.64 \frac{\text{m}}{\text{s}^2}\right) t^2 .$$

This is solved using the quadratic formula. The positive root is $t = 6.80$ s.

(b) Running through the calculation of part (a) with $f = 42.0$ N instead of $f = 62$ N results in $t = 6.76$ s.

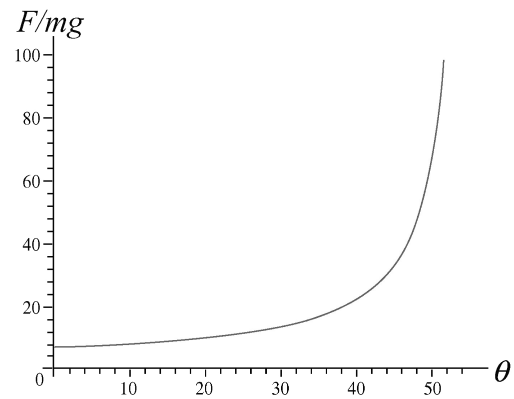
94. (a) The x component of $\vec{F}$ tries to move the crate while its y component indirectly contributes to the inhibiting effects of friction (by increasing the normal force). Newton's second law implies

$$x \text{ direction: } F\cos\theta - f_s = 0$$

$$y \text{ direction: } F_N - F\sin\theta - mg = 0.$$

To be "on the verge of sliding" means $f_s = f_{s,\max} = \mu_s F_N$ (Eq. 6-1). Solving these equations for F (actually, for the ratio of F to mg) yields

$$\frac{F}{mg} = \frac{\mu_s}{\cos\theta - \mu_s \sin\theta} .$$



This is plotted on the right (θ in degrees).

(b) The denominator of our expression (for F/mg) vanishes when

$$\cos\theta - \mu_s \sin\theta = 0 \quad \Rightarrow \quad \theta_{\text{inf}} = \tan^{-1}\left(\frac{1}{\mu_s}\right)$$

For $\mu_s = 0.70$, we obtain $\theta_{\text{inf}} = \tan^{-1}\left(\frac{1}{\mu_s}\right) = 55°$.

(c) Reducing the coefficient means increasing the angle by the condition in part (b).

(d) For $\mu_s = 0.60$ we have $\theta_{\text{inf}} = \tan^{-1}\left(\frac{1}{\mu_s}\right) = 59°$.

95. The car is in “danger of sliding” down when

$$\mu_s = \tan\theta = \tan 35.0° = 0.700.$$

This value represents a 3.4% decrease from the given 0.725 value.

96. For the m_2 = 1.0 kg block, application of Newton's laws result in

$$F\cos\theta - T - f_k = m_2 a \qquad x \text{ axis}$$
$$F_N - F\sin\theta - m_2 g = 0 \qquad y \text{ axis}$$

Since $f_k = \mu_k F_N$, these equations can be combined into an equation to solve for a:

$$F(\cos\theta - \mu_k \sin\theta) - T - \mu_k m_2 g = m_2 a$$

Similarly (but without the applied push) we analyze the m_1= 2.0 kg block:

$$T - f_k' = m_1 a \qquad x \text{ axis}$$
$$F_N' - m_1 g = 0 \qquad y \text{ axis}$$

Using $f_k = \mu_k \ F_N'$, the equations can be combined:

$$T - \mu_k m_1 g = m_1 a$$

Subtracting the two equations for a and solving for the tension, we obtain

$$T = \frac{m_1(\cos\theta - \mu_k \sin\theta)}{m_1 + m_2} F = \frac{(2.0\text{ kg})[\cos 35° - (0.20)\sin 35°]}{2.0\text{ kg} + 1.0\text{ kg}}(20\text{ N}) = 9.4\text{ N}.$$

97. (a) The x component of $\vec{F}$ contributes to the motion of the crate while its y component indirectly contributes to the inhibiting effects of friction (by increasing the normal force). Along the y direction, we have $F_N - F\cos\theta - mg = 0$ and along the x direction we have $F\sin\theta - f_k = 0$ (since it is not accelerating, according to the problem). Also, Eq. 6-2 gives $f_k = \mu_k F_N$. Solving these equations for F yields

$$F = \frac{\mu_k mg}{\sin\theta - \mu_k \cos\theta} .$$

(b) When $\theta < \theta_0 = \tan^{-1} \mu_s$, F will not be able to move the mop head.

98. Consider that the car is "on the verge of sliding out" – meaning that the force of static friction is acting "down the bank" (or "downhill" from the point of view of an ant on the banked curve) with maximum possible magnitude. We first consider the vector sum $\vec{F}$ of the (maximum) static friction force and the normal force. Due to the facts that they are perpendicular and their magnitudes are simply proportional (Eq. 6-1), we find $\vec{F}$ is at angle (measured from the vertical axis) $\phi = \theta + \theta_s$ where $\tan\theta_s = \mu_s$ (compare with Eq. 6-13), and θ is the bank angle. Now, the vector sum of $\vec{F}$ and the vertically downward pull (mg) of gravity must be equal to the (horizontal) centripetal force (mv^2/R), which leads to a surprisingly simple relationship:

$$\tan\phi = \frac{mv^2/R}{mg} = \frac{v^2}{Rg}\,.$$

Writing this as an expression for the maximum speed, we have

$$v_{\max} = \sqrt{Rg\tan(\theta + \tan^{-1}\mu_s)} = \sqrt{\frac{Rg(\tan\theta + \mu_s)}{1 - \mu_s\tan\theta}}\,.$$

(a) We note that the given speed is (in SI units) roughly 17 m/s. If we do not want the cars to "depend" on the static friction to keep from sliding out (that is, if we want the component "down the back" of gravity to be sufficient), then we can set $\mu_s = 0$ in the above expression and obtain $v = \sqrt{Rg\tan\theta}$. With $R = 150$ m, this leads to $\theta = 11°$.

(b) If, however, the curve is not banked (so $\theta = 0$) then the above expression becomes

$$v = \sqrt{Rg\tan(\tan^{-1}\mu_s)} = \sqrt{Rg\mu_s}$$

Solving this for the coefficient of static friction $\mu_s = 0.19$.

99. Replace f_s with f_k in Fig. 6-5(b) to produce the appropriate force diagram for the first part of this problem (when it is sliding downhill with zero acceleration). This amounts to replacing the static coefficient with the kinetic coefficient in Eq. 6-13: $\mu_k = \tan\theta$. Now (for the second part of the problem, with the block projected uphill) the friction direction is reversed from what is shown in Fig. 6-5(b). Newton's second law for the uphill motion (and Eq. 6-12) leads to

$$-mg\sin\theta - \mu_k mg\cos\theta = ma.$$

Canceling the mass and substituting what we found earlier for the coefficient, we have

$$-g\sin\theta - \tan\theta\, g\cos\theta = a\ .$$

This simplifies to $-2g\sin\theta = a$. Eq. 2-16 then gives the distance to stop: $\Delta x = -v_o^2/2a$.

(a) Thus, the distance up the incline traveled by the block is $\Delta x = v_o^2/(4g\sin\theta)$.

(b) We usually expect $\mu_s > \mu_k$ (see the discussion in section 6-1). Sample Problem 6-2 treats the "angle of repose" (the minimum angle necessary for a stationary block to start sliding downhill): $\mu_s = \tan(\theta_{\text{repose}})$. Therefore, we expect $\theta_{\text{repose}} > \theta$ found in part (a). Consequently, when the block comes to rest, the incline is not steep enough to cause it to start slipping down the incline again.

100. Analysis of forces in the horizontal direction (where there can be no acceleration) leads to the conclusion that $F = F_N$; the magnitude of the normal force is 60 N. The maximum possible static friction force is therefore $\mu_s F_N = 33$ N, and the kinetic friction force (when applicable) is $\mu_k F_N = 23$ N.

(a) In this case, $\vec{P}$ = 34 N upward. Assuming $\vec{f}$ points down, then Newton's second law for the y leads to

$$P - mg - f = ma \ .$$

if we assume $f = f_s$ and $a = 0$, we obtain $f = (34 - 22)$ N = 12 N. This is less than $f_{s,\,\max}$, which shows the consistency of our assumption. The answer is: $\vec{f_s}$ = 12 N down.

(b) In this case, $\vec{P}$ = 12 N upward. The above equation, with the same assumptions as in part (a), leads to $f = (12 - 22)$ N = –10 N. Thus, $|f_s| < f_{s,\,\max}$, justifying our assumption that the block is stationary, but its negative value tells us that our initial assumption about the direction of $\vec{f}$ is incorrect in this case. Thus, the answer is: $\vec{f_s}$ = 10 N up.

(c) In this case, $\vec{P}$ = 48 N upward. The above equation, with the same assumptions as in part (a), leads to $f = (48 - 22)$ N = 26 N. Thus, we again have $f_s < f_{s,\,\max}$, and our answer is: $\vec{f_s}$ = 26 N down.

(d) In this case, $\vec{P}$ = 62 N upward. The above equation, with the same assumptions as in part (a), leads to $f = (62 - 22)$ N = 40 N, which is larger than $f_{s,\,\max}$, -- invalidating our assumptions. Therefore, we take $f = f_k$ and $a \neq 0$ in the above equation; if we wished to find the value of a we would find it to be positive, as we should expect. The answer is: $\vec{f_k}$ = 23 N down.

(e) In this case, $\vec{P}$ = 10 N downward. The above equation (but with P replaced with $-P$) with the same assumptions as in part (a), leads to $f = (-10 - 22)$ N = –32 N. Thus, we have $|f_s| < f_{s,\,\max}$, justifying our assumption that the block is stationary, but its negative value tells us that our initial assumption about the direction of $\vec{f}$ is incorrect in this case. Thus, the answer is: $\vec{f_s}$ = 32 N up.

(f) In this case, $\vec{P}$ = 18 N downward. The above equation (but with P replaced with $-P$) with the same assumptions as in part (a), leads to $f = (-18 - 22)$ N = –40 N, which is larger (in absolute value) than $f_{s,\,\max}$, -- invalidating our assumptions. Therefore, we take $f = f_k$ and $a \neq 0$ in the above equation; if we wished to find the value of a we would find it to be negative, as we should expect. The answer is: $\vec{f_k}$ = 23 N up.

(g) The block moves up the wall in case (d) where $a > 0$.

(h) The block moves down the wall in case (f) where $a < 0$.

(i) The frictional force $\vec{f_s}$ is directed down in cases (a), (c) and (d).

101. (a) The distance traveled by the coin in 3.14 s is $3(2\pi r) = 6\pi(0.050) = 0.94$ m. Thus, its speed is $v = 0.94/3.14 = 0.30$ m/s.

(b) The centripetal acceleration is given by Eq. 6-17:

$$a = \frac{v^2}{r} = \frac{(0.30 \text{ m/s})^2}{0.050 \text{ m}} = 1.8 \text{ m/s}^2 .$$

(c) The acceleration vector (at any instant) is horizontal and points from the coin towards the center of the turntable.

(d) The only horizontal force acting on the coin is static friction f_s and must be large enough to supply the acceleration of part (b) for the $m = 0.0020$ kg coin. Using Newton's second law,

$$f_s = ma = (0.0020 \text{ kg})(1.8 \text{ m/s}^2) = 3.6 \times 10^{-3} \text{ N} .$$

(e) The static friction f_s must point in the same direction as the acceleration (towards the center of the turntable).

(f) We note that the normal force exerted upward on the coin by the turntable must equal the coin's weight (since there is no vertical acceleration in the problem). We also note that if we repeat the computations in parts (a) and (b) for $r' = 0.10$ m, then we obtain $v' = 0.60$ m/s and $a' = 3.6$ m/s^2. Now, if friction is at its maximum at $r = r'$, then, by Eq. 6-1, we obtain

$$\mu_s = \frac{f_{s,\max}}{mg} = \frac{ma'}{mg} = 0.37 .$$

102. (a) The distance traveled in one revolution is $2\pi R = 2\pi(4.6 \text{ m}) = 29$ m. The (constant) speed is consequently $v = (29 \text{ m})/(30 \text{ s}) = 0.96$ m/s.

(b) Newton's second law (using Eq. 6-17 for the magnitude of the acceleration) leads to

$$f_s = m\left(\frac{v^2}{R}\right) = m(0.20)$$

in SI units. Noting that $F_N = mg$ in this situation, the maximum possible static friction is $f_{s,\text{max}} = \mu_s mg$ using Eq. 6-1. Equating this with $f_s = m(0.20)$ we find the mass m cancels and we obtain $\mu_s = 0.20/9.8 = 0.021$.

103. (a) The box doesn't move until $t = 2.8$ s, which is when the applied force $\vec{F}$ reaches a magnitude of $F = (1.8)(2.8) = 5.0$ N, implying therefore that $f_{s,\,\max} = 5.0$ N. Analysis of the vertical forces on the block leads to the observation that the normal force magnitude equals the weight $F_N = mg = 15$ N. Thus, $\mu_s = f_{s,\,\max}/F_N = 0.34$.

(b) We apply Newton's second law to the horizontal x axis (positive in the direction of motion):

$$F - f_k = ma \;\Rightarrow\; 1.8t - f_k = (1.5)(1.2t - 2.4)$$

Thus, we find $f_k = 3.6$ N. Therefore, $\mu_k = f_k / F_N = 0.24$.

104. We note that $F_N = mg$ in this situation, so $f_k = \mu_k mg = (0.32)\ (220\ \text{N}) = 70.4\ \text{N}$ and $f_{s,\max} = \mu_s mg = (0.41)\ (220\ \text{N}) = 90.2\ \text{N}$.

(a) The person needs to push at least as hard as the static friction maximum if he hopes to start it moving. Denoting his force as P, this means a value of P slightly larger than 90.2 N is sufficient. Rounding to two figures, we obtain $P = 90$ N.

(b) Constant velocity (zero acceleration) implies the push equals the kinetic friction, so $P = 70\ \text{N}$.

(c) Applying Newton's second law, we have

$$P - f_k = ma \Rightarrow a = \frac{\mu_s mg - \mu_k mg}{m}$$

which simplifies to $a = g(\mu_s - \mu_k) = 0.88\ \text{m/s}^2$.

105. Probably the most appropriate picture in the textbook to represent the situation in this problem is in the previous chapter: Fig. 5-9. We adopt the familiar axes with $+x$ rightward and $+y$ upward, and refer to the 85 N horizontal push of the worker as P (and assume it to be rightward). Applying Newton's second law to the x axis and y axis, respectively, produces

$$P - f_k = ma$$
$$F_N - mg = 0.$$

Using $v^2 = v_0^2 + 2a\Delta x$ we find a = 0.36 m/s^2. Consequently, we obtain f_k = 71 N and F_N = 392 N. Therefore, $\mu_k = f_k / F_N = 0.18$.

106. (a) The centripetal force is given by Eq. 6-18:

$$F = \frac{mv^2}{R} = \frac{(1.00\text{ kg})(465\text{ m/s})^2}{6.40\times10^6\text{ m}} = 0.0338\,\text{N}.$$

(b) Calling downward (towards the center of Earth) the positive direction, Newton's second law leads to

$$mg - T = ma$$

where mg = 9.80 N and ma = 0.034 N, calculated in part (a). Thus, the tension in the cord by which the body hangs from the balance is T = 9.80 N – 0.03 N = 9.77 N. Thus, this is the reading for a standard kilogram mass, of the scale at the equator of the spinning Earth.

107. Except for replacing f_s with f_k, Fig 6-5 in the textbook is appropriate. With that figure in mind, we choose uphill as the $+x$ direction. Applying Newton's second law to the x axis, we have

$$f_k - W\sin\theta = ma \quad \text{where} \quad m = \frac{W}{g},$$

and where $W = 40$ N, $a = +0.80$ m/s^2 and $\theta = 25°$. Thus, we find $f_k = 20$ N. Along the y axis, we have

$$\sum \vec{F}_y = 0 \Rightarrow F_N = W\cos\theta$$

so that $\mu_k = f_k / F_N = 0.56$.

108. The assumption that there is no slippage indicates that we are dealing with static friction f_s, and it is this force that is responsible for "pushing" the luggage along as the belt moves. Thus, Fig. 6-5 in the textbook is appropriate for this problem -- *if* one reverses the arrow indicating the direction of motion (and removes the word "impending"). The mass of the box is m = 69/9.8 = 7.0 kg. Applying Newton's law to the x axis leads to

$$f_s - mg\sin\theta = ma$$

where θ = 2.5° and uphill is the positive direction.

(a) Interpreting "temporarily at rest" (which is not meant to be the same thing as "momentarily at rest") to mean that the box is at equilibrium, we have a = 0 and, consequently, $f_s = mg\sin\theta$ = 3.0 N. It is positive and therefore pointed uphill.

(b) Constant speed in a one-dimensional setting implies that the velocity is constant -- thus, a = 0 again. We recover the answer f_s = 3.0 N uphill, which we obtained in part (a).

(c) Early in the problem, the direction of motion of the luggage was given: downhill. Thus, an increase in that speed indicates a downhill acceleration $a = -0.20$ m/s^2. We now solve for the friction and obtain

$$f_s = ma + mg\sin\theta = 1.6 \text{ N},$$

which is positive -- therefore, uphill.

(d) A decrease in the (downhill) speed indicates the acceleration vector points uphill; thus, $a = +0.20$ m/s^2. We solve for the friction and obtain

$$f_s = ma + mg\sin\theta = 4.4 \text{ N},$$

which is positive -- therefore, uphill.

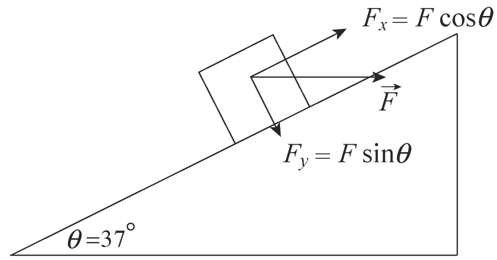
(e) The situation is similar to the one described in part (c), but with $a = -0.57$ m/s^2. Now,

$$f_s = ma + mg\sin\theta = -1.0 \text{ N},$$

or $|f_s| = 1.0\text{ N}$. Since f_s is negative, the direction is downhill.

(f) From the above, the only case where f_s is directed downhill is (e).

109. We resolve this horizontal force into appropriate components.



(a) Applying Newton's second law to the x (directed uphill) and y (directed away from the incline surface) axes, we obtain

$$F\cos\theta - f_k - mg\sin\theta = ma$$
$$F_N - F\sin\theta - mg\cos\theta = 0.$$

Using $f_k = \mu_k F_N$, these equations lead to

$$a = \frac{F}{m}(\cos\theta - \mu_k \sin\theta) - g(\sin\theta + \mu_k \cos\theta)$$

which yields $a = -2.1$ m/s^2, or $|a| = 2.1$ m/s^2, for $\mu_k = 0.30$, $F = 50$ N and $m = 5.0$ kg.

(b) The direction of $\vec{a}$ is down the plane.

(c) With $v_0 = +4.0$ m/s and $v = 0$, Eq. 2-16 gives

$$\Delta x = -\frac{(4.0 \text{ m/s})^2}{2(-2.1 \text{ m/s}^2)} = 3.9 \text{ m}.$$

(d) We expect $\mu_s \geq \mu_k$; otherwise, an object started into motion would immediately start decelerating (before it gained any speed)! In the minimal expectation case, where $\mu_s = 0.30$, the maximum possible (downhill) static friction is, using Eq. 6-1,

$$f_{s,\max} = \mu_s F_N = \mu_s(F\sin\theta + mg\cos\theta)$$

which turns out to be 21 N. But in order to have no acceleration along the x axis, we must have

$$f_s = F\cos\theta - mg\sin\theta = 10 \text{ N}$$

(the fact that this is positive reinforces our suspicion that $\vec{f}_s$ points downhill). Since the f_s needed to remain at rest is less than $f_{s,\max}$ then it stays at that location.

Chapter 7

1. (a) The change in kinetic energy for the meteorite would be

$$\Delta K = K_f - K_i = -K_i = -\frac{1}{2}m_i v_i^2 = -\frac{1}{2}\left(4\times10^6 \text{ kg}\right)\left(15\times10^3 \text{ m/s}\right)^2 = -5\times10^{14} \text{ J},$$

or $|\Delta K| = 5\times10^{14}$ J. The negative sign indicates that kinetic energy is lost.

(b) The energy loss in units of megatons of TNT would be

$$-\Delta K = \left(5\times10^{14} \text{ J}\right)\left(\frac{1 \text{ megaton TNT}}{4.2\times10^{15} \text{ J}}\right) = 0.1 \text{megaton TNT}.$$

(c) The number of bombs N that the meteorite impact would correspond to is found by noting that megaton = 1000 kilotons and setting up the ratio:

$$N = \frac{0.1\times1000 \text{kiloton TNT}}{13 \text{kiloton TNT}} = 8.$$

2. With speed v = 11200 m/s, we find

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(2.9\times10^5 \text{ kg})\ (11200 \text{ m/s})^2 = 1.8\times10^{13} \text{ J}.$$

3. (a) From Table 2-1, we have $v^2 = v_0^2 + 2a\Delta x$. Thus,

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{\left(2.4\times10^7 \text{ m/s}\right)^2 + 2\ \left(3.6\times10^{15} \text{ m/s}^2\right)\left(0.035 \text{ m}\right)} = 2.9\times10^7 \text{ m/s}.$$

(b) The initial kinetic energy is

$$K_i = \frac{1}{2}mv_0^2 = \frac{1}{2}\ \left(1.67\times10^{-27} \text{ kg}\right)\left(2.4\times10^7 \text{ m/s}\right)^2 = 4.8\times10^{-13} \text{ J}.$$

The final kinetic energy is

$$K_f = \frac{1}{2}mv^2 = \frac{1}{2}\ \left(1.67\times10^{-27} \text{ kg}\right)\left(2.9\times10^7 \text{ m/s}\right)^2 = 6.9\times10^{-13} \text{ J}.$$

The change in kinetic energy is $\Delta K = 6.9 \times 10^{-13}$ J – 4.8×10^{-13} J = 2.1×10^{-13} J.

4. The work done by the applied force $\vec{F}_a$ is given by $W = \vec{F}_a \cdot \vec{d} = F_a d \cos\phi$. From Fig. 7-24, we see that $W = 25$ J when $\phi = 0$ and $d = 5.0$ cm. This yields the magnitude of $\vec{F}_a$:

$$F_a = \frac{W}{d} = \frac{25\text{ J}}{0.050\text{ m}} = 5.0\times10^2\text{ N}.$$

(a) For $\phi = 64°$, we have $W = F_a d\cos\phi = (5.0\times10^2\text{ N})(0.050\text{ m})\cos 64° = 11$ J.

(b) For $\phi = 147°$, we have $W = F_a d\cos\phi = (5.0\times10^2\text{ N})(0.050\text{ m})\cos 147° = -21$ J.

5. We denote the mass of the father as m and his initial speed v_i. The initial kinetic energy of the father is

$$K_i = \frac{1}{2} K_{\text{son}}$$

and his final kinetic energy (when his speed is $v_f = v_i + 1.0$ m/s) is $K_f = K_{\text{son}}$. We use these relations along with Eq. 7-1 in our solution.

(a) We see from the above that $K_i = \frac{1}{2} K_f$ which (with SI units understood) leads to

$$\frac{1}{2} m v_i^2 = \frac{1}{2} \left[\frac{1}{2} m \ \left(v_i + 1.0 \text{ m/s}\right)^2 \right].$$

The mass cancels and we find a second-degree equation for v_i:

$$\frac{1}{2} v_i^2 - v_i - \frac{1}{2} = 0.$$

The positive root (from the quadratic formula) yields $v_i = 2.4$ m/s.

(b) From the first relation above $\left(K_i = \frac{1}{2} K_{\text{son}}\right)$, we have

$$\frac{1}{2} m v_i^2 = \frac{1}{2} \left(\frac{1}{2} \ (m/2) \ v_{\text{son}}^2 \right)$$

and (after canceling m and one factor of 1/2) are led to $v_{\text{son}} = 2 v_i = 4.8 \text{ m/s}$.

6. We apply the equation $x(t) = x_0 + v_0 t + \frac{1}{2}at^2$, found in Table 2-1. Since at $t = 0$ s, $x_0 = 0$ and $v_0 = 12\ \text{m/s}$, the equation becomes (in unit of meters)

$$x(t) = 12t + \tfrac{1}{2}at^2 .$$

With $x = 10\ \text{m}$ when $t = 1.0\ \text{s}$, the acceleration is found to be $a = -4.0\ \text{m/s}^2$. The fact that $a < 0$ implies that the bead is decelerating. Thus, the position is described by $x(t) = 12t - 2.0t^2$. Differentiating x with respect to t then yields

$$v(t) = \frac{dx}{dt} = 12 - 4.0t .$$

Indeed at t =3.0 s, $v(t = 3.0) = 0$ and the bead stops momentarily. The speed at $t = 10\ \text{s}$ is $v(t = 10) = -28\ \text{m/s}$, and the corresponding kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.8\times 10^{-2}\text{kg})(-28\ \text{m/s})^2 = 7.1\ \text{J}.$$

7. By the work-kinetic energy theorem,

$$W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(2.0\,\text{kg})\left((6.0\,\text{m/s})^2 - (4.0\,\text{m/s})^2\right) = 20\ \text{J}.$$

We note that the *directions* of $\vec{v}_f$ and $\vec{v}_i$ play no role in the calculation.

8. Eq. 7-8 readily yields

$$W = F_x \Delta x + F_y \Delta y = (2.0 \text{ N})\cos(100^\circ)(3.0 \text{ m}) + (2.0 \text{ N})\sin(100^\circ)(4.0 \text{ m}) = 6.8 \text{ J}.$$

9. Since this involves constant-acceleration motion, we can apply the equations of Table 2-1, such as $x = v_0 t + \frac{1}{2}at^2$ (where $x_0 = 0$). We choose to analyze the third and fifth points, obtaining

$$0.2\,\text{m} = v_0(1.0\ \text{s}) + \frac{1}{2}a\ (1.0\ \text{s})^2$$
$$0.8\,\text{m} = v_0(2.0\ \text{s}) + \frac{1}{2}a\ (2.0\ \text{s})^2$$

Simultaneous solution of the equations leads to $v_0 = 0$ and $a = 0.40\,\text{m/s}^2$. We now have two ways to finish the problem. One is to compute force from $F = ma$ and then obtain the work from Eq. 7-7. The other is to find ΔK as a way of computing W (in accordance with Eq. 7-10). In this latter approach, we find the velocity at $t = 2.0\,\text{s}$ from $v = v_0 + at$ (so $v = 0.80\,\text{m/s}$). Thus,

$$W = \Delta K = \frac{1}{2}(3.0\,\text{kg})(0.80\,\text{m/s})^2 = 0.96\ \text{J}.$$

10. Using Eq. 7-8 (and Eq. 3-23), we find the work done by the water on the ice block:

$$W = \vec{F} \cdot \vec{d} = \left[(210\text{ N})\,\hat{\text{i}} - (150\text{ N})\,\hat{\text{j}}\right] \cdot \left[(15\text{ m})\,\hat{\text{i}} - (12\text{ m})\hat{\text{j}}\right] = (210\text{ N})(15\text{ m}) + (-150\text{ N})(-12\text{ m})$$
$$= 5.0 \times 10^3\text{ J}.$$

11. We choose $+x$ as the direction of motion (so $\vec{a}$ and $\vec{F}$ are negative-valued).

(a) Newton's second law readily yields $\vec{F} = (85\,\text{kg})(-2.0\,\text{m/s}^2)$ so that

$$F = |\vec{F}| = 1.7\times 10^2\ \text{N}.$$

(b) From Eq. 2-16 (with $v = 0$) we have

$$0 = v_0^2 + 2a\Delta x \ \Rightarrow\ \Delta x = -\frac{(37\,\text{m/s})^2}{2(-2.0\,\text{m/s}^2)} = 3.4\times 10^2\ \text{m}.$$

Alternatively, this can be worked using the work-energy theorem.

(c) Since $\vec{F}$ is opposite to the direction of motion (so the angle ϕ between $\vec{F}$ and $\vec{d} = \Delta x$ is 180°) then Eq. 7-7 gives the work done as $W = -F\Delta x = -5.8\times 10^4\ \text{J}$.

(d) In this case, Newton's second law yields $\vec{F} = (85\,\text{kg})(-4.0\,\text{m/s}^2)$ so that $F = |\vec{F}| = 3.4\times 10^2\ \text{N}$.

(e) From Eq. 2-16, we now have

$$\Delta x = -\frac{(37\,\text{m/s})^2}{2(-4.0\,\text{m/s}^2)} = 1.7\times 10^2\ \text{m}.$$

(f) The force $\vec{F}$ is again opposite to the direction of motion (so the angle ϕ is again 180°) so that Eq. 7-7 leads to $W = -F\Delta x = -5.8\times 10^4$ J. The fact that this agrees with the result of part (c) provides insight into the concept of work.

12. The change in kinetic energy can be written as

$$\Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}m(2a\Delta x) = ma\Delta x$$

where we have used $v_f^2 = v_i^2 + 2a\Delta x$ from Table 2-1. From Fig. 7-27, we see that $\Delta K = (0-30)\ \text{J} = -30\ \text{J}$ when $\Delta x = +5\ \text{m}$. The acceleration can then be obtained as

$$a = \frac{\Delta K}{m\Delta x} = \frac{(-30\ \text{J})}{(8.0\ \text{kg})(5.0\ \text{m})} = -0.75\ \text{m/s}^2.$$

The negative sign indicates that the mass is decelerating. From the figure, we also see that when $x = 5\ \text{m}$ the kinetic energy becomes zero, implying that the mass comes to rest momentarily. Thus,

$$v_0^2 = v^2 - 2a\Delta x = 0 - 2(-0.75\ \text{m/s}^2)(5.0\ \text{m}) = 7.5\ \text{m}^2/\text{s}^2,$$

or $v_0 = 2.7\ \text{m/s}$. The speed of the object when $x = -3.0$ m is

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{7.5\ \text{m}^2/\text{s}^2 + 2(-0.75\ \text{m/s}^2)(-3.0\ \text{m})} = \sqrt{12}\ \text{m/s} = 3.5\ \text{m/s}.$$

13. (a) The forces are constant, so the work done by any one of them is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{d}$ is the displacement. Force $\vec{F}_1$ is in the direction of the displacement, so

$$W_1 = F_1 d \cos\phi_1 = (5.00\,\text{N})(3.00\,\text{m})\cos 0° = 15.0\ \text{J}.$$

Force $\vec{F}_2$ makes an angle of 120° with the displacement, so

$$W_2 = F_2 d \cos\phi_2 = (9.00\,\text{N})(3.00\,\text{m})\cos 120° = -13.5\ \text{J}.$$

Force $\vec{F}_3$ is perpendicular to the displacement, so

$$W_3 = F_3 d \cos\phi_3 = 0 \text{ since } \cos 90° = 0.$$

The net work done by the three forces is

$$W = W_1 + W_2 + W_3 = 15.0\ \text{J} - 13.5\ \text{J} + 0 = +1.50\ \text{J}.$$

(b) If no other forces do work on the box, its kinetic energy increases by 1.50 J during the displacement.

14. (a) From Eq. 7-6, $F = W/x = 3.00$ N (this is the slope of the graph).

(b) Eq. 7-10 yields $K = K_i + W = 3.00 \text{ J} + 6.00 \text{ J} = 9.00$ J.

15. Using the work-kinetic energy theorem, we have

$$\Delta K = W = \vec{F} \cdot \vec{d} = Fd\cos\phi$$

In addition, $F = 12\ \text{N}$ and $d = \sqrt{(2.00\ \text{m})^2 + (-4.00\ \text{m})^2 + (3.00\ \text{m})^2} = 5.39\ \text{m}$.

(a) If $\Delta K = +30.0\ \text{J}$, then

$$\phi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left(\frac{30.0\ \text{J}}{(12.0\ \text{N})(5.39\ \text{m})}\right) = 62.3°\,.$$

(b) $\Delta K = -30.0\ \text{J}$, then

$$\phi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left(\frac{-30.0\ \text{J}}{(12.0\ \text{N})(5.39\ \text{m})}\right) = 118°$$

16. The forces are all constant, so the total work done by them is given by $W = F_{\text{net}}\Delta x$, where F_{net} is the magnitude of the net force and Δx is the magnitude of the displacement. We add the three vectors, finding the x and y components of the net force:

$$\begin{aligned} F_{\text{net}\,x} &= -F_1 - F_2\sin 50.0° + F_3\cos 35.0° = -3.00\,\text{N} - (4.00\,\text{N})\sin 35.0° + (10.0\,\text{N})\cos 35.0° \\ &= 2.13\,\text{N} \end{aligned}$$

$$\begin{aligned} F_{\text{net}\,y} &= -F_2\cos 50.0° + F_3\sin 35.0° = -(4.00\,\text{N})\,\cos 50.0° + (10.0\,\text{N})\sin 35.0° \\ &= 3.17\,\text{N}. \end{aligned}$$

The magnitude of the net force is

$$F_{\text{net}} = \sqrt{F_{\text{net}\,x}^2 + F_{\text{net}\,y}^2} = \sqrt{(2.13\ \text{N})^2 + (3.17\ \text{N})^2} = 3.82\,\text{N}.$$

The work done by the net force is

$$W = F_{\text{net}}d = (3.82\,\text{N})(4.00\,\text{m}) = 15.3\ \text{J}$$

where we have used the fact that $\vec{d} \,||\, \vec{F}_{\text{net}}$ (which follows from the fact that the canister started from rest and moved horizontally under the action of horizontal forces — the resultant effect of which is expressed by $\vec{F}_{\text{net}}$).

17. (a) We use $\vec{F}$ to denote the upward force exerted by the cable on the astronaut. The force of the cable is upward and the force of gravity is mg downward. Furthermore, the acceleration of the astronaut is $g/10$ upward. According to Newton's second law, $F - mg = mg/10$, so $F = 11\ mg/10$. Since the force $\vec{F}$ and the displacement $\vec{d}$ are in the same direction, the work done by $\vec{F}$ is

$$W_F = Fd = \frac{11mgd}{10} = \frac{11\,(72\text{ kg})(9.8\text{ m/s}^2)(15\text{ m})}{10} = 1.164\times10^4\text{ J}$$

which (with respect to significant figures) should be quoted as 1.2×10^4 J.

(b) The force of gravity has magnitude mg and is opposite in direction to the displacement. Thus, using Eq. 7-7, the work done by gravity is

$$W_g = -mgd = -\,(72\text{ kg})(9.8\text{ m/s}^2)(15\text{ m}) = -1.058\times10^4\text{ J}$$

which should be quoted as $-\,1.1 \times 10^4$ J.

(c) The total work done is $W = 1.164\times10^4\text{ J} - 1.058\times10^4\text{ J} = 1.06\times10^3\text{ J}$. Since the astronaut started from rest, the work-kinetic energy theorem tells us that this (which we round to $1.1\times10^3\text{ J}$) is her final kinetic energy.

(d) Since $K = \frac{1}{2}mv^2$, her final speed is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.06\times10^3\text{ J})}{72\text{ kg}}} = 5.4\text{ m/s}.$$

18. In both cases, there is no acceleration, so the lifting force is equal to the weight of the object.

(a) Eq. 7-8 leads to $W = \vec{F} \cdot \vec{d} = (360\,\text{kN})(0.10\,\text{m}) = 36$ kJ.

(b) In this case, we find $W = (4000 \text{ N})(0.050 \text{ m}) = 2.0\times10^{2}$ J .

19. (a) We use F to denote the magnitude of the force of the cord on the block. This force is upward, opposite to the force of gravity (which has magnitude Mg). The acceleration is $\vec{a} = g/4$ downward. Taking the downward direction to be positive, then Newton's second law yields

$$\vec{F}_{\text{net}} = m\vec{a} \Rightarrow Mg - F = M\left(\frac{g}{4}\right)$$

so $F = 3Mg/4$. The displacement is downward, so the work done by the cord's force is, using Eq. 7-7,

$$W_F = -Fd = -3Mgd/4.$$

(b) The force of gravity is in the same direction as the displacement, so it does work $W_g = Mgd$.

(c) The total work done on the block is $-3Mgd/4 + Mgd = Mgd/4$. Since the block starts from rest, we use Eq. 7-15 to conclude that this $(Mgd/4)$ is the block's kinetic energy K at the moment it has descended the distance d.

(d) Since $K = \frac{1}{2}Mv^2$, the speed is

$$v = \sqrt{\frac{2K}{M}} = \sqrt{\frac{2(Mgd/4)}{M}} = \sqrt{\frac{gd}{2}}$$

at the moment the block has descended the distance d.

20. (a) Using notation common to many vector capable calculators, we have (from Eq. 7-8) W = dot([20.0,0] + [0, −(3.00)(9.8)], [0.500 $\angle$ 30.0º]) = +1.31 J.

(b) Eq. 7-10 (along with Eq. 7-1) then leads to

$$v = \sqrt{2(1.31 \text{ J})/(3.00 \text{ kg})} = 0.935 \text{ m/s}.$$

21. The fact that the applied force $\vec{F}_a$ causes the box to move up a frictionless ramp at a constant speed implies that there is no net change in the kinetic energy: $\Delta K = 0$. Thus, the work done by $\vec{F}_a$ must be equal to the negative work done by gravity: $W_a = -W_g$. Since the box is displaced vertically upward by $h = 0.150$ m, we have

$$W_a = +mgh = (3.00 \text{ kg})(9.80 \text{ m/s}^2)(0.150 \text{ m}) = 4.41 \text{ J}$$

22. From the figure, one may write the kinetic energy (in units of J) as a function of x as

$$K = K_s - 20x = 40 - 20x$$

Since $W = \Delta K = \vec{F}_x \cdot \Delta x$, the component of the force along the force along $+x$ is $F_x = dK/dx = -20$ N. The normal force on the block is $F_N = F_y$, which is related to the gravitational force by

$$mg = \sqrt{F_x^2 + (-F_y)^2} \; .$$

(Note that F_N points in the opposite direction of the component of the gravitational force.) With an initial kinetic energy $K_s = 40.0\text{ J}$ and $v_0 = 4.00\text{ m/s}$, the mass of the block is

$$m = \frac{2K_s}{v_0^2} = \frac{2(40.0\text{ J})}{(4.00\text{ m/s})^2} = 5.00\text{ kg}.$$

Thus, the normal force is

$$F_y = \sqrt{(mg)^2 - F_x^2} = \sqrt{(5.0\text{ kg})^2(9.8\text{ m/s}^2)^2 - (20\text{ N})^2} = 44.7\text{ N} \approx 45\text{ N}.$$

23. Eq. 7-15 applies, but the wording of the problem suggests that it is only necessary to examine the contribution from the rope (which would be the "W_a" term in Eq. 7-15):

$$W_a = -(50 \text{ N})(0.50 \text{ m}) = -25 \text{ J}$$

(the minus sign arises from the fact that the pull from the rope is anti-parallel to the direction of motion of the block). Thus, the kinetic energy would have been 25 J greater if the rope had not been attached (given the same displacement).

24. We use d to denote the magnitude of the spelunker's displacement during each stage. The mass of the spelunker is m = 80.0 kg. The work done by the lifting force is denoted W_i where i = 1, 2, 3 for the three stages. We apply the work-energy theorem, Eq. 17-15.

(a) For stage 1, $W_1 - mgd = \Delta K_1 = \frac{1}{2}mv_1^2$, where $v_1 = 5.00 \text{ m/s}$. This gives

$$W_1 = mgd + \frac{1}{2}mv_1^2 = (80.0 \text{ kg})(9.80 \text{ m/s}^2)(10.0 \text{ m}) + \frac{1}{2}(80.0 \text{ kg})(5.00 \text{ m/s})^2 = 8.84 \times 10^3 \text{ J}.$$

(b) For stage 2, $W_2 - mgd = \Delta K_2 = 0$, which leads to

$$W_2 = mgd = (80.0 \text{ kg})(9.80 \text{ m/s}^2)(10.0 \text{ m}) = 7.84 \times 10^3 \text{ J}.$$

(c) For stage 3, $W_3 - mgd = \Delta K_3 = -\frac{1}{2}mv_1^2$. We obtain

$$W_3 = mgd - \frac{1}{2}mv_1^2 = (80.0 \text{ kg})(9.80 \text{ m/s}^2)(10.0 \text{ m}) - \frac{1}{2}(80.0 \text{ kg})(5.00 \text{ m/s})^2 = 6.84 \times 10^3 \text{ J}.$$

25. (a) The net upward force is given by

$$F + F_N - (m+M)g = (m+M)a$$

where m = 0.250 kg is the mass of the cheese, M = 900 kg is the mass of the elevator cab, F is the force from the cable, and $F_N = 3.00\ \text{N}$ is the normal force on the cheese. On the cheese alone, we have

$$F_N - mg = ma \quad \Rightarrow \quad a = \frac{3.00\ \text{N} - (0.250\ \text{kg})(9.80\ \text{m/s}^2)}{0.250\ \text{kg}} = 2.20\ \text{m/s}^2 .$$

Thus the force from the cable is $F = (m+M)(a+g) - F_N = 1.08\times10^4\ \text{N}$, and the work done by the cable on the cab is

$$W = Fd_1 = (1.80\times10^4\ \text{N})(2.40\ \text{m}) = 2.59\times10^4\ \text{J}.$$

(b) If $W = 92.61\ \text{kJ}$ and $d_2 = 10.5\ \text{m}$, the magnitude of the normal force is

$$F_N = (m+M)g - \frac{W}{d_2} = (0.250\ \text{kg} + 900\ \text{kg})(9.80\ \text{m/s}^2) - \frac{9.261\times10^4\ \text{J}}{10.5\ \text{m}} = 2.45\ \text{N}.$$

26. The spring constant is k = 100 N/m and the maximum elongation is x_i = 5.00 m. Using Eq. 7-25 with x_f = 0, the work is found to be

$$W = \frac{1}{2}kx_i^2 = \frac{1}{2}(100 \text{ N/m})(5.00 \text{ m})^2 = 1.25\times10^3 \text{ J}.$$

27. From Eq. 7-25, we see that the work done by the spring force is given by

$$W_s = \frac{1}{2}k(x_i^2 - x_f^2).$$

The fact that 360 N of force must be applied to pull the block to $x = +\,4.0$ cm implies that the spring constant is

$$k = \frac{360\text{ N}}{4.0\text{ cm}} = 90\text{ N/cm} = 9.0\times10^3\text{ N/m}.$$

(a) When the block moves from $x_i = +5.0\text{ cm}$ to $x = +3.0\text{ cm}$, we have

$$W_s = \frac{1}{2}(9.0\times10^3\text{ N/m})[(0.050\text{ m})^2 - (0.030\text{ m})^2] = 7.2\text{ J}.$$

(b) Moving from $x_i = +5.0\text{ cm}$ to $x = -3.0\text{ cm}$, we have

$$W_s = \frac{1}{2}(9.0\times10^3\text{ N/m})[(0.050\text{ m})^2 - (-0.030\text{ m})^2] = 7.2\text{ J}.$$

(c) Moving from $x_i = +5.0\text{ cm}$ to $x = -5.0\text{ cm}$, we have

$$W_s = \frac{1}{2}(9.0\times10^3\text{ N/m})[(0.050\text{ m})^2 - (-0.050\text{ m})^2] = 0\text{ J}.$$

(d) Moving from $x_i = +5.0\text{ cm}$ to $x = -9.0\text{ cm}$, we have

$$W_s = \frac{1}{2}(9.0\times10^3\text{ N/m})[(0.050\text{ m})^2 - (-0.090\text{ m})^2] = -25\text{ J}.$$

28. We make use of Eq. 7-25 and Eq. 7-28 since the block is stationary before and after the displacement. The work done by the applied force can be written as

$$W_a = -W_s = \frac{1}{2}k(x_f^2 - x_i^2)\,.$$

The spring constant is $k = (80\text{ N})/(2.0\text{ cm}) = 4.0\times10^3\text{N/m}$. With $W_a = 4.0\text{ J}$, and $x_i = -2.0\text{ cm}$, we have

$$x_f = \pm\sqrt{\frac{2W_a}{k} + x_i^2} = \pm\sqrt{\frac{2(4.0\text{ J})}{(4.0\times10^3\text{ N/m})} + (-0.020\text{ m})^2} = \pm 0.049\text{ m} = \pm 4.9\text{ cm}.$$

29. (a) As the body moves along the x axis from x_i = 3.0 m to x_f = 4.0 m the work done by the force is

$$W = \int_{x_i}^{x_f} F_x \; dx = \int_{x_i}^{x_f} -6x \; dx = -3(x_f^2 - x_i^2) = -3\;(4.0^2 - 3.0^2) = -21 \text{ J}.$$

According to the work-kinetic energy theorem, this gives the change in the kinetic energy:

$$W = \Delta K = \frac{1}{2} m\left(v_f^2 - v_i^2\right)$$

where v_i is the initial velocity (at x_i) and v_f is the final velocity (at x_f). The theorem yields

$$v_f = \sqrt{\frac{2W}{m} + v_i^2} = \sqrt{\frac{2(-21 \text{ J})}{2.0 \text{ kg}} + (8.0 \text{ m/s})^2} = 6.6 \text{ m/s}.$$

(b) The velocity of the particle is v_f = 5.0 m/s when it is at $x = x_f$. The work-kinetic energy theorem is used to solve for x_f. The net work done on the particle is $W = -3\left(x_f^2 - x_i^2\right)$, so the theorem leads to

$$-3\left(x_f^2 - x_i^2\right) = \frac{1}{2} m \left(v_f^2 - v_i^2\right).$$

Thus,

$$x_f = \sqrt{-\frac{m}{6}\left(v_f^2 - v_i^2\right) + x_i^2} = \sqrt{-\frac{2.0 \text{ kg}}{6 \text{ N/m}}\left((5.0 \text{ m/s})^2 - (8.0 \text{ m/s})^2\right) + (3.0 \text{ m})^2} = 4.7 \text{ m}.$$

30. The work done by the spring force is given by Eq. 7-25:

$$W_s = \frac{1}{2}k(x_i^2 - x_f^2) .$$

Since $F_x = -kx$, the slope in Fig. 7-36 corresponds to the spring constant k. Its value is given by $k = 80\ \text{N/cm} = 8.0\times10^3\ \text{N/m}$.

(a) When the block moves from $x_i = +8.0\ \text{cm}$ to $x = +5.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3\ \text{N/m})[(0.080\ \text{m})^2 - (0.050\ \text{m})^2] = 15.6\ \text{J} \approx 16\ \text{J}.$$

(b) Moving from $x_i = +8.0\ \text{cm}$ to $x = -5.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3\ \text{N/m})[(0.080\ \text{m})^2 - (-0.050\ \text{m})^2] = 15.6\ \text{J} \approx 16\ \text{J}.$$

(c) Moving from $x_i = +8.0\ \text{cm}$ to $x = -8.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3\ \text{N/m})[(0.080\ \text{m})^2 - (-0.080\ \text{m})^2] = 0\ \text{J}.$$

(d) Moving from $x_i = +8.0\ \text{cm}$ to $x = -10.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3\ \text{N/m})[(0.080\ \text{m})^2 - (-0.10\ \text{m})^2] = -14.4\ \text{J} \approx -14\ \text{J}.$$

31. The work done by the spring force is given by Eq. 7-25: $W_s = \frac{1}{2}k(x_i^2 - x_f^2)$.

The spring constant k can be deduced from Fig. 7-37 which shows the amount of work done to pull the block from 0 to x = 3.0 cm. The parabola $W_a = kx^2/2$ contains (0,0), (2.0 cm, 0.40 J) and (3.0 cm, 0.90 J). Thus, we may infer from the data that $k = 2.0\times10^3$ N/m.

(a) When the block moves from $x_i = +5.0$ cm to $x = +4.0$ cm, we have

$$W_s = \frac{1}{2}(2.0\times10^3\text{ N/m})[(0.050\text{ m})^2 - (0.040\text{ m})^2] = 0.90\text{ J}.$$

(b) Moving from $x_i = +5.0$ cm to $x = -2.0$ cm, we have

$$W_s = \frac{1}{2}(2.0\times10^3\text{ N/m})[(0.050\text{ m})^2 - (-0.020\text{ m})^2] = 2.1\text{ J}.$$

(c) Moving from $x_i = +5.0$ cm to $x = -5.0$ cm, we have

$$W_s = \frac{1}{2}(2.0\times10^3\text{ N/m})[(0.050\text{ m})^2 - (-0.050\text{ m})^2] = 0\text{ J}.$$

32. Hooke's law and the work done by a spring is discussed in the chapter. We apply work-kinetic energy theorem, in the form of $\Delta K = W_a + W_s$, to the points in Figure 7-38 at x = 1.0 m and x = 2.0 m, respectively. The "applied" work W_a is that due to the constant force $\vec{P}$.

$$4 \text{ J} = P(1.0 \text{ m}) - \frac{1}{2}k(1.0 \text{ m})^2$$
$$0 = P(2.0 \text{ m}) - \frac{1}{2}k(2.0 \text{ m})^2$$

(a) Simultaneous solution leads to P = 8.0 N.

(b) Similarly, we find k = 8.0 N/m.

33. (a) This is a situation where Eq. 7-28 applies, so we have

$$Fx = \tfrac{1}{2}kx^2 \;\Rightarrow\; (3.0\text{ N})\,x = \tfrac{1}{2}(50\text{ N/m})x^2$$

which (other than the trivial root) gives $x = (3.0/25)$ m = 0.12 m.

(b) The work done by the applied force is $W_a = Fx = (3.0\text{ N})(0.12\text{ m}) = 0.36$ J.

(c) Eq. 7-28 immediately gives $W_s = -W_a = -0.36$ J.

(d) With $K_f = K$ considered variable and $K_i = 0$, Eq. 7-27 gives $K = Fx - \tfrac{1}{2}kx^2$. We take the derivative of K with respect to x and set the resulting expression equal to zero, in order to find the position x_c which corresponds to a maximum value of K:

$$x_c = \frac{F}{k} = (3.0/50)\text{ m} = 0.060\text{ m}.$$

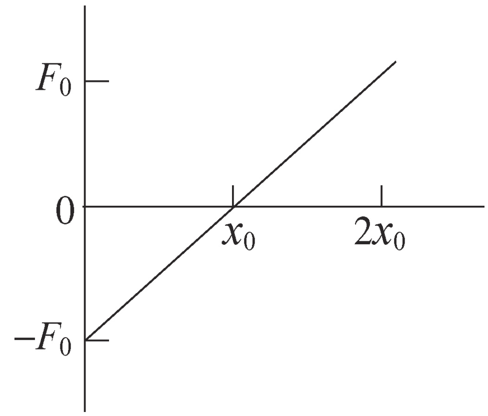
We note that x_c is also the point where the applied and spring forces "balance."

(e) At x_c we find $K = K_{max} = 0.090$ J.

34. From Eq. 7-32, we see that the "area" in the graph is equivalent to the work done. Finding that area (in terms of rectangular [length $\times$ width] and triangular [$\frac{1}{2}$ base $\times$ height] areas) we obtain

$$W = W_{0<x<2} + W_{2<x<4} + W_{4<x<6} + W_{6<x<8} = (20 + 10 + 0 - 5)\ \text{J} = 25\ \text{J}.$$

35. (a) The graph shows F as a function of x assuming x_0 is positive. The work is negative as the object moves from $x = 0$ to $x = x_0$ and positive as it moves from $x = x_0$ to $x = 2x_0$.



Since the area of a triangle is (base)(altitude)/2, the work done from $x = 0$ to $x = x_0$ is $-(x_0)(F_0)/2$ and the work done from $x = x_0$ to $x = 2x_0$ is

$$(2x_0 - x_0)(F_0)/2 = (x_0)(F_0)/2$$

The total work is the sum, which is zero.

(b) The integral for the work is

$$W = \int_0^{2x_0} F_0\left(\frac{x}{x_0} - 1\right)dx = F_0\left(\frac{x^2}{2x_0} - x\right)\Bigg|_0^{2x_0} = 0.$$

36. According to the graph the acceleration a varies linearly with the coordinate x. We may write $a = \alpha x$, where α is the slope of the graph. Numerically,

$$\alpha = \frac{20 \text{ m/s}^2}{8.0 \text{ m}} = 2.5 \text{ s}^{-2}.$$

The force on the brick is in the positive x direction and, according to Newton's second law, its magnitude is given by $F = ma = m\alpha x$. If x_f is the final coordinate, the work done by the force is

$$W = \int_0^{x_f} F\, dx = m\alpha \int_0^{x_f} x\, dx = \frac{m\alpha}{2} x_f^2 = \frac{(10 \text{ kg})(2.5 \text{ s}^{-2})}{2}(8.0 \text{ m})^2 = 8.0\times 10^2 \text{ J}.$$

37. We choose to work this using Eq. 7-10 (the work-kinetic energy theorem). To find the initial and final kinetic energies, we need the speeds, so

$$v = \frac{dx}{dt} = 3.0 - 8.0t + 3.0t^2$$

in SI units. Thus, the initial speed is v_i = 3.0 m/s and the speed at t = 4 s is v_f = 19 m/s. The change in kinetic energy for the object of mass m = 3.0 kg is therefore

$$\Delta K = \frac{1}{2}m\ \left(v_f^2 - v_i^2\right) = 528 \text{ J}$$

which we round off to two figures and (using the work-kinetic energy theorem) conclude that the work done is $W = 5.3\times10^2$ J.

38. Using Eq. 7-32, we find

$$W = \int_{0.25}^{1.25} e^{-4x^2}\, dx = 0.21 \text{ J}$$

where the result has been obtained numerically. Many modern calculators have that capability, as well as most math software packages that a great many students have access to.

39. (a) We first multiply the vertical axis by the mass, so that it becomes a graph of the applied force. Now, adding the triangular and rectangular “areas” in the graph (for $0 \le x \le 4$) gives 42 J for the work done.

(b) Counting the “areas” under the axis as negative contributions, we find (for $0 \le x \le 7$) the work to be 30 J at $x = 7.0$ m.

(c) And at $x = 9.0$ m, the work is 12 J.

(d) Eq. 7-10 (along with Eq. 7-1) leads to speed $v = 6.5$ m/s at $x = 4.0$ m. Returning to the original graph (where a was plotted) we note that (since it started from rest) it has received acceleration(s) (up to this point) only in the $+x$ direction and consequently must have a velocity vector pointing in the $+x$ direction at $x = 4.0$ m.

(e) Now, using the result of part (b) and Eq. 7-10 (along with Eq. 7-1) we find the speed is 5.5 m/s at $x = 7.0$ m. Although it has experienced some deceleration during the $0 \le x \le 7$ interval, its velocity vector still points in the $+x$ direction.

(f) Finally, using the result of part (c) and Eq. 7-10 (along with Eq. 7-1) we find its speed $v = 3.5$ m/s at $x = 9.0$ m. It certainly has experienced a significant amount of deceleration during the $0 \le x \le 9$ interval; nonetheless, its velocity vector *still* points in the $+x$ direction.

40. (a) Using the work-kinetic energy theorem

$$K_f = K_i + \int_0^{2.0} (2.5 - x^2)\ dx = 0 + (2.5)(2.0) - \frac{1}{3}(2.0)^3 = 2.3\ \text{J}.$$

(b) For a variable end-point, we have K_f as a function of x, which could be differentiated to find the extremum value, but we recognize that this is equivalent to solving $F = 0$ for x:

$$F = 0 \ \Rightarrow\ 2.5 - x^2 = 0.$$

Thus, K is extremized at $x = \sqrt{2.5} \approx 1.6\ \text{m}$ and we obtain

$$K_f = K_i + \int_0^{\sqrt{2.5}} (2.5 - x^2) dx = 0 + (2.5)(\sqrt{2.5}) - \frac{1}{3}\ (\sqrt{2.5})^3 = 2.6\ \text{J}.$$

Recalling our answer for part (a), it is clear that this extreme value is a maximum.

41. As the body moves along the x axis from $x_i = 0$ m to $x_f = 3.00$ m the work done by the force is

$$W = \int_{x_i}^{x_f} F_x \; dx = \int_{x_i}^{x_f} (cx - 3.00x^2)dx = \left(\frac{c}{2}x^2 - x^3 \right)_0^3 = \frac{c}{2}(3.00)^2 - (3.00)^3$$
$$= 4.50c - 27.0.$$

However, $W = \Delta K = (11.0 - 20.0) = -9.00$ J from the work-kinetic energy theorem. Thus,

$$4.50c - 27.0 = -9.00$$

or $c = 4.00$ N/m .

42. We solve the problem using the work-kinetic energy theorem which states that the change in kinetic energy is equal to the work done by the applied force, $\Delta K = W$. In our problem, the work done is $W = Fd$, where F is the tension in the cord and d is the length of the cord pulled as the cart slides from x_1 to x_2. From Fig. 7-42, we have

$$\begin{aligned} d &= \sqrt{x_1^2+h^2} - \sqrt{x_2^2+h^2} = \sqrt{(3.00\text{ m})^2+(1.20\text{ m})^2} - \sqrt{(1.00\text{ m})^2+(1.20\text{ m})^2} \\ &= 3.23\text{ m} - 1.56\text{ m} = 1.67\text{ m} \end{aligned}$$

which yields $\Delta K = Fd = (25.0\text{ N})(1.67\text{ m}) = 41.7\text{ J}$.

43. The power associated with force $\vec{F}$ is given by $P = \vec{F} \cdot \vec{v}$, where $\vec{v}$ is the velocity of the object on which the force acts. Thus,

$$P = \vec{F} \cdot \vec{v} = Fv\cos\phi = (122\ \text{N})(5.0\ \text{m/s})\cos 37° = 4.9\times10^2\ \text{W}.$$

44. Recognizing that the force in the cable must equal the total weight (since there is no acceleration), we employ Eq. 7-47:

$$P = Fv\cos\theta = mg\left(\frac{\Delta x}{\Delta t}\right)$$

where we have used the fact that $\theta = 0°$ (both the force of the cable and the elevator's motion are upward). Thus,

$$P = (3.0\times10^{3}\ \text{kg})(9.8\ \text{m/s}^{2})\left(\frac{210\ \text{m}}{23\ \text{s}}\right) = 2.7\times10^{5}\ \text{W}.$$

45. (a) The power is given by $P = Fv$ and the work done by $\vec{F}$ from time t_1 to time t_2 is given by

$$W = \int_{t_1}^{t_2} P\,\mathrm{d}t = \int_{t_1}^{t_2} Fv\,\mathrm{d}t.$$

Since $\vec{F}$ is the net force, the magnitude of the acceleration is $a = F/m$, and, since the initial velocity is $v_0 = 0$, the velocity as a function of time is given by $v = v_0 + at = (F/m)t$. Thus

$$W = \int_{t_1}^{t_2} (F^2/m)t\,\mathrm{d}t = \frac{1}{2}(F^2/m)(t_2^2 - t_1^2).$$

For $t_1 = 0$ and $t_2 = 1.0\,\mathrm{s}$,

$$W = \frac{1}{2}\left(\frac{(5.0\text{ N})^2}{15\text{ kg}}\right)(1.0\text{ s})^2 = 0.83\text{ J}.$$

(b) For $t_1 = 1.0\,\mathrm{s}$, and $t_2 = 2.0\,\mathrm{s}$,

$$W = \frac{1}{2}\left(\frac{(5.0\text{ N})^2}{15\text{ kg}}\right)[(2.0\text{ s})^2 - (1.0\text{ s})^2] = 2.5\text{ J}.$$

(c) For $t_1 = 2.0\,\mathrm{s}$ and $t_2 = 3.0\,\mathrm{s}$,

$$W = \frac{1}{2}\left(\frac{(5.0\text{ N})^2}{15\text{ kg}}\right)[(3.0\text{ s})^2 - (2.0\text{ s})^2] = 4.2\text{ J}.$$

(d) Substituting $v = (F/m)t$ into $P = Fv$ we obtain $P = F^2\,t/m$ for the power at any time t. At the end of the third second

$$P = \left(\frac{(5.0\text{ N})^2\ (3.0\text{ s})}{15\text{ kg}}\right) = 5.0\text{ W}.$$

46. (a) Since constant speed implies $\Delta K = 0$, we require $W_a = -W_g$, by Eq. 7-15. Since W_g is the same in both cases (same weight and same path), then $W_a = 9.0\times10^2$ J just as it was in the first case.

(b) Since the speed of 1.0 m/s is constant, then 8.0 meters is traveled in 8.0 seconds. Using Eq. 7-42, and noting that average power is *the* power when the work is being done at a steady rate, we have

$$P = \frac{W}{\Delta t} = \frac{900 \text{ J}}{8.0 \text{ s}} = 1.1\times10^2 \text{ W}.$$

(c) Since the speed of 2.0 m/s is constant, 8.0 meters is traveled in 4.0 seconds. Using Eq. 7-42, with *average power* replaced by *power*, we have

$$P = \frac{W}{\Delta t} = \frac{900 \text{ J}}{4.0 \text{ s}} = 225 \text{ W} \approx 2.3\times10^2 \text{ W}.$$

47. The total work is the sum of the work done by gravity on the elevator, the work done by gravity on the counterweight, and the work done by the motor on the system:

$$W_T = W_e + W_c + W_s.$$

Since the elevator moves at constant velocity, its kinetic energy does not change and according to the work-kinetic energy theorem the total work done is zero. This means $W_e + W_c + W_s = 0$. The elevator moves upward through 54 m, so the work done by gravity on it is

$$W_e = -m_e g d = -(1200 \text{ kg})(9.80 \text{ m/s}^2)(54 \text{ m}) = -6.35 \times 10^5 \text{ J}.$$

The counterweight moves downward the same distance, so the work done by gravity on it is

$$W_c = m_c g d = (950 \text{ kg})(9.80 \text{ m/s}^2)(54 \text{ m}) = 5.03\times10^5 \text{ J}.$$

Since $W_T = 0$, the work done by the motor on the system is

$$W_s = -W_e - W_c = 6.35\times10^5 \text{ J} - 5.03\times10^5 \text{ J} = 1.32\times10^5 \text{ J}.$$

This work is done in a time interval of $\Delta t = 3.0 \text{ min} = 180 \text{ s}$, so the power supplied by the motor to lift the elevator is

$$P = \frac{W_s}{\Delta t} = \frac{1.32 \times 10^5 \text{ J}}{180 \text{ s}} = 7.4 \times 10^2 \text{ W}.$$

48. (a) Using Eq. 7-48 and Eq. 3-23, we obtain

$$P = \vec{F} \cdot \vec{v} = (4.0\,\text{N})(-2.0\ \text{m/s}) + (9.0\ \text{N})(4.0\ \text{m/s}) = 28\ \text{W}.$$

(b) We again use Eq. 7-48 and Eq. 3-23, but with a one-component velocity: $\vec{v} = v\hat{\text{j}}$.

$$P = \vec{F} \cdot \vec{v} \quad \Rightarrow -12\ \text{W} = (-2.0\ \text{N})v.$$

which yields $v = 6$ m/s.

49. (a) Eq. 7-8 yields

$$
\begin{aligned}
W &= F_x \Delta x + F_y \Delta y + F_z \Delta z \\
&= (2.00 \text{ N})(7.5 \text{ m} - 0.50 \text{ m}) + (4.00 \text{ N})(12.0 \text{ m} - 0.75 \text{ m}) + (6.00 \text{ N})(7.2\text{m} - 0.20 \text{ m}) \\
&= 101 \text{ J} \approx 1.0\times 10^2 \text{ J}.
\end{aligned}
$$

(b) Dividing this result by 12 s (see Eq. 7-42) yields $P = 8.4$ W.

50. (a) Since the force exerted by the spring on the mass is zero when the mass passes through the equilibrium position of the spring, the rate at which the spring is doing work on the mass at this instant is also zero.

(b) The rate is given by $P = \vec{F} \cdot \vec{v} = -Fv$, where the minus sign corresponds to the fact that $\vec{F}$ and $\vec{v}$ are anti-parallel to each other. The magnitude of the force is given by

$$F = kx = (500 \text{ N/m})(0.10 \text{ m}) = 50 \text{ N},$$

while v is obtained from conservation of energy for the spring-mass system:

$$E = K + U = 10 \text{ J} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}(0.30 \text{ kg})v^2 + \frac{1}{2}(500 \text{ N/m})(0.10 \text{ m})^2$$

which gives $v = 7.1$ m/s. Thus,

$$P = -Fv = -(50 \text{ N})(7.1 \text{ m/s}) = -3.5 \times 10^2 \text{ W}.$$

51. (a) The object's displacement is

$$\vec{d} = \vec{d}_f - \vec{d}_i = (-8.00 \text{ m})\hat{\text{i}} + (6.00 \text{ m})\hat{\text{j}} + (2.00 \text{ m})\hat{\text{k}} .$$

Thus, Eq. 7-8 gives

$$W = \vec{F} \cdot \vec{d} = (3.00 \text{ N})(-8.00 \text{ m}) + (7.00 \text{ N})(6.00 \text{ m}) + (7.00 \text{ N})(2.00 \text{ m}) = 32.0 \text{ J}.$$

(b) The average power is given by Eq. 7-42:

$$P_{\text{avg}} = \frac{W}{t} = \frac{32.0}{4.00} = 8.00 \text{ W}.$$

(c) The distance from the coordinate origin to the initial position is

$$d_i = \sqrt{(3.00 \text{ m})^2 + (-2.00 \text{ m})^2 + (5.00 \text{ m})^2} = 6.16 \text{ m},$$

and the magnitude of the distance from the coordinate origin to the final position is

$$d_f = \sqrt{(-5.00 \text{ m})^2 + (4.00 \text{ m})^2 + (7.00 \text{ m})^2} = 9.49 \text{ m} .$$

Their scalar (dot) product is

$$\vec{d}_i \cdot \vec{d}_f = (3.00 \text{ m})(-5.00 \text{ m}) + (-2.00 \text{ m})(4.00 \text{ m}) + (5.00 \text{ m})(7.00 \text{ m}) = 12.0 \text{ m}^2.$$

Thus, the angle between the two vectors is

$$\phi = \cos^{-1}\left(\frac{\vec{d}_i \cdot \vec{d}_f}{d_i d_f}\right) = \cos^{-1}\left(\frac{12.0}{(6.16)(9.49)}\right) = 78.2°.$$

52. According to the problem statement, the power of the car is

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = mv\frac{dv}{dt} = \text{constant.}$$

The condition implies $dt = mvdv/P$, which can be integrated to give

$$\int_0^T dt = \int_0^{v_T} \frac{mvdv}{P} \quad \Rightarrow \; T = \frac{mv_T^2}{2P}$$

where v_T is the speed of the car at $t = T$. On the other hand, the total distance traveled can be written as

$$L = \int_0^T vdt = \int_0^{v_T} v\frac{mvdv}{P} = \frac{m}{P}\int_0^{v_T} v^2 dv = \frac{mv_T^3}{3P}.$$

By squaring the expression for L and substituting the expression for T, we obtain

$$L^2 = \left(\frac{mv_T^3}{3P}\right)^2 = \frac{8P}{9m}\left(\frac{mv_T^2}{2P}\right)^3 = \frac{8PT^3}{9m}$$

which implies that

$$PT^3 = \frac{9}{8}mL^2 = \text{constant.}$$

Differentiating the above equation gives $dPT^3 + 3PT^2 dT = 0,$ or $dT = -\dfrac{T}{3P}dP.$

53. (a) We set up the ratio

$$\frac{50 \text{ km}}{1 \text{ km}} = \left(\frac{E}{1 \text{ megaton}}\right)^{1/3}$$

and find $E = 50^3 \approx 1 \times 10^5$ megatons of TNT.

(b) We note that 15 kilotons is equivalent to 0.015 megatons. Dividing the result from part (a) by 0.013 yields about ten million bombs.

54. (a) The compression of the spring is d = 0.12 m. The work done by the force of gravity (acting on the block) is, by Eq. 7-12,

$$W_1 = mgd = (0.25 \text{ kg}) \left(9.8 \text{ m/s}^2\right) (0.12 \text{ m}) = 0.29 \text{ J}.$$

(b) The work done by the spring is, by Eq. 7-26,

$$W_2 = -\frac{1}{2}kd^2 = -\frac{1}{2} \ (250 \text{ N/m}) \ (0.12 \text{ m})^2 = -1.8 \text{ J}.$$

(c) The speed v_i of the block just before it hits the spring is found from the work-kinetic energy theorem (Eq. 7-15):

$$\Delta K = 0 - \frac{1}{2}mv_i^2 = W_1 + W_2$$

which yields

$$v_i = \sqrt{\frac{(-2)(W_1 + W_2)}{m}} = \sqrt{\frac{(-2)(0.29 \text{ J} - 1.8 \text{ J})}{0.25 \text{ kg}}} = 3.5 \text{ m/s}.$$

(d) If we instead had $v_i' = 7 \text{ m/s}$, we reverse the above steps and solve for d'. Recalling the theorem used in part (c), we have

$$0 - \frac{1}{2}mv_i'^2 = W_1' + W_2' = mgd' - \frac{1}{2}kd'^2$$

which (choosing the positive root) leads to

$$d' = \frac{mg + \sqrt{m^2g^2 + mkv_i'^2}}{k}$$

which yields d' = 0.23 m. In order to obtain this result, we have used more digits in our intermediate results than are shown above (so $v_i = \sqrt{12.048} \text{ m/s} = 3.471 \text{ m/s}$ and v_i' = 6.942 m/s).

55. One approach is to assume a “path” from $\vec{r}_i$ to $\vec{r}_f$ and do the line-integral accordingly. Another approach is to simply use Eq. 7-36, which we demonstrate:

$$W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy = \int_2^{-4} (2x) dx + \int_3^{-3} (3)\ dy$$

with SI units understood. Thus, we obtain W = 12 J – 18 J = – 6 J.

56. (a) The force of the worker on the crate is constant, so the work it does is given by $W_F = \vec{F} \cdot \vec{d} = Fd\cos\phi$, where $\vec{F}$ is the force, $\vec{d}$ is the displacement of the crate, and ϕ is the angle between the force and the displacement. Here F = 210 N, d = 3.0 m, and ϕ = 20°. Thus,

$$W_F = (210 \text{ N})\,(3.0 \text{ m}) \cos 20° = 590 \text{ J}.$$

(b) The force of gravity is downward, perpendicular to the displacement of the crate. The angle between this force and the displacement is 90° and cos 90° = 0, so the work done by the force of gravity is zero.

(c) The normal force of the floor on the crate is also perpendicular to the displacement, so the work done by this force is also zero.

(d) These are the only forces acting on the crate, so the total work done on it is 590 J.

57. There is no acceleration, so the lifting force is equal to the weight of the object. We note that the person's pull $\vec{F}$ is equal (in magnitude) to the tension in the cord.

(a) As indicated in the *hint*, tension contributes twice to the lifting of the canister: $2T = mg$. Since $|\vec{F}| = T$, we find $|\vec{F}| = 98\,\text{N}$.

(b) To rise 0.020 m, two segments of the cord (see Fig. 7-44) must shorten by that amount. Thus, the amount of string pulled down at the left end (this is the magnitude of $\vec{d}$, the downward displacement of the hand) is $d = 0.040$ m.

(c) Since (at the left end) both $\vec{F}$ and $\vec{d}$ are downward, then Eq. 7-7 leads to

$$W = \vec{F} \cdot \vec{d} = (98\text{ N})(0.040\text{ m}) = 3.9\text{ J}.$$

(d) Since the force of gravity $\vec{F}_g$ (with magnitude mg) is opposite to the displacement $\vec{d}_c = 0.020\text{ m}$ (up) of the canister, Eq. 7-7 leads to

$$W = \vec{F}_g \cdot \vec{d}_c = -\ (196\text{ N})(0.020\text{ m}) = -3.9\text{ J}.$$

This is consistent with Eq. 7-15 since there is no change in kinetic energy.

58. With SI units understood, Eq. 7-8 leads to $W = (4.0)(3.0) - c(2.0) = 12 - 2c$.

(a) If $W = 0$, then $c = 6.0$ N.

(b) If $W = 17$ J, then $c = -2.5$ N.

(c) If $W = -18$ J, then $c = 15$ N.

59. Using Eq. 7-8, we find

$$W = \vec{F} \cdot \vec{d} = (F \cos\theta\ \hat{\mathrm{i}} + \mathrm{F}\ \sin\theta\ \hat{\mathrm{j}}) \cdot (x\hat{\mathrm{i}} + y\hat{\mathrm{j}}) = Fx\cos\theta + Fy\sin\theta$$

where x = 2.0 m, y = –4.0 m, F = 10 N, and $\theta = 150°$. Thus, we obtain W = –37 J. Note that the given mass value (2.0 kg) is not used in the computation.

60. The acceleration is constant, so we may use the equations in Table 2-1. We choose the direction of motion as $+x$ and note that the displacement is the same as the distance traveled, in this problem. We designate the force (assumed singular) along the x direction acting on the m = 2.0 kg object as F.

(a) With v_0 = 0, Eq. 2-11 leads to $a = v/t$. And Eq. 2-17 gives $\Delta x = \frac{1}{2}vt$. Newton's second law yields the force $F = ma$. Eq. 7-8, then, gives the work:

$$W = F\Delta x = m\left(\frac{v}{t}\right)\left(\frac{1}{2}vt\right) = \frac{1}{2}mv^2$$

as we expect from the work-kinetic energy theorem. With v = 10 m/s, this yields $W = 1.0\times10^2$ J.

(b) Instantaneous power is defined in Eq. 7-48. With t = 3.0 s, we find

$$P = Fv = m\left(\frac{v}{t}\right)v = 67 \text{ W}.$$

(c) The velocity at $t' = 1.5\,\text{s}$ is $v' = at' = 5.0\ \text{m/s}$. Thus, $P' = Fv' = 33$ W.

61. The total weight is $(100)(660\ \text{N}) = 6.60 \times 10^4$ N, and the words “raises … at constant speed” imply zero acceleration, so the lift-force is equal to the total weight. Thus

$$P = Fv = (6.60 \times 10^4)(150\ \text{m}/60.0\ \text{s}) = 1.65 \times 10^5\ \text{W}.$$

62. (a) The force $\vec{F}$ of the incline is a combination of normal and friction force which is serving to "cancel" the tendency of the box to fall downward (due to its 19.6 N weight). Thus, $\vec{F} = mg$ upward. In this part of the problem, the angle ϕ between the belt and $\vec{F}$ is 80°. From Eq. 7-47, we have

$$P = Fv\cos\phi = (19.6\text{ N})(0.50\text{ m/s})\cos 80° = 1.7\text{ W}.$$

(b) Now the angle between the belt and $\vec{F}$ is 90°, so that $P = 0$.

(c) In this part, the angle between the belt and $\vec{F}$ is 100°, so that

$$P = (19.6\text{ N})(0.50\text{ m/s})\cos 100° = -1.7\text{ W}.$$

63. (a) In 10 min the cart moves

$$d = \left(6.0\ \frac{\text{mi}}{\text{h}}\right)\left(\frac{5280\ \text{ft/mi}}{60\ \text{min/h}}\right)(10\ \text{min}) = 5280\ \text{ft}$$

so that Eq. 7-7 yields

$$W = Fd\cos\phi = (40\ \text{lb})(5280\ \text{ft})\cos 30° = 1.8\times10^5\ \text{ft}\cdot\text{lb}.$$

(b) The average power is given by Eq. 7-42, and the conversion to horsepower (hp) can be found on the inside back cover. We note that 10 min is equivalent to 600 s.

$$P_{\text{avg}} = \frac{1.8\times10^5\ \text{ft}\cdot\text{lb}}{600\ \text{s}} = 305\ \text{ft}\cdot\text{lb/s}$$

which (upon dividing by 550) converts to $P_{\text{avg}} = 0.55$ hp.

64. Using Eq. 7-7, we have $W = Fd \cos \phi = 1504 \text{ J}$. Then, by the work-kinetic energy theorem, we find the kinetic energy $K_f = K_i + W = 0 + 1504$ J. The answer is therefore 1.5 kJ.

65. (a) To hold the crate at equilibrium in the final situation, $\vec{F}$ must have the same magnitude as the horizontal component of the rope's tension $T \sin\theta$, where θ is the angle between the rope (in the final position) and vertical:

$$\theta = \sin^{-1}\left(\frac{4.00}{12.0}\right) = 19.5°.$$

But the vertical component of the tension supports against the weight: $T \cos\theta = mg$. Thus, the tension is

$$T = (230 \text{ kg})(9.80 \text{ m/s}^2)/\cos 19.5° = 2391 \text{ N}$$

and $F = (2391 \text{ N}) \sin 19.5° = 797 \text{ N}$.

An alternative approach based on drawing a vector triangle (of forces) in the final situation provides a quick solution.

(b) Since there is no change in kinetic energy, the net work on it is zero.

(c) The work done by gravity is $W_g = \vec{F}_g \cdot \vec{d} = -mgh$, where $h = L(1 - \cos\theta)$ is the vertical component of the displacement. With $L = 12.0$ m, we obtain $W_g = -1547$ J which should be rounded to three figures: –1.55 kJ.

(d) The tension vector is everywhere perpendicular to the direction of motion, so its work is zero (since cos 90° = 0).

(e) The implication of the previous three parts is that the work due to $\vec{F}$ is $-W_g$ (so the net work turns out to be zero). Thus, $W_F = -W_g = 1.55$ kJ.

(f) Since $\vec{F}$ does not have constant magnitude, we cannot expect Eq. 7-8 to apply.

66. From Eq. 7-32, we see that the "area" in the graph is equivalent to the work done. We find the area in terms of rectangular [length × width] and triangular [$\frac{1}{2}$ base × height] areas and use the work-kinetic energy theorem appropriately. The initial point is taken to be $x = 0$, where $v_0 = 4.0$ m/s.

(a) With $K_i = \frac{1}{2}mv_0^2 = 16 \text{ J}$, we have

$$K_3 - K_0 = W_{0<x<1} + W_{1<x<2} + W_{2<x<3} = -4.0 \text{ J}$$

so that K_3 (the kinetic energy when $x = 3.0$ m) is found to equal 12 J.

(b) With SI units understood, we write $W_{3<x<x_f}$ as $F_x\Delta x = (-4.0 \text{ N})(x_f - 3.0 \text{ m})$ and apply the work-kinetic energy theorem:

$$\begin{aligned} K_{x_f} - K_3 &= W_{3<x<x_f} \\ K_{x_f} - 12 &= (-4)(x_f - 3.0) \end{aligned}$$

so that the requirement $K_{xf} = 8.0 \text{ J}$ leads to $x_f = 4.0 \text{ m}$.

(c) As long as the work is positive, the kinetic energy grows. The graph shows this situation to hold until $x = 1.0$ m. At that location, the kinetic energy is

$$K_1 = K_0 + W_{0<x<1} = 16 \text{ J} + 2.0 \text{ J} = 18 \text{ J}.$$

67. (a) Noting that the x component of the third force is F_{3x} = (4.00 N)cos(60º), we apply Eq. 7-8 to the problem:

$$W = [5.00 \text{ N} - 1.00 \text{ N} + (4.00 \text{ N})\cos 60^\circ](0.20 \text{ m}) = 1.20 \text{ J}.$$

(b) Eq. 7-10 (along with Eq. 7-1) then yields $v = \sqrt{2W/m} = 1.10$ m/s.

68. (a) In the work-kinetic energy theorem, we include both the work due to an applied force W_a and work done by gravity W_g in order to find the latter quantity.

$$\Delta K = W_a + W_g \quad \Rightarrow \quad 30\text{ J} = (100\text{ N})(1.8\text{ m})\cos 180° + W_g$$

leading to $W_g = 2.1\times10^2$ J.

(b) The value of W_g obtained in part (a) still applies since the weight and the path of the child remain the same, so $\Delta K = W_g = 2.1\times10^2$ J.

69. (a) Eq. 7-6 gives $W_a = Fd = (209 \text{ N})(1.50 \text{ m}) \approx 314 \text{ J}$.

(b) Eq. 7-12 leads to $W_g = (25.0 \text{ kg})(9.80 \text{ m/s}^2)(1.50 \text{ m})\cos(115°) \approx -155 \text{ J}$.

(c) The angle between the normal force and the direction of motion remains 90º at all times, so the work it does is zero.

(d) The total work done on the crate is $W_T = 314 \text{ J} - 155 \text{ J} = 158 \text{ J}$.

70. After converting the speed to meters-per-second, we find

$$K = \frac{1}{2} m\mathrm{v}^2 = 667 \text{ kJ}.$$

71. (a) Hooke's law and the work done by a spring is discussed in the chapter. Taking absolute values, and writing that law in terms of differences ΔF and Δx, we analyze the first two pictures as follows:

$$\begin{aligned} |\Delta F| &= k|\Delta x| \\ 240\ \text{N} - 110\ \text{N} &= k(60\ \text{mm} - 40\ \text{mm}) \end{aligned}$$

which yields $k = 6.5$ N/mm. Designating the relaxed position (as read by that scale) as x_o we look again at the first picture:

$$110\ \text{N} = k(40\ \text{mm} - x_o)$$

which (upon using the above result for k) yields $x_o = 23$ mm.

(b) Using the results from part (a) to analyze that last picture, we find

$$W = k(30\,\text{mm} - x_o) = 45\ \text{N}\ .$$

72. (a) Using Eq. 7-8 and SI units, we find

$$W = \vec{F} \cdot \vec{d} = (2\,\hat{\mathrm{i}} - 4\,\hat{\mathrm{j}}) \cdot (8\,\hat{\mathrm{i}} + c\,\hat{\mathrm{j}}) = 16 - 4c$$

which, if equal zero, implies $c = 16/4 = 4$ m.

(b) If $W > 0$ then $16 > 4c$, which implies $c < 4$ m.

(c) If $W < 0$ then $16 < 4c$, which implies $c > 4$ m.

73. A convenient approach is provided by Eq. 7-48.

$$P = F\,v = (1800 \text{ kg} + 4500 \text{ kg})(9.8 \text{ m/s}^2)(3.80 \text{ m/s}) = 235 \text{ kW}.$$

Note that we have set the applied force equal to the weight in order to maintain constant velocity (zero acceleration).

74. (a) The component of the force of gravity exerted on the ice block (of mass *m*) along the incline is *mg* sin θ, where $\theta = \sin^{-1}(0.91/1.5)$ gives the angle of inclination for the inclined plane. Since the ice block slides down with uniform velocity, the worker must exert a force $\vec{F}$ "uphill" with a magnitude equal to *mg* sin θ. Consequently,

$$F = mg\sin\theta = (45\text{ kg})(9.8\text{ m/s}^2)\left(\frac{0.91\text{m}}{1.5\text{m}}\right) = 2.7\times10^2\text{ N}.$$

(b) Since the "downhill" displacement is opposite to $\vec{F}$, the work done by the worker is

$$W_1 = -\left(2.7\times10^2\text{ N}\right)(1.5\text{ m}) = -4.0\times10^2\text{ J}.$$

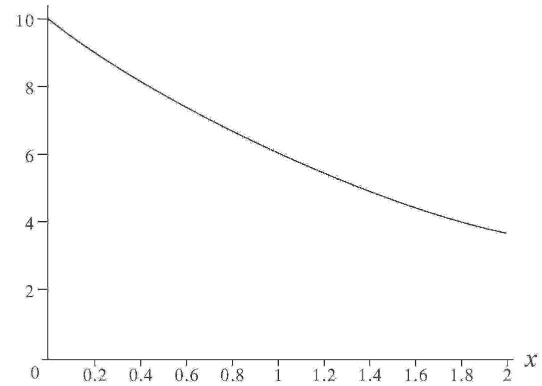
(c) Since the displacement has a vertically downward component of magnitude 0.91 m (in the same direction as the force of gravity), we find the work done by gravity to be

$$W_2 = (45\text{ kg})\left(9.8\text{ m}/\text{s}^2\right)(0.91\text{ m}) = 4.0\times10^2\text{ J}.$$

(d) Since $\vec{F}_N$ is perpendicular to the direction of motion of the block, and $\cos 90° = 0$, work done by the normal force is $W_3 = 0$ by Eq. 7-7.

(e) The resultant force $\vec{F}_{\text{net}}$ is zero since there is no acceleration. Thus, its work is zero, as can be checked by adding the above results $W_1 + W_2 + W_3 = 0$.

75. (a) The plot of the function (with SI units understood) is shown below.



Estimating the area under the curve allows for a range of answers. Estimates from 11 J to 14 J are typical.

(b) Evaluating the work analytically (using Eq. 7-32), we have

$$W = \int_0^2 10e^{-x/2}dx = -20e^{-x/2}\Big|_0^2 = 12.6 \text{ J} \approx 13 \text{ J}.$$

76. (a) Eq. 7-10 (along with Eq. 7-1 and Eq. 7-7) leads to

$$v_f = \left(2\frac{d}{m}F\cos\theta\right)^{1/2} = (\cos\theta)^{1/2},$$

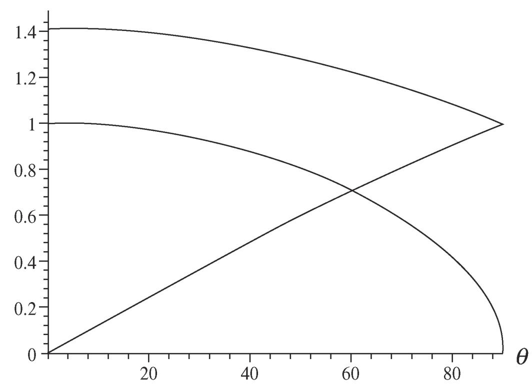
where we have substituted $F = 2.0$ N, $m = 4.0$ kg and $d = 1.0$ m.

(b) With $v_i = 1$, those same steps lead to $v_f = (1 + \cos\theta)^{1/2}$.

(c) Replacing θ with 180º – θ, and still using $v_i = 1$, we find

$$v_f = [1 + \cos(180^\circ - \theta)]^{1/2} = (1 - \cos\theta)^{1/2}.$$

(d) The graphs are shown on the right. Note that as θ is increased in parts (a) and (b) the force provides less and less of a positive acceleration, whereas in part (c) the force provides less and less of a deceleration (as its θ value increases). The highest curve (which slowly decreases from 1.4 to 1) is the curve for part (b); the other decreasing curve (starting at 1 and ending at 0) is for part (a). The rising curve is for part (c); it is equal to 1 where $\theta = 90$º.



77. (a) We can easily fit the curve to a concave-downward parabola: $x = \frac{1}{10}t(10 - t)$, from which (by taking two derivatives) we find the acceleration to be a = –0.20 m/s^2. The (constant) force is therefore $F = ma$ = –0.40 N, with a corresponding work given by $W = Fx = \frac{2}{50}t(t - 10)$. It also follows from the x expression that v_o = 1.0 m/s. This means that $K_i = \frac{1}{2}mv^2$ = 1.0 J. Therefore, when t = 1.0 s, Eq. 7-10 gives $K = K_i + W$ = 0.64 J ≈ 0.6 J, where the second significant figure is not to be taken too seriously.

(b) At t = 5.0 s, the above method gives K = 0.

(c) Evaluating the $W = \frac{2}{50}t(t - 10)$ expression at t = 5.0 s and t = 1.0 s, and subtracting, yields –0.6 J. This can also be inferred from the answers for parts (a) and (b).

78. The problem indicates that SI units are understood, so the result (of Eq. 7-23) is in Joules. Done numerically, using features available on many modern calculators, the result is roughly 0.47 J. For the interested student it might be worthwhile to quote the "exact" answer (in terms of the "error function"):

$$\int_{.15}^{1.2} e^{-2x^2}\, dx = \tfrac{1}{4}\sqrt{2\pi}\,\left[\mathrm{erf}(6\sqrt{2}\,/5) - \mathrm{erf}(3\sqrt{2}\,/20)\right].$$

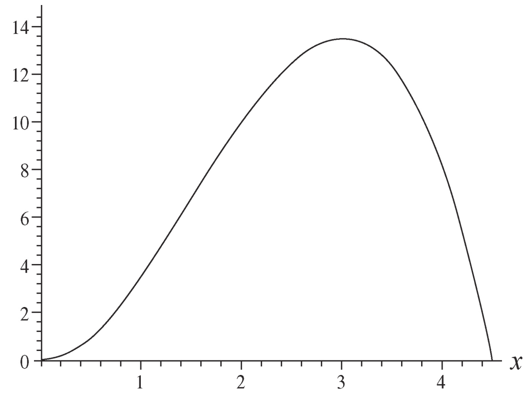
79. (a) To estimate the area under the curve between x = 1 m and x = 3 m (which should yield the value for the work done), one can try "counting squares" (or half-squares or thirds of squares) between the curve and the axis. Estimates between 5 J and 8 J are typical for this (crude) procedure.

(b) Eq. 7-32 gives

$$\int_1^3 \frac{a}{x^2}\, dx = \frac{a}{3} - \frac{a}{1} = 6 \text{ J}$$

where a = –9 N·m^2 is given in the problem statement.

80. (a) Using Eq. 7-32, the work becomes $W = \frac{9}{2}x^2 - x^3$ (SI units understood). The plot is shown below:



(b) We see from the graph that its peak value occurs at $x = 3.00$ m. This can be verified by taking the derivative of W and setting equal to zero, or simply by noting that this is where the force vanishes.

(c) The maximum value is $W = \frac{9}{2}(3.00)^2 - (3.00)^3 = 13.50$ J.

(d) We see from the graph (or from our analytic expression) that $W = 0$ at $x = 4.50$ m.

(e) The case is at rest when $v = 0$. Since $W = \Delta K = mv^2/2$, the condition implies $W = 0$. This happens at $x = 4.50$ m.

Chapter 8

1. (a) Noting that the vertical displacement is 10.0 m – 1.50 m = 8.50 m downward (same direction as $\vec{F}_g$), Eq. 7-12 yields

$$W_g = mgd\cos\phi = (2.00\text{ kg})(9.80\text{ m/s}^2)(8.50\text{ m})\cos 0° = 167\text{ J.}$$

(b) One approach (which is fairly trivial) is to use Eq. 8-1, but we feel it is instructive to instead calculate this as ΔU where $U = mgy$ (with upwards understood to be the $+y$ direction). The result is

$$\Delta U = mg(y_f - y_i) = (2.00\text{ kg})(9.80\text{ m/s}^2)(1.50\text{ m} - 10.0\text{ m}) = -167\text{ J.}$$

(c) In part (b) we used the fact that $U_i = mgy_i$ =196 J.

(d) In part (b), we also used the fact $U_f = mgy_f = 29$ J.

(e) The computation of W_g does not use the new information (that U = 100 J at the ground), so we again obtain W_g = 167 J.

(f) As a result of Eq. 8-1, we must again find $\Delta U = -W_g = -167$ J.

(g) With this new information (that U_0 = 100 J where y = 0) we have

$$U_i = mgy_i + U_0 = 296\text{ J.}$$

(h) With this new information (that U_0 = 100 J where y = 0) we have

$$U_f = mgy_f + U_0 = 129\text{ J.}$$

We can check part (f) by subtracting the new U_i from this result.

2. (a) The only force that does work on the ball is the force of gravity; the force of the rod is perpendicular to the path of the ball and so does no work. In going from its initial position to the lowest point on its path, the ball moves vertically through a distance equal to the length *L* of the rod, so the work done by the force of gravity is

$$W = mgL = (0.341\ \text{kg})(9.80\ \text{m/s}^2)(0.452\ \text{m}) = 1.51\ \text{J}\ .$$

(b) In going from its initial position to the highest point on its path, the ball moves vertically through a distance equal to *L*, but this time the displacement is upward, opposite the direction of the force of gravity. The work done by the force of gravity is

$$W = -mgL = -(0.341\ \text{kg})(9.80\ \text{m/s}^2)(0.452\ \text{m}) = -1.51\ \text{J}.$$

(c) The final position of the ball is at the same height as its initial position. The displacement is horizontal, perpendicular to the force of gravity. The force of gravity does no work during this displacement.

(d) The force of gravity is conservative. The change in the gravitational potential energy of the ball-Earth system is the negative of the work done by gravity:

$$\Delta U = -mgL = -(0.341\ \text{kg})(9.80\ \text{m/s}^2)(0.452\ \text{m}) = -1.51\ \text{J}$$

as the ball goes to the lowest point.

(e) Continuing this line of reasoning, we find

$$\Delta U = +mgL = (0.341\ \text{kg})(9.80\ \text{m/s}^2)(0.452\ \text{m}) = 1.51\ \text{J}$$

as it goes to the highest point.

(f) Continuing this line of reasoning, we have $\Delta U = 0$ as it goes to the point at the same height.

(g) The change in the gravitational potential energy depends only on the initial and final positions of the ball, not on its speed anywhere. The change in the potential energy is the *same* since the initial and final positions are the same.

3. (a) The force of gravity is constant, so the work it does is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{F}$ is the force and $\vec{d}$ is the displacement. The force is vertically downward and has magnitude *mg*, where *m* is the mass of the flake, so this reduces to $W = mgh$, where *h* is the height from which the flake falls. This is equal to the radius *r* of the bowl. Thus

$$W = mgr = (2.00 \times 10^{-3}\ \text{kg})(9.8\ \text{m/s}^2)(22.0 \times 10^{-2}\,\text{m}) = 4.31 \times 10^{-3}\,\text{J}.$$

(b) The force of gravity is conservative, so the change in gravitational potential energy of the flake-Earth system is the negative of the work done: $\Delta U = -W = -4.31 \times 10^{-3}$ J.

(c) The potential energy when the flake is at the top is greater than when it is at the bottom by $|\Delta U|$. If $U = 0$ at the bottom, then $U = +4.31 \times 10^{-3}$ J at the top.

(d) If $U = 0$ at the top, then $U = -4.31 \times 10^{-3}$ J at the bottom.

(e) All the answers are proportional to the mass of the flake. If the mass is doubled, all answers are doubled.

4. We use Eq. 7-12 for W_g and Eq. 8-9 for U.

(a) The displacement between the initial point and A is horizontal, so ϕ = 90.0° and $W_g = 0$ (since cos 90.0° = 0).

(b) The displacement between the initial point and B has a vertical component of $h/2$ downward (same direction as $\vec{F}_g$), so we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = \frac{1}{2}mgh = \frac{1}{2}(825\text{ kg})(9.80\text{ m/s}^2)(42.0\text{ m}) = 1.70\times10^5\text{ J}.$$

(c) The displacement between the initial point and C has a vertical component of h downward (same direction as $\vec{F}_g$), so we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = mgh = (825\text{ kg})(9.80\text{ m/s}^2)(42.0\text{ m}) = 3.40\times10^5\text{ J}.$$

(d) With the reference position at C, we obtain

$$U_B = \frac{1}{2}mgh = \frac{1}{2}(825\text{ kg})(9.80\text{ m/s}^2)(42.0\text{ m}) = 1.70\times10^5\text{ J}$$

(e) Similarly, we find

$$U_A = mgh = (825\text{ kg})(9.80\text{ m/s}^2)(42.0\text{ m}) = 3.40\times10^5\text{ J}$$

(f) All the answers are proportional to the mass of the object. If the mass is doubled, all answers are doubled.

5. The potential energy stored by the spring is given by $U = \frac{1}{2}kx^2$, where k is the spring constant and x is the displacement of the end of the spring from its position when the spring is in equilibrium. Thus

$$k = \frac{2U}{x^2} = \frac{2(25\,\text{J})}{(0.075\,\text{m})^2} = 8.9\times10^3\ \text{N/m}.$$

6. (a) The force of gravity is constant, so the work it does is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{F}$ is the force and $\vec{d}$ is the displacement. The force is vertically downward and has magnitude mg, where m is the mass of the snowball. The expression for the work reduces to $W = mgh$, where h is the height through which the snowball drops. Thus

$$W = mgh = (1.50 \text{ kg})(9.80 \text{ m/s}^2)(12.5 \text{ m}) = 184 \text{ J} .$$

(b) The force of gravity is conservative, so the change in the potential energy of the snowball-Earth system is the negative of the work it does: $\Delta U = -W = -184$ J.

(c) The potential energy when it reaches the ground is less than the potential energy when it is fired by $|\Delta U|$, so $U = -184$ J when the snowball hits the ground.

7. The main challenge for students in this type of problem seems to be working out the trigonometry in order to obtain the height of the ball (relative to the low point of the swing) $h = L - L\cos\theta$ (for angle θ measured from vertical as shown in Fig. 8-34). Once this relation (which we will not derive here since we have found this to be most easily illustrated at the blackboard) is established, then the principal results of this problem follow from Eq. 7-12 (for W_g) and Eq. 8-9 (for U).

(a) The vertical component of the displacement vector is downward with magnitude h, so we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = mgh = mgL(1-\cos\theta)$$
$$= (5.00\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m})(1-\cos 30°) = 13.1\text{ J}$$

(b) From Eq. 8-1, we have $\Delta U = -W_g = -mgL(1 - \cos\theta) = -13.1$ J.

(c) With $y = h$, Eq. 8-9 yields $U = mgL(1 - \cos\theta) = 13.1$ J.

(d) As the angle increases, we intuitively see that the height h increases (and, less obviously, from the mathematics, we see that $\cos\theta$ decreases so that $1 - \cos\theta$ increases), so the answers to parts (a) and (c) increase, and the absolute value of the answer to part (b) also increases.

8. We use Eq. 7-12 for W_g and Eq. 8-9 for U.

(a) The displacement between the initial point and Q has a vertical component of $h - R$ downward (same direction as $\vec{F}_g$), so (with $h = 5R$) we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = 4mgR = 4(3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.15\ \text{J}.$$

(b) The displacement between the initial point and the top of the loop has a vertical component of $h - 2R$ downward (same direction as $\vec{F}_g$), so (with $h = 5R$) we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = 3mgR = 3(3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.11\ \text{J}.$$

(c) With $y = h = 5R$, at P we find

$$U = 5mgR = 5(3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.19\ \text{J}.$$

(d) With $y = R$, at Q we have

$$U = mgR = (3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.038\ \text{J}$$

(e) With $y = 2R$, at the top of the loop, we find

$$U = 2mgR = 2(3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.075\ \text{J}$$

(f) The new information $(v_i \neq 0)$ is not involved in any of the preceding computations; the above results are unchanged.

9. We neglect any work done by friction. We work with SI units, so the speed is converted: $v = 130(1000/3600) = 36.1$ m/s.

(a) We use Eq. 8-17: $K_f + U_f = K_i + U_i$ with $U_i = 0$, $U_f = mgh$ and $K_f = 0$. Since $K_i = \frac{1}{2}mv^2$, where v is the initial speed of the truck, we obtain

$$\frac{1}{2}mv^2 = mgh \quad \Rightarrow \quad h = \frac{v^2}{2g} = \frac{(36.1\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = 66.5\ \text{m}.$$

If L is the length of the ramp, then $L \sin 15° = 66.5$ m so that $L = (66.5\ \text{m})/\sin 15° = 257$ m. Therefore, the ramp must be about 2.6×10^2 m long if friction is negligible.

(b) The answers do not depend on the mass of the truck. They remain the same if the mass is reduced.

(c) If the speed is decreased, h and L both decrease (note that h is proportional to the square of the speed and that L is proportional to h).

10. We use Eq. 8-17, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).

(a) In the solution to exercise 2 (to which this problem refers), we found $U_i = mgy_i$ = 196J and $U_f = mgy_f$ = 29.0 J (assuming the reference position is at the ground). Since $K_i = 0$ in this case, we have

$$0+196\text{ J} = K_f + 29.0\text{ J}$$

which gives K_f = 167 J and thus leads to

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(167\text{ J})}{2.00\text{ kg}}} = 12.9\text{ m/s}.$$

(b) If we proceed algebraically through the calculation in part (a), we find $K_f = -\Delta U = mgh$ where $h = y_i - y_f$ and is positive-valued. Thus,

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{2gh}$$

as we might also have derived from the equations of Table 2-1 (particularly Eq. 2-16). The fact that the answer is independent of mass means that the answer to part (b) is identical to that of part (a), i.e., $v = 12.9$ m/s .

(c) If $K_i \neq 0$, then we find $K_f = mgh + K_i$ (where K_i is necessarily positive-valued). This represents a larger value for K_f than in the previous parts, and thus leads to a larger value for v.

11. (a) If K_i is the kinetic energy of the flake at the edge of the bowl, K_f is its kinetic energy at the bottom, U_i is the gravitational potential energy of the flake-Earth system with the flake at the top, and U_f is the gravitational potential energy with it at the bottom, then $K_f + U_f = K_i + U_i$.

Taking the potential energy to be zero at the bottom of the bowl, then the potential energy at the top is $U_i = mgr$ where $r = 0.220$ m is the radius of the bowl and m is the mass of the flake. $K_i = 0$ since the flake starts from rest. Since the problem asks for the speed at the bottom, we write $\frac{1}{2}mv^2$ for K_f. Energy conservation leads to

$$W_g = \vec{F}_g \cdot \vec{d} = mgh = mgL\,(1-\cos\theta)\ .$$

The speed is $v = \sqrt{2gr} = 2.08\ \text{m/s}$.

(b) Since the expression for speed does not contain the mass of the flake, the speed would be the same, 2.08 m/s, regardless of the mass of the flake.

(c) The final kinetic energy is given by $K_f = K_i + U_i - U_f$. Since K_i is greater than before, K_f is greater. This means the final speed of the flake is greater.

12. We use Eq. 8-18, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).

(a) In the solution to Problem 4 we found $\Delta U = mgL$ as it goes to the highest point. Thus, we have

$$\Delta K + \Delta U = 0$$
$$K_{\text{top}} - K_0 + mgL = 0$$

which, upon requiring $K_{\text{top}} = 0$, gives $K_0 = mgL$ and thus leads to

$$v_0 = \sqrt{\frac{2K_0}{m}} = \sqrt{2gL} = \sqrt{2(9.80 \text{ m/s}^2)(0.452 \text{ m})} = 2.98 \text{ m/s} .$$

(b) We also found in the Problem 4 that the potential energy change is $\Delta U = -mgL$ in going from the initial point to the lowest point (the bottom). Thus,

$$\Delta K + \Delta U = 0$$
$$K_{\text{bottom}} - K_0 - mgL = 0$$

which, with $K_0 = mgL$, leads to $K_{\text{bottom}} = 2mgL$. Therefore,

$$v_{\text{bottom}} = \sqrt{\frac{2K_{\text{bottom}}}{m}} = \sqrt{4gL} = \sqrt{4(9.80 \text{ m/s}^2)(0.452 \text{ m})} = 4.21 \text{ m/s} .$$

(c) Since there is no change in height (going from initial point to the rightmost point), then $\Delta U = 0$, which implies $\Delta K = 0$. Consequently, the speed is the same as what it was initially,

$$v_{\text{right}} = v_0 = 2.98 \text{ m/s} .$$

(d) It is evident from the above manipulations that the results do not depend on mass. Thus, a different mass for the ball must lead to the same results.

13. We use Eq. 8-17, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).

(a) In Problem 4, we found $U_A = mgh$ (with the reference position at C). Referring again to Fig. 8-33, we see that this is the same as U_0 which implies that $K_A = K_0$ and thus that

$$v_A = v_0 = 17.0 \text{ m/s}.$$

(b) In the solution to Problem 4, we also found $U_B = mgh/2$. In this case, we have

$$\begin{aligned} K_0 + U_0 &= K_B + U_B \\ \frac{1}{2}mv_0^2 + mgh &= \frac{1}{2}mv_B^2 + mg\left(\frac{h}{2}\right) \end{aligned}$$

which leads to

$$v_B = \sqrt{v_0^2 + gh} = \sqrt{(17.0 \text{ m/s})^2 + (9.80 \text{ m/s}^2)(42.0 \text{ m})} = 26.5 \text{ m/s}.$$

(c) Similarly,

$$v_C = \sqrt{v_0^2 + 2gh} = \sqrt{(17.0 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(42.0 \text{ m})} = 33.4 \text{ m/s}.$$

(d) To find the "final" height, we set $K_f = 0$. In this case, we have

$$\begin{aligned} K_0 + U_0 &= K_f + U_f \\ \frac{1}{2}mv_0^2 + mgh &= 0 + mgh_f \end{aligned}$$

which yields $h_f = h + \dfrac{v_0^2}{2g} = 42.0 \text{ m} + \dfrac{(17.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 56.7 \text{ m}.$

(e) It is evident that the above results do not depend on mass. Thus, a different mass for the coaster must lead to the same results.

14. We use Eq. 8-18, representing the conservation of mechanical energy. We choose the reference position for computing U to be at the ground below the cliff; it is also regarded as the "final" position in our calculations.

(a) Using Eq. 8-9, the initial potential energy is given by $U_i = mgh$ where $h = 12.5$ m and $m = 1.50$ kg. Thus, we have

$$K_i + U_i = K_f + U_f$$
$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv^2 + 0$$

which leads to the speed of the snowball at the instant before striking the ground:

$$v = \sqrt{\frac{2}{m}\left(\frac{1}{2}mv_i^2 + mgh\right)} = \sqrt{v_i^2 + 2gh}$$

where $v_i = 14.0$ m/s is the magnitude of its initial velocity (not just one component of it). Thus we find $v = 21.0$ m/s.

(b) As noted above, v_i is the magnitude of its initial velocity and not just one component of it; therefore, there is no dependence on launch angle. The answer is again 21.0 m/s.

(c) It is evident that the result for v in part (a) does not depend on mass. Thus, changing the mass of the snowball does not change the result for v.

15. We take the reference point for gravitational potential energy at the position of the marble when the spring is compressed.

(a) The gravitational potential energy when the marble is at the top of its motion is $U_g = mgh$, where h = 20 m is the height of the highest point. Thus,

$$U_g = \left(5.0\times10^{-3}\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right)\left(20\ \text{m}\right) = 0.98\ \text{J}.$$

(b) Since the kinetic energy is zero at the release point and at the highest point, then conservation of mechanical energy implies $\Delta U_g + \Delta U_s = 0$, where ΔU_s is the change in the spring's elastic potential energy. Therefore, $\Delta U_s = -\Delta U_g = -0.98$ J.

(c) We take the spring potential energy to be zero when the spring is relaxed. Then, our result in the previous part implies that its initial potential energy is U_s = 0.98 J. This must be $\frac{1}{2}kx^2$, where k is the spring constant and x is the initial compression. Consequently,

$$k = \frac{2U_s}{x^2} = \frac{2(0.98\ \text{J})}{(0.080\ \text{m})^2} = 3.1\times10^2\ \text{N/m} = 3.1\ \text{N/cm}.$$

16. We use Eq. 8-18, representing the conservation of mechanical energy. The reference position for computing U is the lowest point of the swing; it is also regarded as the "final" position in our calculations.

(a) In the solution to problem 7, we found $U = mgL(1 - \cos\theta)$ at the position shown in Fig. 8-34 (which we consider to be the initial position). Thus, we have

$$K_i + U_i = K_f + U_f$$

$$0 + mgL(1 - \cos\theta) = \frac{1}{2}mv^2 + 0$$

which leads to

$$v = \sqrt{\frac{2mgL(1 - \cos\theta)}{m}} = \sqrt{2gL(1 - \cos\theta)}.$$

Plugging in $L = 2.00$ m and $\theta = 30.0°$ we find $v = 2.29$ m/s.

(b) It is evident that the result for v does not depend on mass. Thus, a different mass for the ball must not change the result.

17. We use Eq. 8-18, representing the conservation of mechanical energy (which neglects friction and other dissipative effects). The reference position for computing U (and height h) is the lowest point of the swing; it is also regarded as the "final" position in our calculations.

(a) Careful examination of the figure leads to the trigonometric relation $h = L - L\cos\theta$ when the angle is measured from vertical as shown. Thus, the gravitational potential energy is $U = mgL(1 - \cos\theta_0)$ at the position shown in Fig. 8-34 (the initial position). Thus, we have

$$K_0 + U_0 = K_f + U_f$$

$$\frac{1}{2}mv_0^2 + mgL\left(1 - \cos\theta_0\right) = \frac{1}{2}mv^2 + 0$$

which leads to

$$v = \sqrt{\frac{2}{m}\left[\frac{1}{2}mv_0^2 + mgL(1-\cos\theta_0)\right]} = \sqrt{v_0^2 + 2gL(1-\cos\theta_0)}$$

$$= \sqrt{(8.00\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(1.25\text{ m})(1-\cos 40°)} = 8.35\text{ m/s}.$$

(b) We look for the initial speed required to barely reach the horizontal position — described by $v_h = 0$ and $\theta = 90°$ (or $\theta = -90°$, if one prefers, but since $\cos(-\phi) = \cos\phi$, the sign of the angle is not a concern).

$$K_0 + U_0 = K_h + U_h$$

$$\frac{1}{2}mv_0^2 + mgL\left(1 - \cos\theta_0\right) = 0 + mgL$$

which yields

$$v_0 = \sqrt{2gL\cos\theta_0} = \sqrt{2(9.80\text{ m/s}^2)(1.25\text{ m})\cos 40°} = 4.33\text{ m/s}.$$

(c) For the cord to remain straight, then the centripetal force (at the top) must be (at least) equal to gravitational force:

$$\frac{mv_t^2}{r} = mg \Rightarrow mv_t^2 = mgL$$

where we recognize that $r = L$. We plug this into the expression for the kinetic energy (at the top, where $\theta = 180°$).

$$K_0 + U_0 = K_t + U_t$$

$$\frac{1}{2}mv_0^2 + mgL\left(1 - \cos\theta_0\right) = \frac{1}{2}mv_t^2 + mg\left(1 - \cos 180°\right)$$

$$\frac{1}{2}mv_0^2 + mgL\left(1 - \cos\theta_0\right) = \frac{1}{2}(mgL) + mg(2L)$$

which leads to

$$v_0 = \sqrt{gL(3+2\cos\theta_0)} = \sqrt{(9.80 \text{ m/s}^2)(1.25 \text{ m})(3+2\cos 40°)} = 7.45 \text{ m/s}.$$

(d) The more initial potential energy there is, the less initial kinetic energy there needs to be, in order to reach the positions described in parts (b) and (c). Increasing θ_0 amounts to increasing U_0, so we see that a greater value of θ_0 leads to smaller results for v_0 in parts (b) and (c).

18. We place the reference position for evaluating gravitational potential energy at the relaxed position of the spring. We use x for the spring's compression, measured positively downwards (so $x > 0$ means it is compressed).

(a) With $x = 0.190$ m, Eq. 7-26 gives

$$W_s = -\frac{1}{2}kx^2 = -7.22 \text{ J} \approx -7.2 \text{ J}$$

for the work done by the spring force. Using Newton's third law, we see that the work done on the spring is 7.2 J.

(b) As noted above, $W_s = -7.2$ J.

(c) Energy conservation leads to

$$K_i + U_i = K_f + U_f$$
$$mgh_0 = -mgx + \frac{1}{2}kx^2$$

which (with $m = 0.70$ kg) yields $h_0 = 0.86$ m.

(d) With a new value for the height $h_0' = 2h_0 = 1.72$ m, we solve for a new value of x using the quadratic formula (taking its positive root so that $x > 0$).

$$mgh_0' = -mgx + \frac{1}{2}kx^2 \;\Rightarrow\; x = \frac{mg + \sqrt{(mg)^2 + 2mgkh_0'}}{k}$$

which yields $x = 0.26$ m.

19. (a) At Q the block (which is in circular motion at that point) experiences a centripetal acceleration v^2/R leftward. We find v^2 from energy conservation:

$$K_P + U_P = K_Q + U_Q$$
$$0 + mgh = \frac{1}{2}mv^2 + mgR$$

Using the fact that $h = 5R$, we find $mv^2 = 8mgR$. Thus, the horizontal component of the net force on the block at Q is

$$F = mv^2/R = 8mg = 8(0.032\text{ kg})(9.8\text{ m/s}^2) = 2.5\text{ N}.$$

and points left (in the same direction as $\vec{a}$).

(b) The downward component of the net force on the block at Q is the downward force of gravity

$$F = mg = (0.032\text{ kg})(9.8\text{ m/s}^2) = 0.31\text{ N}.$$

(c) To barely make the top of the loop, the centripetal force there must equal the force of gravity:

$$\frac{mv_t^2}{R} = mg \Rightarrow mv_t^2 = mgR$$

This requires a different value of h than was used above.

$$K_P + U_P = K_t + U_t$$
$$0 + mgh = \frac{1}{2}mv_t^2 + mgh_t$$
$$mgh = \frac{1}{2}(mgR) + mg(2R)$$

Consequently, $h = 2.5R = (2.5)(0.12\text{ m}) = 0.30\text{ m}$.

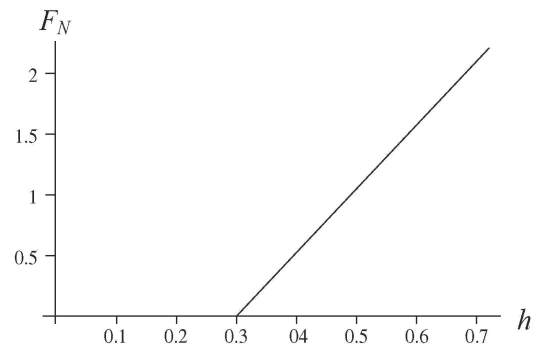
(d) The normal force F_N, for speeds v_t greater than $\sqrt{gR}$ (which are the only possibilities for non-zero F_N — see the solution in the previous part), obeys

$$F_N = \frac{mv_t^2}{R} - mg$$

from Newton's second law. Since v_t^2 is related to h by energy conservation

$$K_P + U_P = K_t + U_t \Rightarrow gh = \frac{1}{2}v_t^2 + 2gR$$

then the normal force, as a function for h (so long as $h \geq 2.5R$ — see solution in previous part), becomes



$$F_N = \frac{2mgh}{R} - 5mg$$

Thus, the graph for $h \geq 2.5R$ consists of a straight line of positive slope $2mg/R$ (which can be set to some convenient values for graphing purposes).

Note that for $h \leq 2.5R$, the normal force is zero.

20. (a) With energy in Joules and length in meters, we have

$$\Delta U = U(x) - U(0) = -\int_0^x (6x' - 12)\,dx' \ .$$

Therefore, with U (0) = 27 J, we obtain $U(x)$ (written simply as U) by integrating and rearranging:

$$U = 27 + 12x - 3x^2 \ .$$

(b) We can maximize the above function by working through the $dU/dx = 0$ condition, or we can treat this as a force equilibrium situation — which is the approach we show.

$$F = 0 \Rightarrow 6x_{eq} - 12 = 0$$

Thus, x_{eq} = 2.0 m, and the above expression for the potential energy becomes U = 39 J.

(c) Using the quadratic formula or using the polynomial solver on an appropriate calculator, we find the negative value of x for which U = 0 to be x = –1.6 m.

(d) Similarly, we find the positive value of x for which U = 0 to be x = 5.6 m

21. (a) As the string reaches its lowest point, its original potential energy $U = mgL$ (measured relative to the lowest point) is converted into kinetic energy. Thus,

$$mgL = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gL} \; .$$

With $L = 1.20$ m we obtain $v = 4.85 \text{ m/s}$.

(b) In this case, the total mechanical energy is shared between kinetic $\frac{1}{2}mv_b^2$ and potential mgy_b. We note that $y_b = 2r$ where $r = L - d = 0.450$ m. Energy conservation leads to

$$mgL = \frac{1}{2}mv_b^2 + mgy_b$$

which yields $v_b = \sqrt{2gL - 2g(2r)} = 2.42 \text{ m/s}$.

22. We denote m as the mass of the block, $h = 0.40$ m as the height from which it dropped (measured from the relaxed position of the spring), and x the compression of the spring (measured downward so that it yields a positive value). Our reference point for the gravitational potential energy is the initial position of the block. The block drops a total distance $h + x$, and the final gravitational potential energy is $-mg(h + x)$. The spring potential energy is $\frac{1}{2}kx^2$ in the final situation, and the kinetic energy is zero both at the beginning and end. Since energy is conserved

$$K_i + U_i = K_f + U_f$$
$$0 = -mg(h+x) + \frac{1}{2}kx^2$$

which is a second degree equation in x. Using the quadratic formula, its solution is

$$x = \frac{mg \pm \sqrt{(mg)^2 + 2mghk}}{k}\ .$$

Now $mg = 19.6$ N, $h = 0.40$ m, and $k = 1960\ \text{N/m}$, and we choose the positive root so that $x > 0$.

$$x = \frac{19.6 + \sqrt{19.6^2 + 2(19.6)(0.40)(1960)}}{1960} = 0.10\ \text{m}\ .$$

23. Since time does not directly enter into the energy formulations, we return to Chapter 4 (or Table 2-1 in Chapter 2) to find the change of height during this t = 6.0 s flight.

$$\Delta y = v_{0_y} t - \frac{1}{2} g t^2$$

This leads to $\Delta y = -32 \text{ m}$. Therefore $\Delta U = mg\Delta y = -318 \text{ J} \approx -3.2 \times 10^{-2} \text{ J}$.

24. From Chapter 4, we know the height h of the skier's jump can be found from $v_y^2 = 0 = v_{0\,y}^2 - 2gh$ where $v_{0\,y} = v_0 \sin 28°$ is the upward component of the skier's "launch velocity." To find v_0 we use energy conservation.

(a) The skier starts at rest y = 20 m above the point of "launch" so energy conservation leads to

$$mgy = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gy} = 20 \text{ m/s}$$

which becomes the initial speed v_0 for the launch. Hence, the above equation relating h to v_0 yields

$$h = \frac{(v_0 \sin 28°)^2}{2g} = 4.4 \text{ m} .$$

(b) We see that all reference to mass cancels from the above computations, so a new value for the mass will yield the same result as before.

25. (a) To find out whether or not the vine breaks, it is sufficient to examine it at the moment Tarzan swings through the lowest point, which is when the vine — if it didn't break — would have the greatest tension. Choosing upward positive, Newton's second law leads to

$$T - mg = m\frac{v^2}{r}$$

where r = 18.0 m and $m = W/g = 688/9.8 = 70.2$ kg. We find the v^2 from energy conservation (where the reference position for the potential energy is at the lowest point).

$$mgh = \frac{1}{2}mv^2 \Rightarrow v^2 = 2gh$$

where h = 3.20 m. Combining these results, we have

$$T = mg + m\frac{2gh}{r} = mg\left(1 + \frac{2h}{r}\right)$$

which yields 933 N. Thus, the vine does not break.

(b) Rounding to an appropriate number of significant figures, we see the maximum tension is roughly 9.3×10^2 N.

26. (a) We take the reference point for gravitational energy to be at the lowest point of the swing. Let θ be the angle measured from vertical. Then the height y of the pendulum "bob" (the object at the end of the pendulum, which i this problem is the stone) is given by $L(1 - \cos\theta) = y$. Hence, the gravitational potential energy is

$$mg\,y = mgL(1 - \cos\theta).$$

When $\theta = 0°$ (the string at its lowest point) we are told that its speed is 8.0 m/s; its kinetic energy there is therefore 64 J (using Eq. 7-1). At $\theta = 60°$ its mechanical energy is

$$E_{\text{mech}} = \frac{1}{2}\,mv^2 + mgL(1 - \cos\theta)\,.$$

Energy conservation (since there is no friction) requires that this be equal to 64 J. Solving for the speed, we find v = 5.0 m/s.

(b) We now set the above expression again equal to 64 J (with θ being the unknown) but with zero speed (which gives the condition for the maximum point, or "turning point" that it reaches). This leads to $\theta_{\max}$ = 79°.

(c) As observed in our solution to part (a), the total mechanical energy is 64 J.

27. We convert to SI units and choose upward as the $+y$ direction. Also, the relaxed position of the top end of the spring is the origin, so the initial compression of the spring (defining an equilibrium situation between the spring force and the force of gravity) is $y_0 = -0.100$ m and the additional compression brings it to the position $y_1 = -0.400$ m.

(a) When the stone is in the equilibrium ($a = 0$) position, Newton's second law becomes

$$\begin{aligned} \vec{F}_{\text{net}} &= ma \\ F_{\text{spring}} - mg &= 0 \\ -k(-0.100) - (8.00)(9.8) &= 0 \end{aligned}$$

where Hooke's law (Eq. 7-21) has been used. This leads to a spring constant equal to k = 784 N/m.

(b) With the additional compression (and release) the acceleration is no longer zero, and the stone will start moving upwards, turning some of its elastic potential energy (stored in the spring) into kinetic energy. The amount of elastic potential energy at the moment of release is, using Eq. 8-11,

$$U = \frac{1}{2}\,ky_1^2 = \frac{1}{2}(784)(-0.400)^2 = 62.7 \text{ J.}$$

(c) Its maximum height y_2 is beyond the point that the stone separates from the spring (entering free-fall motion). As usual, it is characterized by having (momentarily) zero speed. If we choose the y_1 position as the reference position in computing the gravitational potential energy, then

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ 0 + \frac{1}{2}ky_1^2 &= 0 + mgh \end{aligned}$$

where $h = y_2 - y_1$ is the height above the release point. Thus, mgh (the gravitational potential energy) is seen to be equal to the previous answer, 62.7 J, and we proceed with the solution in the next part.

(d) We find $h = ky_1^2/2mg = 0.800$ m, or 80.0 cm.

28. We take the original height of the box to be the $y = 0$ reference level and observe that, in general, the height of the box (when the box has moved a distance d downhill) is $y = -d \sin 40°$.

(a) Using the conservation of energy, we have

$$K_i + U_i = K + U \Rightarrow 0 + 0 = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kd^2.$$

Therefore, with $d = 0.10$ m, we obtain $v = 0.81$ m/s.

(b) We look for a value of $d \neq 0$ such that $K = 0$.

$$K_i + U_i = K + U \Rightarrow 0 + 0 = 0 + mgy + \frac{1}{2}kd^2.$$

Thus, we obtain $mgd \sin 40° = \frac{1}{2}kd^2$ and find $d = 0.21$ m.

(c) The uphill force is caused by the spring (Hooke's law) and has magnitude $kd = 25.2$ N. The downhill force is the component of gravity $mg \sin 40° = 12.6$ N. Thus, the net force on the box is (25.2 – 12.6) N = 12.6 N uphill, with $a = F/m =$(12.6 N)/(2.0 kg) = 6.3 m/s^2.

(d) The acceleration is up the incline.

29. The reference point for the gravitational potential energy U_g (and height h) is at the block when the spring is maximally compressed. When the block is moving to its highest point, it is first accelerated by the spring; later, it separates from the spring and finally reaches a point where its speed v_f is (momentarily) zero. The x axis is along the incline, pointing uphill (so x_0 for the initial compression is negative-valued); its origin is at the relaxed position of the spring. We use SI units, so $k = 1960$ N/m and $x_0 = -0.200$ m.

(a) The elastic potential energy is $\frac{1}{2}kx_0^2 = 39.2\text{ J}$.

(b) Since initially $U_g = 0$, the change in U_g is the same as its final value mgh where m = 2.00 kg. That this must equal the result in part (a) is made clear in the steps shown in the next part. Thus, $\Delta U_g = U_g = 39.2$ J.

(c) The principle of mechanical energy conservation leads to

$$K_0 + U_0 = K_f + U_f$$
$$0 + \frac{1}{2}kx_0^2 = 0 + mgh$$

which yields $h = 2.00$ m. The problem asks for the distance *along the incline*, so we have $d = h/\sin 30° = 4.00$ m.

30. From the slope of the graph, we find the spring constant

$$k = \frac{\Delta F}{\Delta x} = 0.10\,\mathrm{N/cm} = 10\,\mathrm{N/m}.$$

(a) Equating the potential energy of the compressed spring to the kinetic energy of the cork at the moment of release, we have

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \Rightarrow v = x\sqrt{\frac{k}{m}}$$

which yields v = 2.8 m/s for m = 0.0038 kg and x = 0.055 m.

(b) The new scenario involves some potential energy at the moment of release. With d = 0.015 m, energy conservation becomes

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{2}kd^2 \Rightarrow v = \sqrt{\frac{k}{m}\left(x^2 - d^2\right)}$$

which yields v = 2.7 m/s.

31. We refer to its starting point as A, the point where it first comes into contact with the spring as B, and the point where the spring is compressed $|x| = 0.055$ m as C. Point C is our reference point for computing gravitational potential energy. Elastic potential energy (of the spring) is zero when the spring is relaxed. Information given in the second sentence allows us to compute the spring constant. From Hooke's law, we find

$$k = \frac{F}{x} = \frac{270\ \text{N}}{0.02\ \text{m}} = 1.35\times10^4\ \text{N/m}\ .$$

(a) The distance between points A and B is $\vec{F}_g$ and we note that the total sliding distance $\ell + |x|$ is related to the initial height h of the block (measured relative to C) by

$$\frac{h}{\ell + |x|} = \sin\,\theta$$

where the incline angle θ is 30°. Mechanical energy conservation leads to

$$K_A + U_A = K_C + U_C$$
$$0 + mgh = 0 + \frac{1}{2}kx^2$$

which yields

$$h = \frac{kx^2}{2mg} = \frac{\left(1.35\times10^4\ \text{N/m}\right)\left(0.055\ \text{m}\right)^2}{2\left(12\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right)} = 0.174\ \text{m}\ .$$

Therefore,

$$\ell + |x| = \frac{h}{\sin 30°} = \frac{0.174\ \text{m}}{\sin 30°} = 0.35\ \text{m}\ .$$

(b) From this result, we find $\ell = 0.35 - 0.055 = 0.29\ \text{m}$, which means that $\Delta y = -\ell\sin\theta = -0.15\ \text{m}$ in sliding from point A to point B. Thus, Eq. 8-18 gives

$$\Delta K + \Delta U = 0$$
$$\frac{1}{2}mv_B^2 + mg\Delta h = 0$$

which yields $v_B = \sqrt{-2g\Delta h} = \sqrt{-(9.8)(-0.15)} = 1.7\ \text{m/s}$.

32. The work required is the change in the gravitational potential energy as a result of the chain being pulled onto the table. Dividing the hanging chain into a large number of infinitesimal segments, each of length dy, we note that the mass of a segment is $(m/L)\,dy$ and the change in potential energy of a segment when it is a distance $|y|$ below the table top is

$$dU = (m/L)g|y|\,dy = -(m/L)gy\,dy$$

since y is negative-valued (we have $+y$ upward and the origin is at the tabletop). The total potential energy change is

$$\Delta U = -\frac{mg}{L}\int_{-L/4}^{0} y\,dy = \frac{1}{2}\frac{mg}{L}(L/4)^2 = mgL/32\,.$$

The work required to pull the chain onto the table is therefore

$$W = \Delta U = mgL/32 = (0.012\text{ kg})(9.8\text{ m/s}^2)(0.28\text{ m})/32 = 0.0010\text{ J}.$$

33. All heights h are measured from the lower end of the incline (which is our reference position for computing gravitational potential energy mgh). Our x axis is along the incline, with $+x$ being uphill (so spring compression corresponds to $x > 0$) and its origin being at the relaxed end of the spring. The height that corresponds to the canister's initial position (with spring compressed amount $x = 0.200$ m) is given by $h_1 = (D + x) \sin \theta$, where $\theta = 37°$.

(a) Energy conservation leads to

$$K_1 + U_1 = K_2 + U_2 \quad \Rightarrow \quad 0 + mg(D+x)\sin\theta + \frac{1}{2}kx^2 = \frac{1}{2}mv_2^2 + mgD\sin\theta$$

which yields, using the data $m = 2.00$ kg and $k = 170$N/m,

$$v_2 = \sqrt{2gx\sin\theta + kx^2/m} = 2.40\,\text{m/s}\,.$$

(b) In this case, energy conservation leads to

$$K_1 + U_1 = K_3 + U_3$$
$$0 + mg(D+x)\sin\theta + \frac{1}{2}kx^2 = \frac{1}{2}mv_3^2 + 0$$

which yields $v_3 = \sqrt{2g(D+x)\sin\theta + kx^2/m} = 4.19$ m/s.

34. The distance the marble travels is determined by its initial speed (and the methods of Chapter 4), and the initial speed is determined (using energy conservation) by the original compression of the spring. We denote h as the height of the table, and x as the horizontal distance to the point where the marble lands. Then $x = v_0\, t$ and $h = \frac{1}{2}gt^2$ (since the vertical component of the marble's "launch velocity" is zero). From these we find $x = v_0\sqrt{2h/g}$. We note from this that the distance to the landing point is directly proportional to the initial speed. We denote v_{01} be the initial speed of the first shot and $D_1 = (2.20 - 0.27)$ m = 1.93 m be the horizontal distance to its landing point; similarly, v_{02} is the initial speed of the second shot and D = 2.20 m is the horizontal distance to its landing spot. Then

$$\frac{v_{02}}{v_{01}} = \frac{D}{D_1} \quad \Rightarrow \quad v_{02} = \frac{D}{D_1} v_{01}$$

When the spring is compressed an amount ℓ, the elastic potential energy is $\frac{1}{2}k\ell^2$. When the marble leaves the spring its kinetic energy is $\frac{1}{2}mv_0^2$. Mechanical energy is conserved: $\frac{1}{2}mv_0^2 = \frac{1}{2}k\ell^2$, and we see that the initial speed of the marble is directly proportional to the original compression of the spring. If ℓ_1 is the compression for the first shot and ℓ_2 is the compression for the second, then $v_{02} = (\ell_2/\ell_1)v_{01}$. Relating this to the previous result, we obtain

$$\ell_2 = \frac{D}{D_1}\ell_1 = \left(\frac{2.20 \text{ m}}{1.93 \text{ m}}\right)(1.10 \text{ cm}) = 1.25 \text{ cm}.$$

35. Consider a differential element of length dx at a distance x from one end (the end which remains stuck) of the cord. As the cord turns vertical, its change in potential energy is given by

$$dU = -(\lambda dx)gx$$

where $\lambda = m/h$ is the mass/unit length and the negative sign indicates that the potential energy decreases. Integrating over the entire length, we obtain the total change in the potential energy:

$$\Delta U = \int dU = -\int_0^h \lambda gx dx = -\frac{1}{2}\lambda gh^2 = -\frac{1}{2}mgh.$$

With m=15 g and h = 25 cm, we have $\Delta U = -0.018$ J.

36. Let $\vec{F}_N$ be the normal force of the ice on him and m is his mass. The net inward force is $mg \cos \theta - F_N$ and, according to Newton's second law, this must be equal to mv^2/R, where v is the speed of the boy. At the point where the boy leaves the ice $F_N = 0$, so $g \cos \theta = v^2/R$. We wish to find his speed. If the gravitational potential energy is taken to be zero when he is at the top of the ice mound, then his potential energy at the time shown is

$$U = -mgR(1 - \cos \theta).$$

He starts from rest and his kinetic energy at the time shown is $\frac{1}{2}mv^2$. Thus conservation of energy gives

$$0 = \tfrac{1}{2}mv^2 - mgR(1 - \cos\theta),$$

or $v^2 = 2gR(1 - \cos \theta)$. We substitute this expression into the equation developed from the second law to obtain $g \cos \theta = 2g(1 - \cos \theta)$. This gives $\cos \theta = 2/3$. The height of the boy above the bottom of the mound is

$$h = R \cos \theta = 2R/3 = 2(13.8 \text{ m})/3 = 9.20 \text{ m}.$$

37. (a) The (final) elastic potential energy is

$$U = \frac{1}{2}\,kx^2 = \frac{1}{2}\,(431\ \text{N/m})(0.210\ \text{m})^2 = 9.50\ \text{J}.$$

Ultimately this must come from the original (gravitational) energy in the system mgy (where we are measuring y from the lowest "elevation" reached by the block, so $y = (d + x)\sin(30°)$. Thus,

$$mg(d + x)\sin(30°) = 9.50\ \text{J} \qquad \Rightarrow \qquad d = 0.396\ \text{m}.$$

(b) The block is still accelerating (due to the component of gravity along the incline, $mg\sin(30°)$) for a few moments after coming into contact with the spring (which exerts the Hooke's law force kx), until the Hooke's law force is strong enough to cause the block to being decelerating. This point is reached when

$$kx = mg\sin 30°$$

which leads to $x = 0.0364$ m $= 3.64$ cm; this is long before the block finally stops (36.0 cm before it stops).

38. (a) The force at the equilibrium position $r = r_{\text{eq}}$ is

$$F = -\left.\frac{dU}{dr}\right|_{r=r_{\text{eq}}} = 0 \quad \Rightarrow \quad -\frac{12A}{r_{\text{eq}}^{13}} + \frac{6B}{r_{\text{eq}}^{7}} = 0$$

which leads to the result

$$r_{\text{eq}} = \left(\frac{2A}{B}\right)^{\frac{1}{6}} = 1.12\left(\frac{A}{B}\right)^{\frac{1}{6}}.$$

(b) This defines a minimum in the potential energy curve (as can be verified either by a graph or by taking another derivative and verifying that it is concave upward at this point), which means that for values of r slightly smaller than r_{eq} the slope of the curve is negative (so the force is positive, repulsive).

(c) And for values of r slightly larger than r_{eq} the slope of the curve must be positive (so the force is negative, attractive).

39. From Fig. 8-50, we see that at x = 4.5 m, the potential energy is U_1 = 15 J. If the speed is v = 7.0 m/s, then the kinetic energy is

$$K_1 = mv^2/2 = (0.90\text{ kg})(7.0\text{ m/s})^2/2 = 22\text{ J}.$$

The total energy is $E_1 = U_1 + K_1 = (15 + 22)\text{ J} = 37\text{ J}$.

(a) At x = 1.0 m, the potential energy is U_2 = 35 J. From energy conservation, we have K_2=2.0 J > 0. This means that the particle can reach there with a corresponding speed

$$v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(2.0\text{ J})}{0.90\text{ kg}}} = 2.1\text{ m/s}.$$

(b) The force acting on the particle is related to the potential energy by the negative of the slope:

$$F_x = -\frac{\Delta U}{\Delta x}$$

From the figure we have $F_x = -\dfrac{35\text{ J} - 15\text{ J}}{2\text{ m} - 4\text{ m}} = +10\text{ N}$.

(c) Since the magnitude $F_x > 0$, the force points in the $+x$ direction.

(d) At x = 7.0m, the potential energy is U_3 = 45 J which exceeds the initial total energy E_1. Thus, the particle can never reach there. At the turning point, the kinetic energy is zero. Between x = 5 and 6 m, the potential energy is given by

$$U(x) = 15 + 30(x-5), \qquad 5 \le x \le 6.$$

Thus, the turning point is found by solving $37 = 15 + 30(x-5)$, which yields x = 5.7 m.

(e) At x =5.0 m, the force acting on the particle is

$$F_x = -\frac{\Delta U}{\Delta x} = -\frac{(45-15)\text{ J}}{(6-5)\text{ m}} = -30\text{ N}$$

The magnitude is $|F_x| = 30\text{ N}$.

(f) The fact that $F_x < 0$ indicated that the force points in the $-x$ direction.

40. In this problem, the mechanical energy (the sum of K and U) remains constant as the particle moves.

(a) Since mechanical energy is conserved, $U_B + K_B = U_A + K_A$, the kinetic energy of the particle in region A ($3.00\text{ m} \le x \le 4.00\text{ m}$) is

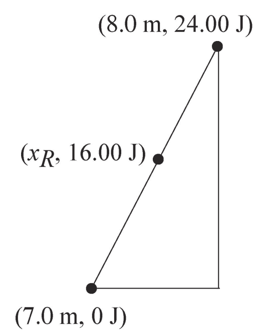
$$K_A = U_B - U_A + K_B = 12.0\text{ J} - 9.00\text{ J} + 4.00\text{ J} = 7.00\text{ J}.$$

With $K_A = mv_A^2/2$, the speed of the particle at $x = 3.5\text{ m}$ (within region A) is

$$v_A = \sqrt{\frac{2K_A}{m}} = \sqrt{\frac{2(7.00\text{ J})}{0.200\text{ kg}}} = 8.37\text{ m/s}.$$

(b) At $x = 6.5\text{ m}$, $U = 0$ and $K = U_B + K_B = 12.0\text{ J} + 4.00\text{ J} = 16.0\text{ J}$ by mechanical energy conservation. Therefore, the speed at this point is

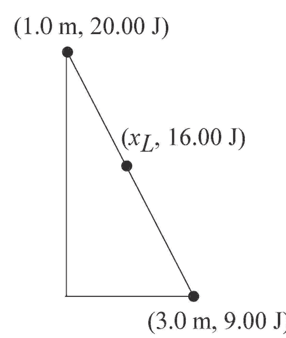
$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(16.0\text{ J})}{0.200\text{ kg}}} = 12.6\text{ m/s}.$$



(c) At the turning point, the speed of the particle is zero. Let the position of the right turning point be x_R. From the figure shown on the right, we find x_R to be

$$\frac{16.00\text{ J} - 0}{x_R - 7.00\text{ m}} = \frac{24.00\text{ J} - 16.00\text{ J}}{8.00\text{ m} - x_R} \quad \Rightarrow x_R = 7.67\text{ m}.$$

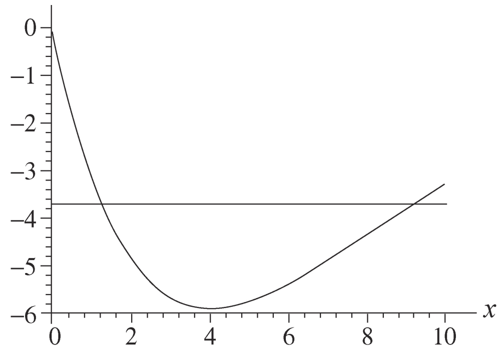
(d) Let the position of the left turning point be x_L. From the figure shown, we find x_L to be



$$\frac{16.00\text{ J} - 20.00\text{ J}}{x_L - 1.00\text{ m}} = \frac{9.00\text{ J} - 16.00\text{ J}}{3.00\text{ m} - x_L} \quad \Rightarrow x_L = 1.73\text{ m}.$$

41. (a) The energy at $x = 5.0$ m is $E = K + U = 2.0 \text{ J} - 5.7 \text{ J} = -3.7$ J.

(b) A plot of the potential energy curve (SI units understood) and the energy E (the horizontal line) is shown for $0 \le x \le 10$ m.



(c) The problem asks for a graphical determination of the turning points, which are the points on the curve corresponding to the total energy computed in part (a). The result for the smallest turning point (determined, to be honest, by more careful means) is $x = 1.3$ m.

(d) And the result for the largest turning point is $x = 9.1$ m.

(e) Since $K = E - U$, then maximizing K involves finding the minimum of U. A graphical determination suggests that this occurs at $x = 4.0$ m, which plugs into the expression $E - U = -3.7 - (-4xe^{-x/4})$ to give $K = 2.16 \text{ J} \approx 2.2 \text{ J}$. Alternatively, one can measure from the graph from the minimum of the U curve up to the level representing the total energy E and thereby obtain an estimate of K at that point.

(f) As mentioned in the previous part, the minimum of the U curve occurs at $x = 4.0$ m.

(g) The force (understood to be in newtons) follows from the potential energy, using Eq. 8-20 (and Appendix E if students are unfamiliar with such derivatives).

$$F = \frac{dU}{dx} = (4 - x)e^{-x/4}$$

(h) This revisits the considerations of parts (d) and (e) (since we are returning to the minimum of $U(x)$) — but now with the advantage of having the analytic result of part (g). We see that the location which produces $F = 0$ is exactly $x = 4.0$ m.

42. Since the velocity is constant, $\vec{a} = 0$ and the horizontal component of the worker's push $F \cos \theta$ (where $\theta = 32°$) must equal the friction force magnitude $f_k = \mu_k F_N$. Also, the vertical forces must cancel, implying

$$W_{\text{applied}} = (8.0\text{N})(0.70\text{m}) = 5.6 \text{ J}$$

which is solved to find $F = 71$ N.

(a) The work done on the block by the worker is, using Eq. 7-7,

$$W = Fd\cos\theta = (71 \text{ N})(9.2 \text{ m})\cos 32° = 5.6\times10^2 \text{ J} .$$

(b) Since $f_k = \mu_k (mg + F \sin \theta)$, we find $\Delta E_{\text{th}} = f_k d = (60\,\text{N})(9.2\,\text{m}) = 5.6\times10^2$ J.

43. (a) Using Eq. 7-8, we have

$$W_{\text{applied}} = (8.0\,\text{N})(0.70\,\text{m}) = 5.6\,\text{J}.$$

(b) Using Eq. 8-31, the thermal energy generated is

$$\Delta E_{\text{th}} = f_k d = (5.0\,\text{N})(0.70\,\text{m}) = 3.5\,\text{J}.$$

44. (a) The work is $W = Fd = (35.0 \text{ N})(3.00 \text{ m}) = 105 \text{ J}$.

(b) The total amount of energy that has gone to thermal forms is (see Eq. 8-31 and Eq. 6-2)

$$\Delta E_{\text{th}} = \mu_k mgd = (0.600)(4.00 \text{ kg})(9.80 \text{ m/s}^2)(3.00 \text{ m}) = 70.6 \text{ J}.$$

If 40.0 J has gone to the block then (70.6 – 40.0) J = 30.6 J has gone to the floor.

(c) Much of the work (105 J) has been "wasted" due to the 70.6 J of thermal energy generated, but there still remains (105 – 70.6) J = 34.4 J which has gone into increasing the kinetic energy of the block. (It has not gone into increasing the potential energy of the block because the floor is presumed to be horizontal.)

45. (a) The work done on the block by the force in the rope is, using Eq. 7-7,

$$W = Fd\cos\theta = (7.68\,\text{N})(4.06\,\text{m})\cos 15.0° = 30.1\,\text{J}.$$

(b) Using f for the magnitude of the kinetic friction force, Eq. 8-29 reveals that the increase in thermal energy is

$$\Delta E_{\text{th}} = fd = (7.42\,\text{N})(4.06\,\text{m}) = 30.1\,\text{J}.$$

(c) We can use Newton's second law of motion to obtain the frictional and normal forces, then use $\mu_k = f/F_N$ to obtain the coefficient of friction. Place the x axis along the path of the block and the y axis normal to the floor. The x and the y component of Newton's second law are

$$\begin{aligned} x:&\quad F\cos\theta - f = 0 \\ y:&\quad F_N + F\sin\theta - mg = 0, \end{aligned}$$

where m is the mass of the block, F is the force exerted by the rope, and θ is the angle between that force and the horizontal. The first equation gives

$$f = F\cos\theta = (7.68\text{ N})\cos 15.0° = 7.42\text{ N}$$

and the second gives

$$F_N = mg - F\sin\theta = (3.57\text{ kg})(9.8\text{ m/s}^2) - (7.68\text{ N})\sin 15.0° = 33.0\text{ N}.$$

Thus,

$$\mu_k = \frac{f}{F_N} = \frac{7.42\text{ N}}{33.0\text{ N}} = 0.225\,.$$

46. Equation 8-33 provides $\Delta E_{th} = -\Delta E_{mec}$ for the energy "lost" in the sense of this problem. Thus,

$$\Delta E_{\text{th}} = \frac{1}{2}m(v_i^2 - v_f^2) + mg(y_i - y_f) = \frac{1}{2}(60\text{ kg})[(24\text{ m/s})^2 - (22\text{ m/s})^2] + (60\text{ kg})(9.8\text{ m/s}^2)(14\text{ m})$$
$$= 1.1\times 10^4\text{ J}.$$

That the angle of 25° is nowhere used in this calculation is indicative of the fact that energy is a scalar quantity.

47. (a) We take the initial gravitational potential energy to be $U_i = 0$. Then the final gravitational potential energy is $U_f = -mgL$, where L is the length of the tree. The change is

$$U_f - U_i = -mgL = -(25\ \text{kg})\left(9.8\ \text{m/s}^2\right)(12\ \text{m}) = -2.9\times10^3\ \text{J}\,.$$

(b) The kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}(25\ \text{kg})(5.6\ \text{m/s})^2 = 3.9\times10^2\ \text{J}$.

(c) The changes in the mechanical and thermal energies must sum to zero. The change in thermal energy is $\Delta E_{\text{th}} = fL$, where f is the magnitude of the average frictional force; therefore,

$$f = -\frac{\Delta K + \Delta U}{L} = -\frac{3.9\times10^2\ \text{J} - 2.9\times10^3\ \text{J}}{12\ \text{m}} = 2.1\times10^2\ \text{N}$$

48. We work this using the English units (with g = 32 ft/s), but for consistency we convert the weight to pounds

$$mg = (9.0)\,\text{oz}\left(\frac{1\,\text{lb}}{16\,\text{oz}}\right) = 0.56\,\text{lb}$$

which implies $m = 0.018\ \text{lb}\cdot\text{s}^2/\text{ft}$ (which can be phrased as 0.018 slug as explained in Appendix D). And we convert the initial speed to feet-per-second

$$v_i = (81.8\,\text{mi/h})\left(\frac{5280\ \text{ft/mi}}{3600\ \text{s/h}}\right) = 120\ \text{ft/s}$$

or a more "direct" conversion from Appendix D can be used. Equation 8-30 provides $\Delta E_{\text{th}} = -\Delta E_{\text{mec}}$ for the energy "lost" in the sense of this problem. Thus,

$$\Delta E_{\text{th}} = \frac{1}{2}m(v_i^2 - v_f^2) + mg(y_i - y_f) = \frac{1}{2}(0.018)(120^2 - 110^2) + 0 = 20\ \text{ft}\cdot\text{lb}.$$

49. We use SI units so m = 0.075 kg. Equation 8-33 provides $\Delta E_{\text{th}} = -\Delta E_{\text{mec}}$ for the energy "lost" in the sense of this problem. Thus,

$$
\begin{aligned}
\Delta E_{\text{th}} &= \frac{1}{2}m(v_i^2 - v_f^2) + mg(y_i - y_f) \\
&= \frac{1}{2}(0.075\text{ kg})[(12\text{ m/s})^2 - (10.5\text{ m/s})^2] + (0.075\text{ kg})(9.8\text{ m/s}^2)(1.1\text{ m} - 2.1\text{ m}) \\
&= 0.53\text{ J}.
\end{aligned}
$$

50. We use Eq. 8-31 to obtain

$$\Delta E_{\text{th}} = f_k d = (10\,\text{N})(5.0\,\text{m}) = 50\,\text{J}$$

and Eq. 7-8 to get

$$W = Fd = (2.0\,\text{N})(5.0\,\text{m}) = 10\,\text{J}.$$

Similarly, Eq. 8-31 gives

$$\begin{aligned} W &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ 10 &= 35 + \Delta U + 50 \end{aligned}$$

which yields $\Delta U = -75$ J. By Eq. 8-1, then, the work done by gravity is $W = -\Delta U = 75$ J.

51. (a) The initial potential energy is

$$U_i = mgy_i = (520 \text{ kg}) \left(9.8 \text{ m/s}^2\right) (300 \text{ m}) = 1.53 \times 10^6 \text{ J}$$

where $+y$ is upward and $y = 0$ at the bottom (so that $U_f = 0$).

(b) Since $f_k = \mu_k F_N = \mu_k mg \cos\theta$ we have $\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta$ from Eq. 8-31. Now, the hillside surface (of length d = 500 m) is treated as an hypotenuse of a 3-4-5 triangle, so $\cos\theta = x/d$ where x = 400 m. Therefore,

$$\Delta E_{\text{th}} = \mu_k mgd \frac{x}{d} = \mu_k mgx = (0.25)(520)(9.8)(400) = 5.1 \times 10^5 \text{ J}.$$

(c) Using Eq. 8-31 (with $W = 0$) we find

$$\begin{aligned} K_f &= K_i + U_i - U_f - \Delta E_{\text{th}} \\ &= 0 + 1.53 \times 10^6 - 0 - 5.1 \times 10^5 \\ &= 0 + 1.02 \times 10^6 \text{ J}. \end{aligned}$$

(d) From $K_f = mv^2/2$, we obtain v = 63 m/s.

52. Energy conservation, as expressed by Eq. 8-33 (with $W = 0$) leads to

$$\Delta E_{\text{th}} = K_i - K_f + U_i - U_f \;\Rightarrow\; f_k d = 0 - 0 + \frac{1}{2}kx^2 - 0$$

$$\Rightarrow\; \mu_k mgd = \frac{1}{2}(200\,\text{N/m})(0.15\,\text{m})^2 \;\Rightarrow\; \mu_k(2.0\,\text{kg})(9.8\,\text{m/s}^2)(0.75\,\text{m}) = 2.25\,\text{J}$$

which yields $\mu_k = 0.15$ as the coefficient of kinetic friction.

53. Since the valley is frictionless, the only reason for the speed being less when it reaches the higher level is the gain in potential energy $\Delta U = mgh$ where h = 1.1 m. Sliding along the rough surface of the higher level, the block finally stops since its remaining kinetic energy has turned to thermal energy $\Delta E_{\text{th}} = f_k d = \mu mgd$, where $\mu = 0.60$. Thus, Eq. 8-33 (with W = 0) provides us with an equation to solve for the distance d:

$$K_i = \Delta U + \Delta E_{\text{th}} = mg\left(h + \mu d\right)$$

where $K_i = mv_i^2/2$ and v_i = 6.0 m/s. Dividing by mass and rearranging, we obtain

$$d = \frac{v_i^2}{2\mu g} - \frac{h}{\mu} = 1.2 \text{ m}.$$

54. (a) An appropriate picture (once friction is included) for this problem is Figure 8-3 in the textbook. We apply equation 8-31, $\Delta E_{\text{th}} = f_k\, d$, and relate initial kinetic energy K_i to the "resting" potential energy U_r:

$$K_i + U_i = f_k d + K_r + U_r \quad \Rightarrow \quad 20.0\text{ J} + 0 = f_k d + 0 + \frac{1}{2}kd^2$$

where f_k = 10.0 N and k = 400 N/m. We solve the equation for d using the quadratic formula or by using the polynomial solver on an appropriate calculator, with d = 0.292 m being the only positive root.

(b) We apply equation 8-31 again and relate U_r to the "second" kinetic energy K_s it has at the unstretched position.

$$K_r + U_r = f_k d + K_s + U_s \quad \Rightarrow \quad \frac{1}{2}kd^2 = f_k d + K_s + 0$$

Using the result from part (a), this yields K_s = 14.2 J.

55. (a) The vertical forces acting on the block are the normal force, upward, and the force of gravity, downward. Since the vertical component of the block's acceleration is zero, Newton's second law requires $F_N = mg$, where m is the mass of the block. Thus $f = \mu_k F_N = \mu_k mg$. The increase in thermal energy is given by $\Delta E_{\text{th}} = fd = \mu_k mgD$, where D is the distance the block moves before coming to rest. Using Eq. 8-29, we have

$$\Delta E_{\text{th}} = (0.25)(3.5\,\text{kg})\left(9.8\,\text{m/s}^2\right)(7.8\,\text{m}) = 67\,\text{J}.$$

(b) The block has its maximum kinetic energy $K_{\max}$ just as it leaves the spring and enters the region where friction acts. Therefore, the maximum kinetic energy equals the thermal energy generated in bringing the block back to rest, 67 J.

(c) The energy that appears as kinetic energy is originally in the form of potential energy in the compressed spring. Thus, $K_{\max} = U_i = \frac{1}{2}kx^2$, where k is the spring constant and x is the compression. Thus,

$$x = \sqrt{\frac{2K_{\max}}{k}} = \sqrt{\frac{2(67\,\text{J})}{640\,\text{N/m}}} = 0.46\,\text{m}.$$

56. We look for the distance along the incline d which is related to the height ascended by $\Delta h = d \sin\theta$. By a force analysis of the style done in Ch. 6, we find the normal force has magnitude $F_N = mg \cos\theta$ which means $f_k = \mu_k mg \cos\theta$. Thus, Eq. 8-33 (with $W = 0$) leads to

$$\begin{aligned} 0 &= K_f - K_i + \Delta U + \Delta E_{\text{th}} \\ &= 0 - K_i + mgd\sin\theta + \mu_k mgd\cos\theta \end{aligned}$$

which leads to

$$d = \frac{K_i}{mg(\sin\theta + \mu_k\cos\theta)} = \frac{128}{(4.0)(9.8)(\sin 30° + 0.30\cos 30°)} = 4.3\,\text{m}.$$

57. Before the launch, the mechanical energy is $\Delta E_{\text{mech},0} = 0$. At the maximum height h where the speed of the beetle vanishes, the mechanical energy is $\Delta E_{\text{mech},1} = mgh$. The change of the mechanical energy is related to the external force by

$$\Delta E_{\text{mech}} = \Delta E_{\text{mech},1} - \Delta E_{\text{mech},0} = mgh = F_{avg} d \cos\phi ,$$

where F_{avg} is the average magnitude of the external force on the beetle.

(a) From the above equation, we have

$$F_{avg} = \frac{mgh}{d\cos\phi} = \frac{(4.0\times10^{-6}\ \text{kg})(9.80\ \text{m/s}^2)(0.30\ \text{m})}{(7.7\times10^{-4}\ \text{m})(\cos 0°)} = 1.5\times10^{-2}\ \text{N}.$$

(b) Dividing the above result by the mass of the beetle, we obtain

$$a = \frac{F_{avg}}{m} = \frac{h}{d\cos\phi}\ g = \frac{(0.30\ \text{m})}{(7.7\times10^{-4}\ \text{m})(\cos 0°)}\ g = 3.8\times10^{2}\, g.$$

58. (a) Using the force analysis shown in Chapter 6, we find the normal force $F_N = mg\cos\theta$ (where mg = 267 N) which means $f_k = \mu_k F_N = \mu_k mg\cos\theta$. Thus, Eq. 8-31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta = (0.10)(267)(6.1)\cos 20° = 1.5\times 10^2\ \text{J}.$$

(b) The potential energy change is

$$\Delta U = mg(-d\sin\theta) = (267\ \text{N})(-6.1\ \text{m})\sin 20° = -5.6\times 10^2\ \text{J}.$$

The initial kinetic energy is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}\left(\frac{267\ \text{N}}{9.8\ \text{m/s}^2}\right)(0.457\ \text{m/s}^2) = 2.8\ \text{J}.$$

Therefore, using Eq. 8-33 (with $W = 0$), the final kinetic energy is

$$K_f = K_i - \Delta U - \Delta E_{\text{th}} = 2.8 - \left(-5.6\times 10^2\right) - 1.5\times 10^2 = 4.1\times 10^2\ \text{J}.$$

Consequently, the final speed is $v_f = \sqrt{2K_f/m} = 5.5\ \text{m/s}$.

59. (a) With $x = 0.075$ m and $k = 320\,\text{N/m}$, Eq. 7-26 yields $W_s = -\frac{1}{2}kx^2 = -0.90$ J. For later reference, this is equal to the negative of ΔU.

(b) Analyzing forces, we find $F_N = mg$ which means $f_k = \mu_k F_N = \mu_k mg$. With $d = x$, Eq. 8-31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgx = (0.25)(2.5)(9.8)(0.075) = 0.46 \text{ J}.$$

(c) Eq. 8-33 (with $W = 0$) indicates that the initial kinetic energy is

$$K_i = \Delta U + \Delta E_{\text{th}} = 0.90 + 0.46 = 1.36 \text{ J}$$

which leads to $v_i = \sqrt{2K_i/m} = 1.0\ \text{m/s}$.

60. This can be worked entirely by the methods of Chapters 2–6, but we will use energy methods in as many steps as possible.

(a) By a force analysis of the style done in Ch. 6, we find the normal force has magnitude $F_N = mg \cos\theta$ (where $\theta = 40°$) which means $f_k = \mu_k F_N = \mu_k mg \cos\theta$ where $\mu_k = 0.15$. Thus, Eq. 8-31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgd \cos\theta.$$

Also, elementary trigonometry leads us to conclude that $\Delta U = mgd \sin\theta$. Eq. 8-33 (with $W = 0$ and $K_f = 0$) provides an equation for determining d:

$$K_i = \Delta U + \Delta E_{\text{th}}$$
$$\frac{1}{2}mv_i^2 = mgd\left(\sin\theta + \mu_k \cos\theta\right)$$

where $v_i = 1.4\,\text{m/s}$. Dividing by mass and rearranging, we obtain

$$d = \frac{v_i^2}{2g(\sin\theta + \mu_k \cos\theta)} = 0.13\,\text{m}.$$

(b) Now that we know where on the incline it stops ($d' = 0.13 + 0.55 = 0.68$ m from the bottom), we can use Eq. 8-33 again (with $W = 0$ and now with $K_i = 0$) to describe the final kinetic energy (at the bottom):

$$K_f = -\Delta U - \Delta E_{\text{th}}$$
$$\frac{1}{2}mv^2 = mgd'\left(\sin\theta - \mu_k \cos\theta\right)$$

which — after dividing by the mass and rearranging — yields

$$v = \sqrt{2gd'\left(\sin\theta - \mu_k \cos\theta\right)} = 2.7\,\text{m/s}.$$

(c) In part (a) it is clear that d increases if μ_k decreases — both mathematically (since it is a positive term in the denominator) and intuitively (less friction — less energy "lost"). In part (b), there are two terms in the expression for v which imply that it should increase if μ_k were smaller: the increased value of $d' = d_0 + d$ and that last factor $\sin\theta - \mu_k \cos\theta$ which indicates that less is being subtracted from $\sin\theta$ when μ_k is less (so the factor itself increases in value).

61. (a) The maximum height reached is h. The thermal energy generated by air resistance as the stone rises to this height is $\Delta E_{\text{th}} = fh$ by Eq. 8-31. We use energy conservation in the form of Eq. 8-33 (with $W = 0$):

$$K_f + U_f + \Delta E_{\text{th}} = K_i + U_i$$

and we take the potential energy to be zero at the throwing point (ground level). The initial kinetic energy is $K_i = \frac{1}{2}mv_0^2$, the initial potential energy is $U_i = 0$, the final kinetic energy is $K_f = 0$, and the final potential energy is $U_f = wh$, where $w = mg$ is the weight of the stone. Thus, $wh + fh = \frac{1}{2}mv_0^2$, and we solve for the height:

$$h = \frac{mv_0^2}{2(w+f)} = \frac{v_0^2}{2g(1+f/w)}.$$

Numerically, we have, with $m = (5.29 \text{ N})/(9.80 \text{ m/s}^2) = 0.54$ kg,

$$h = \frac{(20.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(1+0.265/5.29)} = 19.4 \text{ m/s}.$$

(b) We notice that the force of the air is downward on the trip up and upward on the trip down, since it is opposite to the direction of motion. Over the entire trip the increase in thermal energy is $\Delta E_{\text{th}} = 2fh$. The final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is the speed of the stone just before it hits the ground. The final potential energy is $U_f = 0$. Thus, using Eq. 8-31 (with $W = 0$), we find

$$\frac{1}{2}mv^2 + 2fh = \frac{1}{2}mv_0^2.$$

We substitute the expression found for h to obtain

$$\frac{2fv_0^2}{2g(1+f/w)} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

which leads to

$$v^2 = v_0^2 - \frac{2fv_0^2}{mg(1+f/w)} = v_0^2 - \frac{2fv_0^2}{w(1+f/w)} = v_0^2\left(1 - \frac{2f}{w+f}\right) = v_0^2\,\frac{w-f}{w+f}$$

where w was substituted for mg and some algebraic manipulations were carried out. Therefore,

$$v = v_0\sqrt{\frac{w-f}{w+f}} = (20.0 \text{ m/s})\sqrt{\frac{5.29 \text{ N} - 0.265 \text{ N}}{5.29 \text{ N} + 0.265 \text{ N}}} = 19.0 \text{ m/s}.$$

62. In the absence of friction, we have a simple conversion (as it moves along the inclined ramps) of energy between the kinetic form (Eq. 7-1) and the potential form (Eq. 8-9). Along the horizontal plateaus, however, there is friction which causes some of the kinetic energy to dissipate in accordance with Eq. 8-31 (along with Eq. 6-2 where μ_k = 0.50 and $F_N = mg$ in this situation). Thus, after it slides down a (vertical) distance d it has gained $K = \frac{1}{2}mv^2 = mgd$, some of which ($\Delta E_{\text{th}} = \mu_k mgd$) is dissipated, so that the value of kinetic energy at the end of the first plateau (just before it starts descending towards the lowest plateau) is $K = mgd - \mu_k mgd = 0.5mgd$. In its descent to the lowest plateau, it gains $mgd/2$ more kinetic energy, but as it slides across it "loses" $\mu_k mgd/2$ of it. Therefore, as it starts its climb up the right ramp, it has kinetic energy equal to

$$K = 0.5mgd + mgd/2 - \mu_k mgd/2 = 3\ mgd\ /\ 4.$$

Setting this equal to Eq. 8-9 (to find the height to which it climbs) we get $H = \frac{3}{4}d$. Thus, the block (momentarily) stops on the inclined ramp at the right, at a height of

$$H = 0.75d = 0.75\ (\ 40\ \text{cm}) = 30\ \text{cm}$$

measured from the lowest plateau.

63. The initial and final kinetic energies are zero, and we set up energy conservation in the form of Eq. 8-33 (with $W = 0$) according to our assumptions. Certainly, it can only come to a permanent stop somewhere in the flat part, but the question is whether this occurs during its first pass through (going rightward) or its second pass through (going leftward) or its third pass through (going rightward again), and so on. If it occurs during its first pass through, then the thermal energy generated is $\Delta E_{th} = f_k d$ where $d \le L$ and $f_k = \mu_k mg$. If it occurs during its second pass through, then the total thermal energy is $\Delta E_{th} = \mu_k mg(L + d)$ where we again use the symbol d for how far through the level area it goes during that last pass (so $0 \le d \le L$). Generalizing to the n^{th} pass through, we see that

$$\Delta E_{th} = \mu_k mg[(n - 1)L + d].$$

In this way, we have

$$mgh = \mu_k mg\left((n-1)L + d\right)$$

which simplifies (when $h = L/2$ is inserted) to

$$\frac{d}{L} = 1 + \frac{1}{2\mu_k} - n.$$

The first two terms give $1 + 1/2\mu_k = 3.5$, so that the requirement $0 \le d/L \le 1$ demands that $n = 3$. We arrive at the conclusion that $d/L = \frac{1}{2}$, or

$$d = \frac{1}{2}L = \frac{1}{2}(40 \text{ cm}) = 20 \text{ cm}$$

and that this occurs on its third pass through the flat region.

64. We will refer to the point where it first encounters the “rough region” as point C (this is the point at a height h above the reference level). From Eq. 8-17, we find the speed it has at point C to be

$$v_C = \sqrt{v_A^2 - 2gh} = \sqrt{(8.0)^2 - 2(9.8)(2.0)} = 4.980 \approx 5.0 \text{ m/s}.$$

Thus, we see that its kinetic energy right at the beginning of its “rough slide” (heading uphill towards B) is

$$K_C = \frac{1}{2}\, m(4.980 \text{ m/s})^2 = 12.4m$$

(with SI units understood). Note that we “carry along” the mass (as if it were a known quantity); as we will see, it will cancel out, shortly. Using Eq. 8-37 (and Eq. 6-2 with $F_N = mg\cos\theta$) and $y = d\sin\theta$, we note that if $d < L$ (the block does not reach point B), this kinetic energy will turn entirely into thermal (and potential) energy

$$K_C = mgy + f_k d \quad \Rightarrow \quad 12.4m = mgd\sin\theta + \mu_k mgd\cos\theta.$$

With $\mu_k = 0.40$ and $\theta = 30^\circ$, we find $d = 1.49$ m, which is greater than L (given in the problem as 0.75 m), so our assumption that $d < L$ is incorrect. What is its kinetic energy as it reaches point B? The calculation is similar to the above, but with d replaced by L and the final v^2 term being the unknown (instead of assumed zero):

$$\frac{1}{2}\, m v^2 = K_C - (mgL\sin\theta + \mu_k mgL\cos\theta)\ .$$

This determines the speed with which it arrives at point B:

$$\begin{aligned} v_B &= \sqrt{v_C^2 - 2gL(\sin\theta + \mu_k \cos\theta)} \\ &= \sqrt{(4.98 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(0.75 \text{ m})(\sin 30^\circ + 0.4\cos 30^\circ)} = 3.5 \text{ m/s}. \end{aligned}$$

65. We observe that the last line of the problem indicates that static friction is not to be considered a factor in this problem. The friction force of magnitude f = 4400 N mentioned in the problem is kinetic friction and (as mentioned) is constant (and directed upward), and the thermal energy change associated with it is $\Delta E_{th} = fd$ (Eq. 8-31) where d = 3.7 m in part (a) (but will be replaced by x, the spring compression, in part (b)).

(a) With W = 0 and the reference level for computing $U = mgy$ set at the top of the (relaxed) spring, Eq. 8-33 leads to

$$U_i = K + \Delta E_{\text{th}} \Rightarrow v = \sqrt{2d\left(g - \frac{f}{m}\right)}$$

which yields $v = 7.4\,\text{m/s}$ for m = 1800 kg.

(b) We again utilize Eq. 8-33 (with W = 0), now relating its kinetic energy at the moment it makes contact with the spring to the system energy at the bottom-most point. Using the same reference level for computing $U = mgy$ as we did in part (a), we end up with gravitational potential energy equal to $mg(-x)$ at that bottom-most point, where the spring (with spring constant $k = 1.5\times10^5\ \text{N/m}$) is fully compressed.

$$K = mg(-x) + \frac{1}{2}kx^2 + fx$$

where $K = \frac{1}{2}mv^2 = 4.9\times10^4$ J using the speed found in part (a). Using the abbreviation $\xi = mg - f = 1.3\times10^4$ N, the quadratic formula yields

$$x = \frac{\xi \pm \sqrt{\xi^2 + 2kK}}{k} = 0.90\,\text{m}$$

where we have taken the positive root.

(c) We relate the energy at the bottom-most point to that of the highest point of rebound (a distance d' above the relaxed position of the spring). We assume $d' > x$. We now use the bottom-most point as the reference level for computing gravitational potential energy.

$$\frac{1}{2}kx^2 = mgd' + fd' \Rightarrow d' = \frac{kx^2}{2(mg+d)} = 2.8\,\text{m}.$$

(d) The non-conservative force (§8-1) is friction, and the energy term associated with it is the one that keeps track of the total distance traveled (whereas the potential energy terms, coming as they do from conservative forces, depend on positions — but not on the paths that led to them). We assume the elevator comes to final rest at the equilibrium position of the spring, with the spring compressed an amount d_{eq} given by

$$mg = kd_{\text{eq}} \Rightarrow d_{\text{eq}} = \frac{mg}{k} = 0.12\,\text{m}.$$

In this part, we use that final-rest point as the reference level for computing gravitational potential energy, so the original $U = mgy$ becomes $mg(d_{eq} + d)$. In that final position, then, the gravitational energy is zero and the spring energy is $kd_{eq}^2/2$. Thus, Eq. 8-33 becomes

$$mg\left(d_{eq} + d\right) = \frac{1}{2}kd_{eq}^2 + fd_{total}$$
$$(1800)(9.8)(0.12 + 3.7) = \frac{1}{2}\left(1.5\times10^5\right)(0.12)^2 + (4400)\,d_{total}$$

which yields $d_{total} = 15$ m.

66. (a) Since the speed of the crate of mass *m* increases from 0 to 1.20 m/s relative to the factory ground, the kinetic energy supplied to it is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(300\,\text{kg})(120\,\text{m/s})^2 = 216\,\text{J}.$$

(b) The magnitude of the kinetic frictional force is

$$f = \mu F_N = \mu mg = (0.400)(300\,\text{kg})(9.8\,\text{m/s}^2) = 1.18\times10^3\ \text{N}.$$

(c) Let the distance the crate moved relative to the conveyor belt before it stops slipping be d, then from Eq. 2-16 ($v^2 = 2ad = 2(f\,/\,m)d$) we find

$$\Delta E_{\text{th}} = fd = \frac{1}{2}mv^2 = K.$$

Thus, the total energy that must be supplied by the motor is

$$W = K + \Delta E_{\text{th}} = 2K = (2)(216\,\text{J}) = 432\,\text{J}.$$

(d) The energy supplied by the motor is the work W it does on the system, and must be greater than the kinetic energy gained by the crate computed in part (b). This is due to the fact that part of the energy supplied by the motor is being used to compensate for the energy dissipated ΔE_{th} while it was slipping.

67. (a) The assumption is that the slope of the bottom of the slide is horizontal, like the ground. A useful analogy is that of the pendulum of length R = 12 m that is pulled leftward to an angle θ (corresponding to being at the top of the slide at height h = 4.0 m) and released so that the pendulum swings to the lowest point (zero height) gaining speed $v = 6.2\ \mathrm{m/s}$. Exactly as we would analyze the trigonometric relations in the pendulum problem, we find

$$h = R(1-\cos\theta) \Rightarrow \theta = \cos^{-1}\left(1-\frac{h}{R}\right) = 48°$$

or 0.84 radians. The slide, representing a circular arc of length $s = R\theta$, is therefore (12 m)(0.84) = 10 m long.

(b) To find the magnitude f of the frictional force, we use Eq. 8-31 (with W = 0):

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= \frac{1}{2}mv^2 - mgh + fs \end{aligned}$$

so that (with m = 25 kg) we obtain f = 49 N.

(c) The assumption is no longer that the slope of the bottom of the slide is horizontal, but rather that the slope of the top of the slide is vertical (and 12 m to the left of the center of curvature). Returning to the pendulum analogy, this corresponds to releasing the pendulum from horizontal (at θ_1 = 90° measured from vertical) and taking a snapshot of its motion a few moments later when it is at angle θ_2 with speed v = 6.2 m/s. The difference in height between these two positions is (just as we would figure for the pendulum of length R)

$$\Delta h = R(1-\cos\theta_2) - R(1-\cos\theta_1) = -R\cos\theta_2$$

where we have used the fact that cos θ_1 = 0. Thus, with Δh = –4.0 m, we obtain θ_2 =70.5° which means the arc subtends an angle of $|\Delta\theta|$ = 19.5° or 0.34 radians. Multiplying this by the radius gives a slide length of s' = 4.1 m.

(d) We again find the magnitude f' of the frictional force by using Eq. 8-31 (with W = 0):

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= \frac{1}{2}mv^2 - mgh + f's' \end{aligned}$$

so that we obtain $f' = 1.2 \times 10^2$ N.

68. We use conservation of mechanical energy: the mechanical energy must be the same at the top of the swing as it is initially. Newton's second law is used to find the speed, and hence the kinetic energy, at the top. There the tension force T of the string and the force of gravity are both downward, toward the center of the circle. We notice that the radius of the circle is $r = L - d$, so the law can be written

$$T + mg = mv^2/(L - d),$$

where v is the speed and m is the mass of the ball. When the ball passes the highest point with the least possible speed, the tension is zero. Then

$$mg = m\frac{v^2}{L-d} \Rightarrow v = \sqrt{g(L-d)} .$$

We take the gravitational potential energy of the ball-Earth system to be zero when the ball is at the bottom of its swing. Then the initial potential energy is mgL. The initial kinetic energy is zero since the ball starts from rest. The final potential energy, at the top of the swing, is $2mg(L - d)$ and the final kinetic energy is $\frac{1}{2}mv^2 = \frac{1}{2}mg(L-d)$ using the above result for v. Conservation of energy yields

$$mgL = 2mg(L-d) + \frac{1}{2}mg(L-d) \Rightarrow d = 3L/5 .$$

With $L = 1.20$ m, we have $d = 0.60(1.20 \text{ m}) = 0.72$ m.

Notice that if d is greater than this value, so the highest point is lower, then the speed of the ball is greater as it reaches that point and the ball passes the point. If d is less, the ball cannot go around. Thus the value we found for d is a lower limit.

69. There is the same potential energy change in both circumstances, so we can equate the kinetic energy changes as well:

$$\Delta K_2 = \Delta K_1 \quad \Rightarrow \quad \frac{1}{2}\, m v_B^2 - \frac{1}{2}\, m(4.00 \text{ m/s})^2 = \frac{1}{2}\, m(2.60 \text{ m/s})^2 - \frac{1}{2}\, m(2.00 \text{ m/s})^2$$

which leads to $v_B = 4.33$ m/s.

70. (a) To stretch the spring an external force, equal in magnitude to the force of the spring but opposite to its direction, is applied. Since a spring stretched in the positive x direction exerts a force in the negative x direction, the applied force must be $F = 52.8x + 38.4x^2$, in the $+x$ direction. The work it does is

$$W = \int_{0.50}^{1.00} (52.8x + 38.4x^2)dx = \left(\frac{52.8}{2}x^2 + \frac{38.4}{3}x^3\right)\Bigg|_{0.50}^{1.00} = 31.0\,\text{J}.$$

(b) The spring does 31.0 J of work and this must be the increase in the kinetic energy of the particle. Its speed is then

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(31.0\,\text{J})}{2.17\,\text{kg}}} = 5.35\,\text{m/s}.$$

(c) The force is conservative since the work it does as the particle goes from any point x_1 to any other point x_2 depends only on x_1 and x_2, not on details of the motion between x_1 and x_2.

71. This can be worked entirely by the methods of Chapters 2–6, but we will use energy methods in as many steps as possible.

(a) By a force analysis in the style of Chapter 6, we find the normal force has magnitude $F_N = mg\cos\theta$ (where $\theta = 39°$) which means $f_k = \mu_k mg\cos\theta$ where $\mu_k = 0.28$. Thus, Eq. 8-31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta.$$

Also, elementary trigonometry leads us to conclude that $\Delta U = -mgd\sin\theta$ where $d = 3.7$ m . Since $K_i = 0$, Eq. 8-33 (with $W = 0$) indicates that the final kinetic energy is

$$K_f = -\Delta U - \Delta E_{\text{th}} = mgd\,(\sin\theta - \mu_k\cos\theta)$$

which leads to the speed at the bottom of the ramp

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{2gd\,(\sin\theta - \mu_k\cos\theta)} = 5.5\ \text{m/s}.$$

(b) This speed begins its horizontal motion, where $f_k = \mu_k mg$ and $\Delta U = 0$. It slides a distance d' before it stops. According to Eq. 8-31 (with $W = 0$),

$$\begin{aligned}0 &= \Delta K + \Delta U + \Delta E_{\text{th}}\\ &= 0 - \frac{1}{2}mv^2 + 0 + \mu_k mgd'\\ &= -\frac{1}{2}\left(2gd\,(\sin\theta - \mu_k\cos\theta)\right) + \mu_k gd'\end{aligned}$$

where we have divided by mass and substituted from part (a) in the last step. Therefore,

$$d' = \frac{d(\sin\theta - \mu_k\cos\theta)}{\mu_k} = 5.4\ \text{m}.$$

(c) We see from the algebraic form of the results, above, that the answers do not depend on mass. A 90 kg crate should have the same speed at the bottom and sliding distance across the floor, to the extent that the friction relations in Ch. 6 are accurate. Interestingly, since g does not appear in the relation for d', the sliding distance would seem to be the same if the experiment were performed on Mars!

72. (a) At B the speed is (from Eq. 8-17)

$$v = \sqrt{v_0^2 + 2gh_1} = \sqrt{(7.0 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(6.0 \text{ m})} = 13 \text{ m/s}.$$

(a) Here what matters is the difference in heights (between A and C):

$$v = \sqrt{v_0^2 + 2g(h_1 - h_2)} = \sqrt{(7.0 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(4.0 \text{ m})} = 11.29 \text{ m/s} \approx 11 \text{ m/s}.$$

(c) Using the result from part (b), we see that its kinetic energy right at the beginning of its "rough slide" (heading horizontally towards D) is $\frac{1}{2}\,m(11.29 \text{ m/s})^2 = 63.7m$ (with SI units understood). Note that we "carry along" the mass (as if it were a known quantity); as we will see, it will cancel out, shortly. Using Eq. 8-31 (and Eq. 6-2 with $F_N = mg$) we note that this kinetic energy will turn entirely into thermal energy

$$63.7m = \mu_k mgd$$

if $d < L$. With $\mu_k = 0.70$, we find $d = 9.3$ m, which is indeed less than L (given in the problem as 12 m). We conclude that the block stops before passing out of the "rough" region (and thus does not arrive at point D).

73. (a) By mechanical energy conversation, the kinetic energy as it reaches the floor (which we choose to be the $U = 0$ level) is the sum of the initial kinetic and potential energies:

$$K = K_i + U_i = \frac{1}{2}\,(2.50\text{ kg})(3.00\text{ m/s})^2 + (2.50\text{ kg})(9.80\text{ m/s}^2)(4.00\text{ m}) = 109\text{ J}.$$

For later use, we note that the speed with which it reaches the ground is

$$v = \sqrt{2K/m} \quad = 9.35\text{ m/s}.$$

(b) When the drop in height is 2.00 m instead of 4.00 m, the kinetic energy is

$$K = \frac{1}{2}\,(2.50\text{ kg})(3.00\text{ m/s})^2 + (2.50\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = 60.3\text{ J}.$$

(c) A simple way to approach this is to imagine the can is *launched* from the ground at $t = 0$ with speed 9.35 m/s (see above) and ask of its height and speed at t = 0.200 s, using Eq. 2-15 and Eq. 2-11:

$$y = (9.35\text{ m/s})(0.200\text{ s}) - \frac{1}{2}\,(9.80\text{ m/s}^2)(0.200\text{ s})^2 = 1.67\text{ m},$$

$$v = 9.35\text{ m/s} - (9.80\text{ m/s}^2)(0.200\text{ s}) = 7.39\text{ m/s}.$$

The kinetic energy is

$$K = \frac{1}{2}\,(2.50\text{ kg})\,(7.39\text{ m/s})^2 = 68.2\text{ J}.$$

(d) The gravitational potential energy

$$U = mgy = (2.5\text{ kg})(9.8\text{ m/s}^2)(1.67\text{ m}) = 41.0\text{ J}$$

74. (a) The initial kinetic energy is $K_i = \frac{1}{2}(1.5)(3)^2 = 6.75 \text{ J}$.

(b) The work of gravity is the negative of its change in potential energy. At the highest point, all of K_i has converted into U (if we neglect air friction) so we conclude the work of gravity is –6.75 J.

(c) And we conclude that $\Delta U = 6.75 \text{ J}$.

(d) The potential energy there is $U_f = U_i + \Delta U = 6.75 \text{ J}$.

(e) If $U_f = 0$, then $U_i = U_f - \Delta U = -6.75 \text{ J}$.

(f) Since $mg\Delta y = \Delta U$, we obtain $\Delta y = 0.459 \text{ m}$.

75. We note that if the larger mass (block B, $m_B = 2$ kg) falls $d = 0.25$ m, then the smaller mass (blocks A, $m_A = 1$ kg) must increase its height by $h = d \sin 30°$. Thus, by mechanical energy conservation, the kinetic energy of the system is

$$K_{\text{total}} = m_B g d - m_A g h = 3.7 \text{ J}.$$

76. (a) At the point of maximum height, where y = 140 m, the vertical component of velocity vanishes but the horizontal component remains what it was when it was launched (if we neglect air friction). Its kinetic energy at that moment is

$$K = \frac{1}{2}(0.55\,\text{kg})v_x^2 .$$

Also, its potential energy (with the reference level chosen at the level of the cliff edge) at that moment is $U = mgy$ = 755 J. Thus, by mechanical energy conservation,

$$K = K_i - U = 1550 - 755 \Rightarrow v_x = \sqrt{\frac{2(1550-755)}{0.55}} = 54 \text{ m/s}.$$

(b) As mentioned $v_x = v_{ix}$ so that the initial kinetic energy

$$K_i = \frac{1}{2}m\left(v_{ix}^2 + v_{iy}^2\right)$$

can be used to find v_{iy}. We obtain $v_{iy} = 52 \text{ m/s}$.

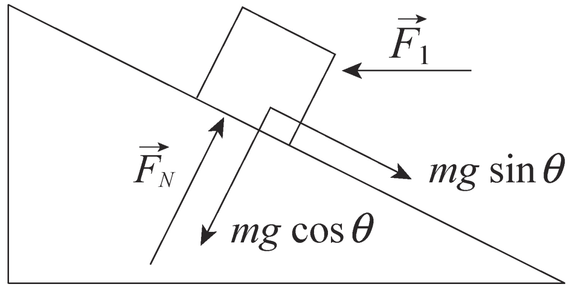
(c) Applying Eq. 2-16 to the vertical direction (with $+y$ upward), we have

$$v_y^2 = v_{iy}^2 - 2g\Delta y \quad \Rightarrow \quad (65 \text{ m/s})^2 = (52 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)\Delta y$$

which yields $\Delta y = -76$ m. The minus sign tells us it is below its launch point.

77. The work done by $\vec{F}$ is the negative of its potential energy change (see Eq. 8-6), so $U_B = U_A - 25 = 15$ J.

78. The free-body diagram for the trunk is shown.



The x and y applications of Newton's second law provide two equations:

$$F_1 \cos\theta - f_k - mg \sin\theta = ma$$
$$F_N - F_1 \sin\theta - mg \cos\theta = 0.$$

(a) The trunk is moving up the incline at constant velocity, so $a = 0$. Using $f_k = \mu_k F_N$, we solve for the push-force F_1 and obtain

$$F_1 = \frac{mg(\sin\theta + \mu_k \cos\theta)}{\cos\theta - \mu_k \sin\theta}.$$

The work done by the push-force $\vec{F}_1$ as the trunk is pushed through a distance ℓ up the inclined plane is therefore

$$\begin{aligned} W_1 = F_1 \ell \cos\theta &= \frac{(mg\ell\cos\theta)(\sin\theta + \mu_k \cos\theta)}{\cos\theta - \mu_k \sin\theta} \\ &= \frac{(50\text{ kg})(9.8\text{ m/s}^2)(6.0\text{ m})(\cos 30°)(\sin 30° + (0.20)\cos 30°)}{\cos 30° - (0.20)\sin 30°} \\ &= 2.2\times10^3\text{ J.} \end{aligned}$$

(b) The increase in the gravitational potential energy of the trunk is

$$\Delta U = mg\ell \sin\theta = (50\,\text{kg})(9.8\,\text{m/s}^2)(6.0\,\text{m})\sin 30° = 1.5\times10^3\text{ J.}$$

Since the speed (and, therefore, the kinetic energy) of the trunk is unchanged, Eq. 8-33 leads to

$$W_1 = \Delta U + \Delta E_{\text{th}}.$$

Thus, using more precise numbers than are shown above, the increase in thermal energy (generated by the kinetic friction) is 2.24×10^3 J – 1.47×10^3 J = 7.7×10^2 J. An alternate way to this result is to use $\Delta E_{\text{th}} = f_k \ell$ (Eq. 8-31).

79. The initial height of the $2M$ block, shown in Fig. 8-64, is the $y = 0$ level in our computations of its value of U_g. As that block drops, the spring stretches accordingly. Also, the kinetic energy K_{sys} is evaluated for the *system* -- that is, for a total moving mass of $3M$.

(a) The conservation of energy, Eq. 8-17, leads to

$$K_i + U_i = K_{sys} + U_{sys} \quad \Rightarrow \quad 0 + 0 = K_{sys} + (2M)g(-0.090) + \frac{1}{2}\, k(0.090)^2 .$$

Thus, with $M = 2.0$ kg, we obtain $K_{sys} = 2.7$ J.

(b) The kinetic energy of the $2M$ block represents a fraction of the total kinetic energy:

$$\frac{K_{2M}}{K_{sys}} = \frac{(2M)v^2/2}{(3M)v^2/2} = \frac{2}{3} .$$

Therefore, $K_{2M} = \frac{2}{3}(2.7 \text{ J}) = 1.8$ J.

(c) Here we let $y = -d$ and solve for d.

$$K_i + U_i = K_{sys} + U_{sys} \quad \Rightarrow \quad 0 + 0 \quad = \quad 0 + (2M)g(-d) + \frac{1}{2}\, kd^2 .$$

Thus, with $M = 2.0$ kg, we obtain $d = 0.39$ m.

80. Sample Problem 8-3 illustrates simple energy conservation in a similar situation, and derives the frequently encountered relationship: $v = \sqrt{2gh}$. In our present problem, the height is related to the distance (on the $\theta = 10°$ slope) d = 920 m by the trigonometric relation $h = d \sin\theta$. Thus,

$$v = \sqrt{2(9.8 \text{ m/s}^2)(920 \text{ m})\sin 10°} = 56 \text{ m/s}.$$

81. Eq. 8-33 gives $mgy_f = K_i + mgy_i - \Delta E_{\text{th}}$, or

$$(0.50 \text{ kg})(9.8 \text{ m/s}^2)(0.80 \text{ m}) = \frac{1}{2}(0.50 \text{ kg})(4.00 \text{ /s})^2 + (0.50 \text{ kg})(9.8 \text{ m/s}^2)(0) - \Delta E_{\text{th}}$$

which yields $\Delta E_{\text{th}} = 4.00 \text{ J} - 3.92 \text{ J} = 0.080 \text{ J}$.

82. (a) The loss of the initial $K = \frac{1}{2}mv^2 = \frac{1}{2}(70\text{ kg})(10\text{ m/s})^2$ is 3500 J, or 3.5 kJ.

(b) This is dissipated as thermal energy; ΔE_{th} = 3500 J = 3.5 kJ.

83. The initial height shown in the figure is the $y = 0$ level in our computations of U_g, and in parts (a) and (b) the heights are y_a =(0.80 m)sin 40°= 0.51 m and y_b = (1.00 m) sin 40° = 0.64 m, respectively.

(a) The conservation of energy, Eq. 8-17, leads to

$$K_i + U_i = K_a + U_a \quad \Rightarrow \quad 16 \text{ J} + 0 = K_a + mgy_a + \frac{1}{2}k(0.20 \text{ m})^2$$

from which we obtain K_a = (16 – 5.0 – 4.0) J = 7.0 J.

(b) Again we use the conservation of energy

$$K_i + U_i = K_b + U_b \quad \Rightarrow \quad K_i + 0 = 0 + mgy_b + \frac{1}{2}\, k(0.40 \text{ m})^2$$

from which we obtain K_i = 6.0 J + 16 J = 22 J.

84. (a) Eq. 8-9 gives $U = (3.2 \text{ kg})(9.8 \text{ m/s}^2)(3.0 \text{ m}) = 94$ J.

(b) The mechanical energy is conserved, so $K = 94$ J.

(c) The speed (from solving Eq. 7-1) is $v = \sqrt{2(94 \text{ J})/(32 \text{ kg})} = 7.7$ m/s.

85. (a) Resolving the gravitational force into components and applying Newton's second law (as well as Eq. 6-2), we find

$$F_{\text{machine}} - mg\sin\theta - \mu_k mg\cos\theta = ma.$$

In the situation described in the problem, we have $a = 0$, so

$$F_{\text{machine}} = mg\sin\theta + \mu_k mg\cos\theta = 372 \text{ N}.$$

Thus, the work done by the machine is $F_{\text{machine}}d = 744 \text{ J} = 7.4 \times 10^2 \text{ J}$.

(b) The thermal energy generated is $\mu_k mg\cos\theta\, d = 240 \text{ J} = 2.4 \times 10^2 \text{ J}$.

86. We use $P = Fv$ to compute the force:

$$F = \frac{P}{v} = \frac{92 \times 10^6\ \text{W}}{(32.5\ \text{knot})\left(1.852\,\dfrac{\text{km/h}}{\text{knot}}\right)\left(\dfrac{1000\ \text{m/km}}{3600\ \text{s/h}}\right)} = 5.5 \times 10^6\ \text{N}.$$

87. Since the speed is constant $\Delta K = 0$ and Eq. 8-33 (an application of the energy conservation concept) implies

$$W_{\text{applied}} = \Delta E_{\text{th}} = \Delta E_{\text{th(cube)}} + \Delta E_{\text{th(floor)}} .$$

Thus, if $W_{\text{applied}} = (15 \text{ N})(3.0 \text{ m}) = 45$ J, and we are told that $\Delta E_{\text{th (cube)}} = 20$ J, then we conclude that $\Delta E_{\text{th (floor)}} = 25$ J.

88. (a) We take the gravitational potential energy of the skier-Earth system to be zero when the skier is at the bottom of the peaks. The initial potential energy is $U_i = mgH$, where m is the mass of the skier, and H is the height of the higher peak. The final potential energy is $U_f = mgh$, where h is the height of the lower peak. The skier initially has a kinetic energy of $K_i = 0$, and the final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is the speed of the skier at the top of the lower peak. The normal force of the slope on the skier does no work and friction is negligible, so mechanical energy is conserved:

$$U_i + K_i = U_f + K_f \quad \Rightarrow \quad mgH = mgh + \frac{1}{2}mv^2$$

Thus,

$$v = \sqrt{2g(H-h)} = \sqrt{2(9.8\ \text{m/s}^2)(850\ \text{m} - 750\ \text{m})} = 44\ \text{m/s}\,.$$

(b) We recall from analyzing objects sliding down inclined planes that the normal force of the slope on the skier is given by $F_N = mg\cos\theta$, where θ is the angle of the slope from the horizontal, 30° for each of the slopes shown. The magnitude of the force of friction is given by $f = \mu_k F_N = \mu_k mg\cos\theta$. The thermal energy generated by the force of friction is $fd = \mu_k mgd\cos\theta$, where d is the total distance along the path. Since the skier gets to the top of the lower peak with no kinetic energy, the increase in thermal energy is equal to the decrease in potential energy. That is, $\mu_k mgd\cos\theta = mg(H - h)$. Consequently,

$$\mu_k = \frac{H-h}{d\cos\theta} = \frac{(850\ \text{m} - 750\ \text{m})}{(3.2\times10^3\ \text{m})\cos 30°} = 0.036\,.$$

89. To swim at constant velocity the swimmer must push back against the water with a force of 110 N. Relative to him the water is going at 0.22 m/s toward his rear, in the same direction as his force. Using Eq. 7-48, his power output is obtained:

$$P = \vec{F} \cdot \vec{v} = Fv = (110\,\text{N})(0.22\,\text{m/s}) = 24\,\text{W}.$$

90. The initial kinetic energy of the automobile of mass m moving at speed v_i is $K_i = \frac{1}{2}mv_i^2$, where m = 16400/9.8 = 1673 kg. Using Eq. 8-31 and Eq. 8-33, this relates to the effect of friction force f in stopping the auto over a distance d by $K_i = fd$, where the road is assumed level (so $\Delta U = 0$). With

$$v_i = (113 \text{ km/h}) = (113 \text{ km/h})(1000 \text{ m/km})(1 \text{ h}/3600 \text{ s}) = 31.4 \text{ m/s},$$

we obtain

$$d = \frac{K_i}{f} = \frac{mv_i^2}{2f} = \frac{(1673\,\text{kg})(31.4 \text{ m/s})^2}{2(8230\,\text{N})} = 100\,\text{m}.$$

91. With the potential energy reference level set at the point of throwing, we have (with SI units understood)

$$\Delta E = mgh - \frac{1}{2}mv_0^2 = m\left((9.8)(8.1) - \frac{1}{2}(14)^2\right)$$

which yields $\Delta E = -12$ J for $m = 0.63$ kg. This "loss" of mechanical energy is presumably due to air friction.

92. (a) The (internal) energy the climber must convert to gravitational potential energy is

$$\Delta U = mgh = (90\text{ kg})(9.80\text{ m/s}^2)(8850\text{ m}) = 7.8\times10^6\text{ J}.$$

(b) The number of candy bars this corresponds to is

$$N = \frac{7.8\times10^6\text{ J}}{1.25\times10^6\text{ J/bar}} \approx 6.2\,\text{bars}.$$

93. (a) The acceleration of the sprinter is (using Eq. 2-15)

$$a = \frac{2\Delta x}{t^2} = \frac{(2)(7.0\,\text{m})}{(1.6\,\text{s})^2} = 5.47\,\text{m/s}^2\,.$$

Consequently, the speed at t = 1.6s is $v = at = (5.47\,\text{m/s}^2)(1.6\,\text{s}) = 8.8\,\text{m/s}$. Alternatively, Eq. 2-17 could be used.

(b) The kinetic energy of the sprinter (of weight w and mass $m = w/g$) is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{w}{g}\right)v^2 = \frac{1}{2}\left(670\text{ N}/(9.8\text{ m/s}^2)\right)(8.8\text{ m/s})^2 = 2.6\times10^3\text{ J}.$$

(c) The average power is

$$P_{\text{avg}} = \frac{\Delta K}{\Delta t} = \frac{2.6\times10^3\text{ J}}{1.6\text{ s}} = 1.6\times10^3\text{ W}.$$

94. We note that in one second, the block slides d = 1.34 m up the incline, which means its height increase is $h = d \sin\theta$ where

$$\theta = \tan^{-1}\left(\frac{30}{40}\right) = 37°.$$

We also note that the force of kinetic friction in this inclined plane problem is $f_k = \mu_k mg\cos\theta$, where μ_k = 0.40 and m = 1400 kg. Thus, using Eq. 8-31 and Eq. 8-33, we find

$$W = mgh + f_k d = mgd\left(\sin\theta + \mu_k \cos\theta\right)$$

or $W = 1.69 \times 10^4$ J for this one-second interval. Thus, the power associated with this is

$$P = \frac{1.69\times10^4 \text{ J}}{1 \text{ s}} = 1.69\times10^4 \text{ W} \approx 1.7\times10^4 \text{ W}.$$

95. (a) The initial kinetic energy is $K_i = (1.5\ \text{kg})(20\ \text{m/s})^2/2 = 300\ \text{J}$.

(b) At the point of maximum height, the vertical component of velocity vanishes but the horizontal component remains what it was when it was "shot" (if we neglect air friction). Its kinetic energy at that moment is

$$K = \frac{1}{2}(1.5\ \text{kg})\left[(20\ \text{m/s})\cos 34°\right]^2 = 206\ \text{J}.$$

Thus, $\Delta U = K_i - K = 300\ \text{J} - 206\ \text{J} = 93.8\ \text{J}$.

(c) Since $\Delta U = mg\Delta y$, we obtain

$$\Delta y = \frac{94\ \text{J}}{(1.5\ \text{kg})(9.8\ \text{m/s}^2)} = 6.38\ \text{m}.$$

96. From Eq. 8-6, we find (with SI units understood)

$$U(\xi) = -\int_0^{\xi} \left(-3x - 5x^2\right) dx = \frac{3}{2}\xi^2 + \frac{5}{3}\xi^3 .$$

(a) Using the above formula, we obtain $U(2) \approx 19$ J.

(b) When its speed is $v = 4$ m/s, its mechanical energy is $\frac{1}{2}mv^2 + U(5)$. This must equal the energy at the origin:

$$\frac{1}{2}mv^2 + U(5) = \frac{1}{2}mv_o^2 + U(0)$$

so that the speed at the origin is

$$v_o = \sqrt{v^2 + \frac{2}{m}\left(U(5) - U(0)\right)}.$$

Thus, with $U(5) = 246$ J, $U(0) = 0$ and $m = 20$ kg, we obtain $v_o = 6.4$ m/s.

(c) Our original formula for U is changed to

$$U(x) = -8 + \tfrac{3}{2}x^2 + \tfrac{5}{3}x^3$$

in this case. Therefore, $U(2) = 11$ J. But we still have $v_o = 6.4$ m/s since that calculation only depended on the difference of potential energy values (specifically, $U(5) - U(0)$).

97. Eq. 8-8 leads directly to $\Delta y = \dfrac{68000\ \text{J}}{(9.4\ \text{kg})(9.8\ \text{m/s}^2)} = 738\ \text{m}$.

98. Since the period T is $(2.5 \text{ rev/s})^{-1} = 0.40$ s, then Eq. 4-33 leads to $v = 3.14$ m/s. The frictional force has magnitude (using Eq. 6-2)

$$f = \mu_k F_N = (0.320)(180 \text{ N}) = 57.6 \text{ N}.$$

The power dissipated by the friction must equal that supplied by the motor, so Eq. 7-48 gives $P = (57.6 \text{ N})(3.14 \text{ m/s}) = 181$ W.

99. (a) In the initial situation, the elongation was (using Eq. 8-11)

$$x_i = \sqrt{2(1.44)/3200} = 0.030 \text{ m (or 3.0 cm)}.$$

In the next situation, the elongation is only 2.0 cm (or 0.020 m), so we now have less stored energy (relative to what we had initially). Specifically,

$$\Delta U = \frac{1}{2}\,(3200 \text{ N/m})(0.020 \text{ m})^2 - 1.44 \text{ J} = -0.80 \text{ J}.$$

(b) The elastic stored energy for $|x| = 0.020$ m, does not depend on whether this represents a stretch or a compression. The answer is the same as in part (a), $\Delta U = -0.80$ J.

(c) Now we have $|x| = 0.040$ m which is greater than x_i, so this represents an increase in the potential energy (relative to what we had initially). Specifically,

$$\Delta U = \frac{1}{2}\,(3200 \text{ N/m})(0.040 \text{ m})^2 - 1.44 \text{ J} = +1.12 \text{ J} \;\approx 1.1 \text{ J}.$$

100. (a) At the highest point, the velocity $v = v_x$ is purely horizontal and is equal to the horizontal component of the launch velocity (see section 4-6): $v_{ox} = v_o \cos\theta$, where $\theta = 30°$ in this problem. Eq. 8-17 relates the kinetic energy at the highest point to the launch kinetic energy:

$$K_o \;= mgy + \frac{1}{2}\,mv^2 = \frac{1}{2}\,mv_{ox}{}^2 + \frac{1}{2}\,mv_{oy}{}^2.$$

with y = 1.83 m. Since the $mv_{ox}{}^2/2$ term on the left-hand side cancels the mv$^2/2$ term on the right-hand side, this yields $v_{oy} = \sqrt{2gy} \approx 6$ m/s. With $v_{oy} = v_o \sin\theta$, we obtain

$$v_o = 11.98 \text{ m/s} \approx 12 \text{ m/s}.$$

(b) Energy conservation (including now the energy stored elastically in the spring, Eq. 8-11) also applies to the motion along the muzzle (through a distance d which corresponds to a vertical height increase of $d\sin\theta$):

$$\frac{1}{2}\,kd^2 = K_o + mg\,d\sin\theta \quad \Rightarrow \quad d = 0.11 \text{ m}.$$

101. (a) We implement Eq. 8-37 as

$$K_f = K_i + mgy_i - f_k d = 0 + (60 \text{ kg})(9.8 \text{ m/s}^2)(4.0 \text{ m}) - 0 = 2.35 \times 10^3 \text{ J}.$$

(b) Now it applies with a nonzero thermal term:

$$K_f = K_i + mgy_i - f_k d = 0 + (60 \text{ kg})(9.8 \text{ m/s}^2)(4.0 \text{ m}) - (500 \text{ N})(4.0 \text{ m}) = 352 \text{ J}.$$

102. (a) We assume his mass is between $m_1 = 50$ kg and $m_2 = 70$ kg (corresponding to a weight between 110 lb and 154 lb). His increase in gravitational potential energy is therefore in the range

$$m_1 gh \le \Delta U \le m_2 gh \quad \Rightarrow \quad 2\times10^5 \le \Delta U \le 3\times10^5$$

in SI units (J), where $h = 443$ m.

(b) The problem only asks for the amount of internal energy which converts into gravitational potential energy, so this result is the same as in part (a). But if we were to consider his *total* internal energy "output" (much of which converts to heat) we can expect that external climb is quite different from taking the stairs.

103. We use SI units so $m = 0.030$ kg and $d = 0.12$ m.

(a) Since there is no change in height (and we assume no changes in elastic potential energy), then $\Delta U = 0$ and we have

$$\Delta E_{\text{mech}} = \Delta K = -\frac{1}{2}mv_0^2 = -3.8\times10^3 \text{ J}.$$

where $v_0 = 500$ m/s and the final speed is zero.

(b) By Eq. 8-33 (with $W = 0$) we have $\Delta E_{\text{th}} = 3.8 \times 10^3$ J, which implies

$$f = \frac{\Delta E_{\text{th}}}{d} = 3.1\times10^4 \text{ N}$$

using Eq. 8-31 with f_k replaced by f (effectively generalizing that equation to include a greater variety of dissipative forces than just those obeying Eq. 6-2).

104. We work this in SI units and convert to horsepower in the last step. Thus,

$$v = (80\,\text{km/h})\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right) = 22.2\,\text{m/s}.$$

The force F_P needed to propel the car (of weight w and mass $m = w/g$) is found from Newton's second law:

$$F_{\text{net}} = F_P - F = ma = \frac{wa}{g}$$

where $F = 300 + 1.8v^2$ in SI units. Therefore, the power required is

$$P = \vec{F}_P \cdot \vec{v} = \left(F + \frac{wa}{g}\right)v = \left(300 + 1.8(22.2)^2 + \frac{(12000)(0.92)}{9.8}\right)(22.2) = 5.14\times10^4\ \text{W}$$

$$= \left(5.14\times10^4\ \text{W}\right)\left(\frac{1\ \text{hp}}{746\ \text{W}}\right) = 69\ \text{hp}.$$

105. (a) With $P = 1.5$ MW $= 1.5 \times 10^6$ W (assumed constant) and $t = 6.0$ min $= 360$ s, the work-kinetic energy theorem becomes

$$W = Pt = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right).$$

The mass of the locomotive is then

$$m = \frac{2Pt}{v_f^2 - v_i^2} = \frac{(2)\left(1.5\times10^6\ \mathrm{W}\right)(360\,\mathrm{s})}{(25\,\mathrm{m/s})^2 - (10\,\mathrm{m/s})^2} = 2.1\times10^6\ \mathrm{kg}.$$

(b) With t arbitrary, we use $Pt = \frac{1}{2}m\left(v^2 - v_i^2\right)$ to solve for the speed $v = v(t)$ as a function of time and obtain

$$v(t) = \sqrt{v_i^2 + \frac{2Pt}{m}} = \sqrt{(10)^2 + \frac{(2)\left(1.5\times10^6\right)t}{2.1\times10^6}} = \sqrt{100 + 1.5t}$$

in SI units (v in m/s and t in s).

(c) The force $F(t)$ as a function of time is

$$F(t) = \frac{P}{v(t)} = \frac{1.5\times10^6}{\sqrt{100+1.5t}}$$

in SI units (F in N and t in s).

(d) The distance d the train moved is given by

$$d = \int_0^t v(t')\,dt' = \int_0^{360}\left(100 + \frac{3}{2}t\right)^{1/2} dt = \frac{4}{9}\left(100 + \frac{3}{2}t\right)^{3/2}\Bigg|_0^{360} = 6.7\times10^3\ \mathrm{m}.$$

106. We take the bottom of the incline to be the $y = 0$ reference level. The incline angle is $\theta = 30°$. The distance along the incline d (measured from the bottom) is related to height y by the relation $y = d \sin \theta$.

(a) Using the conservation of energy, we have

$$K_0 + U_0 = K_{\text{top}} + U_{\text{top}} \Rightarrow \frac{1}{2}mv_0^2 + 0 = 0 + mgy$$

with $v_0 = 5.0\,\text{m/s}$. This yields $y = 1.3$ m, from which we obtain $d = 2.6$ m.

(b) An analysis of forces in the manner of Chapter 6 reveals that the magnitude of the friction force is $f_k = \mu_k mg \cos \theta$. Now, we write Eq. 8-33 as

$$\begin{aligned} K_0 + U_0 &= K_{\text{top}} + U_{\text{top}} + f_k d \\ \frac{1}{2}mv_0^2 + 0 &= 0 + mgy + f_k d \\ \frac{1}{2}mv_0^2 &= mgd \sin\theta + \mu_k mgd \cos\theta \end{aligned}$$

which — upon canceling the mass and rearranging — provides the result for d:

$$d = \frac{v_0^2}{2g(\mu_k \cos\theta + \sin\theta)} = 1.5 \text{ m}.$$

(c) The thermal energy generated by friction is $f_k d = \mu_k mgd \cos \theta = 26$ J.

(d) The slide back down, from the height $y = 1.5 \sin 30°$ is also described by Eq. 8-33. With ΔE_{th} again equal to 26 J, we have

$$K_{\text{top}} + U_{\text{top}} = K_{\text{bot}} + U_{\text{bot}} + f_k d \Rightarrow 0 + mgy = \frac{1}{2}mv_{\text{bot}}^2 + 0 + 26$$

from which we find $v_{\text{bot}} = 2.1 \text{ m/s}$.

107. (a) The effect of a (sliding) friction is described in terms of energy dissipated as shown in Eq. 8-31. We have

$$\Delta E = K + \frac{1}{2}k(0.08)^2 - \frac{1}{2}k(0.10)^2 = -f_k\,(0.02)$$

where distances are in meters and energies are in Joules. With k = 4000 N/m and $f_k = 80$ N, we obtain K = 5.6 J.

(b) In this case, we have d = 0.10 m. Thus,

$$\Delta E = K + 0 - \frac{1}{2}k(0.10)^2 = -f_k(0.10)$$

which leads to K = 12 J.

(c) We can approach this two ways. One way is to examine the dependence of energy on the variable d:

$$\Delta E = K + \frac{1}{2}k(d_0 - d)^2 - \frac{1}{2}kd_0^2 = -f_k d$$

where d_0 = 0.10 m, and solving for K as a function of d:

$$K = -\frac{1}{2}kd^2 + (kd_0)d - f_k d\,.$$

In this first approach, we could work through the $\frac{dK}{dd} = 0$ condition (or with the special capabilities of a graphing calculator) to obtain the answer $K_{\max} = \frac{1}{2k}(kd_0 - f_k)^2$. In the second (and perhaps easier) approach, we note that K is maximum where v is maximum — which is where $a = 0 \Rightarrow$ equilibrium of forces. Thus, the second approach simply solves for the equilibrium position

$$\left|F_{\text{spring}}\right| = f_k \Rightarrow kx = 80.$$

Thus, with k = 4000 N/m we obtain x = 0.02 m. But $x = d_0 - d$ so this corresponds to d = 0.08 m. Then the methods of part (a) lead to the answer $K_{\max}$ = 12.8 J ≈ 13 J.

108. We assume his initial kinetic energy (when he jumps) is negligible. Then, his initial gravitational potential energy measured relative to where he momentarily stops is what becomes the elastic potential energy of the stretched net (neglecting air friction). Thus,

$$U_{\text{net}} = U_{\text{grav}} = mgh$$

where h = 11.0 m + 1.5 m = 12.5 m. With m = 70 kg, we obtain U_{net} = 8580 J $\approx 8.6 \times 10^3$ J.

109. The connection between angle θ (measured from vertical) and height h (measured from the lowest point, which is our choice of reference position in computing the gravitational potential energy mgh) is given by $h = L(1 - \cos\theta)$ where L is the length of the pendulum.

(a) Using this formula (or simply using intuition) we see the initial height is $h_1 = 2L$, and of course $h_2 = 0$. We use energy conservation in the form of Eq. 8-17.

$$K_1 + U_1 = K_2 + U_2$$
$$0 + mg\,(2L) = \frac{1}{2}mv^2 + 0$$

This leads to $v = 2\sqrt{gL}$. With $L = 0.62$ m, we have

$$v = 2\sqrt{(9.8\text{ m/s}^2)(0.62\text{ m})} = 4.9\text{ m/s}.$$

(b) The ball is in circular motion with the center of the circle above it, so $\vec{a} = v^2/r$ upward, where $r = L$. Newton's second law leads to

$$T - mg = m\frac{v^2}{r} \Rightarrow T = m\left(g + \frac{4gL}{L}\right) = 5\,mg.$$

With $m = 0.092$ kg, the tension is given by $T = 4.5$ N.

(c) The pendulum is now started (with zero speed) at $\theta_i = 90°$ (that is, $h_i = L$), and we look for an angle θ such that $T = mg$. When the ball is moving through a point at angle θ, then Newton's second law applied to the axis along the rod yields

$$T - mg\cos\theta = m\frac{v^2}{r}$$

which (since $r = L$) implies $v^2 = gL(1 - \cos\theta)$ at the position we are looking for. Energy conservation leads to

$$K_i + U_i = K + U$$
$$0 + mgL = \frac{1}{2}mv^2 + mgL\,(1 - \cos\theta)$$
$$gL = \frac{1}{2}(gL(1 - \cos\theta)) + gL\,(1 - \cos\theta)$$

where we have divided by mass in the last step. Simplifying, we obtain

$$\theta = \cos^{-1}\left(\frac{1}{3}\right) = 71°.$$

(d) Since the angle found in (c) is independent of the mass, the result remains the same if the mass of the ball is changed.

110. We take her original elevation to be the $y = 0$ reference level and observe that the top of the hill must consequently have $y_A = R(1 - \cos 20°) = 1.2$ m, where R is the radius of the hill. The mass of the skier is 600/9.8 = 61 kg.

(a) Applying energy conservation, Eq. 8-17, we have

$$K_B + U_B = K_A + U_A \Rightarrow K_B + 0 = K_A + mgy_A.$$

Using $K_B = \frac{1}{2}(61\,\text{kg})(8.0\,\text{m/s})^2$, we obtain $K_A = 1.2 \times 10^3$ J. Thus, we find the speed at the hilltop is

$$v = \sqrt{2K/m} = 6.4 \text{ m/s}.$$

Note: one might wish to check that the skier stays in contact with the hill — which is indeed the case, here. For instance, at A we find $v^2/r \approx 2$ m/s^2 which is considerably less than g.

(b) With $K_A = 0$, we have

$$K_B + U_B = K_A + U_A \Rightarrow K_B + 0 = 0 + mgy_A$$

which yields $K_B = 724$ J, and the corresponding speed is $v = \sqrt{2K/m} = 4.9$ m/s.

(c) Expressed in terms of mass, we have

$$K_B + U_B = K_A + U_A \Rightarrow$$
$$\frac{1}{2}mv_B^2 + mgy_B = \frac{1}{2}mv_A^2 + mgy_A \,.$$

Thus, the mass m cancels, and we observe that solving for speed does not depend on the value of mass (or weight).

111. (a) At the top of its flight, the vertical component of the velocity vanishes, and the horizontal component (neglecting air friction) is the same as it was when it was thrown. Thus,

$$K_{\text{top}} = \frac{1}{2}mv_x^2 = \frac{1}{2}(0.050\,\text{kg})\left((8.0\,\text{m/s})\cos 30°\right)^2 = 1.2\text{ J}.$$

(b) We choose the point 3.0 m below the window as the reference level for computing the potential energy. Thus, equating the mechanical energy when it was thrown to when it is at this reference level, we have (with SI units understood)

$$mgy_0 + K_0 = K$$
$$m(9.8)(3.0) + \frac{1}{2}m(8.0)^2 = \frac{1}{2}mv^2$$

which yields (after canceling m and simplifying) $v = 11$ m/s.

(c) As mentioned, m cancels — and is therefore not relevant to that computation.

(d) The v in the kinetic energy formula is the magnitude of the velocity vector; it does not depend on the direction.

112. (a) The rate of change of the gravitational potential energy is

$$\frac{dU}{dt} = mg\frac{dy}{dt} = -mg|v| = -(68)(9.8)(59) = -3.9\times10^4 \text{ J/s}.$$

Thus, the gravitational energy is being reduced at the rate of 3.9×10^4 W.

(b) Since the velocity is constant, the rate of change of the kinetic energy is zero. Thus the rate at which the mechanical energy is being dissipated is the same as that of the gravitational potential energy (3.9×10^4 W).

113. The water has gained

$$\Delta K = \frac{1}{2}\ (10\ \text{kg})(13\ \text{m/s})^2 - \frac{1}{2}\ (10\ \text{kg})(3.2\ \text{m/s})^2 = 794\ \text{J}$$

of kinetic energy, and it has lost $\Delta U = (10\ \text{kg})(9.8\ \text{m/s}^2)(15\ \text{m}) = 1470\ \text{J}$.

of potential energy (the lack of agreement between these two values is presumably due to transfer of energy into thermal forms). The ratio of these values is 0.54 = 54%. The mass of the water cancels when we take the ratio, so that the assumption (stated at the end of the problem: m = 10 kg) is not needed for the final result.

114. (a) The integral (see Eq. 8-6, where the value of U at $x = \infty$ is required to vanish) is straightforward. The result is $U(x) = -Gm_1m_2/x$.

(b) One approach is to use Eq. 8-5, which means that we are effectively doing the integral of part (a) all over again. Another approach is to use our result from part (a) (and thus use Eq. 8-1). Either way, we arrive at

$$W = \frac{G\,m_1\,m_2}{x_1} - \frac{G\,m_1\,m_2}{x_1 + d} = \frac{G\,m_1\,m_2\,d}{x_1(x_1 + d)} \ .$$

115. (a) During one second, the decrease in potential energy is

$$-\Delta U = mg(-\Delta y) = (5.5\times10^6\ \text{kg})\left(9.8\,\text{m/s}^2\right)(50\ \text{m}) = 2.7\times10^9\ \text{J}$$

where $+y$ is upward and $\Delta y = y_f - y_i$.

(b) The information relating mass to volume is not needed in the computation. By Eq. 8-40 (and the SI relation W = J/s), the result follows:

$$P = (2.7\times10^9\ \text{J})/(1\ \text{s}) = 2.7\times10^9\ \text{W}.$$

(c) One year is equivalent to $24 \times 365.25 = 8766$ h which we write as 8.77 kh. Thus, the energy supply rate multiplied by the cost and by the time is

$$(2.7\times10^9\ \text{W})(8.77\ \text{kh})\left(\frac{1\ \text{cent}}{1\ \text{kWh}}\right) = 2.4\times10^{10}\,\text{cents} = \$2.4\times10^8.$$

116. (a) The kinetic energy K of the automobile of mass m at $t = 30$ s is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1500\,\text{kg})\left((72\,\text{km/h})\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right)\right)^2 = 3.0\times10^5\,\text{J}.$$

(b) The average power required is

$$P_{\text{avg}} = \frac{\Delta K}{\Delta t} = \frac{3.0\times10^5\,\text{J}}{30\,\text{s}} = 1.0\times10^4\,\text{W}.$$

(c) Since the acceleration a is constant, the power is $P = Fv = mav = ma(at) = ma^2t$ using Eq. 2-11. By contrast, from part (b), the average power is $P_{\text{avg}} = \frac{mv^2}{2t}$ which becomes $\frac{1}{2}ma^2t$ when $v = at$ is again utilized. Thus, the instantaneous power at the end of the interval is twice the average power during it: $P = 2P_{\text{avg}} = (2)(1.0\times10^4\,\text{W}) = 2.0\times10^4\,\text{W}$.

117. (a) The remark in the problem statement that the forces can be associated with potential energies is illustrated as follows: the work from $x = 3.00$ m to $x = 2.00$ m is

$$W = F_2\,\Delta x = (5.00\text{ N})(-1.00\text{ m}) = -5.00\text{ J},$$

so the potential energy at $x = 2.00$ m is $U_2 = +5.00$ J.

(b) Now, it is evident from the problem statement that $E_{\max} = 14.0$ J, so the kinetic energy at $x = 2.00$ m is

$$K_2 = E_{\max} - U_2 = 14.0 - 5.00 = 9.00\text{ J}.$$

(c) The work from $x = 2.00$ m to $x = 0$ is $W = F_1\,\Delta x = (3.00\text{ N})(-2.00\text{ m}) = -6.00$ J, so the potential energy at $x = 0$ is

$$U_0 = 6.00\text{ J} + U_2 = (6.00 + 5.00)\text{ J} = 11.0\text{ J}.$$

(d) Similar reasoning to that presented in part (a) then gives

$$K_0 = E_{\max} - U_0 = (14.0 - 11.0)\text{ J} = 3.00\text{ J}.$$

(e) The work from $x = 8.00$ m to $x = 11.0$ m is $W = F_3\,\Delta x = (-4.00\text{ N})(3.00\text{ m}) = -12.0$ J, so the potential energy at $x = 11.0$ m is $U_{11} = 12.0$ J.

(f) The kinetic energy at $x = 11.0$ m is therefore

$$K_{11} = E_{\max} - U_{11} = (14.0 - 12.0)\text{ J} = 2.00\text{ J}.$$

(g) Now we have $W = F_4\,\Delta x = (-1.00\text{ N})(1.00\text{ m}) = -1.00$ J, so the potential energy at $x = 12.0$ m is

$$U_{12} = 1.00\text{ J} + U_{11} = (1.00 + 12.0)\text{ J} = 13.0\text{ J}.$$

(h) Thus, the kinetic energy at $x = 12.0$ m is

$$K_{12} = E_{\max} - U_{12} = (14.0 - 13.0) = 1.00\text{ J}.$$

(i) There is no work done in this interval (from $x = 12.0$ m to $x = 13.0$ m) so the answers are the same as in part (g): $U_{12} = 13.0$ J.

(j) There is no work done in this interval (from $x = 12.0$ m to $x = 13.0$ m) so the answers are the same as in part (h): $K_{12} = 1.00$ J.

(k) Although the plot is not shown here, it would look like a "potential well" with piecewise-sloping sides: from $x = 0$ to $x = 2$ (SI units understood) the graph if U is a decreasing line segment from 11 to 5, and from $x = 2$ to $x = 3$, it then heads down to zero, where it stays until $x = 8$, where it starts increasing to a value of 12 (at $x = 11$), and then in another positive-slope line segment it increases to a value of 13 (at $x = 12$). For $x > 12$ its value does not change (this is the "top of the well").

(l) The particle can be thought of as "falling" down the $0 < x < 3$ slopes of the well, gaining kinetic energy as it does so, and certainly is able to reach $x = 5$. Since $U = 0$ at $x = 5$, then it's initial potential energy (11 J) has completely converted to kinetic: now $K = 11.0$ J.

(m) This is not sufficient to climb up and out of the well on the large x side ($x > 8$), but does allow it to reach a "height" of 11 at $x = 10.8$ m. As discussed in section 8-5, this is a "turning point" of the motion.

(n) Next it "falls" back down and rises back up the small x slopes until it comes back to its original position. Stating this more carefully, when it is (momentarily) stopped at $x = 10.8$ m it is accelerated to the left by the force $\vec{F}_3$; it gains enough speed as a result that it eventually is able to return to $x = 0$, where it stops again.

118. (a) At $x = 5.00$ m the potential energy is zero, and the kinetic energy is

$$K = \frac{1}{2}\, mv^2 = \frac{1}{2}\,(2.00\text{ kg})(3.45\text{ m/s})^2 = 11.9\text{ J}.$$

The total energy, therefore, is great enough to reach the point $x = 0$ where $U = 11.0$ J, with a little "left over" (11.9 J – 11.0 J = 0.9025 J). This is the kinetic energy at $x = 0$, which means the speed there is

$$v = \sqrt{2(0.9025\text{ J})/(2\text{ kg})} = 0.950\text{ m/s}.$$

It has now come to a stop, therefore, so it has not encountered a turning point.

(b) The total energy (11.9 J) is equal to the potential energy (in the scenario where it is initially moving rightward) at $x = 10.9756 \approx 11.0$ m. This point may be found by interpolation or simply by using the work-kinetic-energy theorem:

$$K_f = K_i + W = 0 \quad\Rightarrow\quad 11.9025 + (-4)d = 0 \quad\Rightarrow\quad d = 2.9756 \approx 2.98$$

(which when added to $x = 8.00$ [the point where F_3 begins to act] gives the correct result). This provides a turning point for the particle's motion.

119. (a) During the final $d = 12$ m of motion, we use

$$K_1 + U_1 = K_2 + U_2 + f_k d$$
$$\frac{1}{2}mv^2 + 0 = 0 + 0 + f_k d$$

where $v = 4.2$ m/s. This gives $f_k = 0.31$ N. Therefore, the thermal energy change is $f_k d = 3.7$ J.

(b) Using $f_k = 0.31$ N we obtain $f_k d_{\text{total}} = 4.3$ J for the thermal energy generated by friction; here, $d_{\text{total}} = 14$ m.

(c) During the initial $d' = 2$ m of motion, we have

$$K_0 + U_0 + W_{\text{app}} = K_1 + U_1 + f_k d' \Rightarrow 0 + 0 + W_{\text{app}} = \frac{1}{2}mv^2 + 0 + f_k d'$$

which essentially combines Eq. 8-31 and Eq. 8-33. This leads to the result $W_{\text{app}} = 4.3$ J, and — reasonably enough — is the same as our answer in part (b).

120. (a) The table shows that the force is $+(3.0\text{ N})\hat{\text{i}}$ while the displacement is in the $+x$ direction ($\vec{d} = +(3.0\text{ m})\hat{\text{i}}$), and it is $-(3.0\text{ N})\hat{\text{i}}$ while the displacement is in the $-x$ direction. Using Eq. 7-8 for each part of the trip, and adding the results, we find the work done is 18 J. This is not a conservative force field; if it had been, then the net work done would have been zero (since it returned to where it started).

(b) This, however, is a conservative force field, as can be easily verified by calculating that the net work done here is zero.

(c) The two integrations that need to be performed are each of the form $\int 2x\,dx$ so that we are adding two equivalent terms, where each equals x^2 (evaluated at $x = 4$, minus its value at $x = 1$). Thus, the work done is $2(4^2 - 1^2) = 30$ J.

(d) This is another conservative force field, as can be easily verified by calculating that the net work done here is zero.

(e) The forces in (b) and (d) are conservative.

121. We use Eq. 8-20.

(a) The force at $x = 2.0$ m is

$$F = -\frac{dU}{dx} \approx -\frac{-(17.5\text{ J}) - (-2.8\text{ J})}{4.0\text{ m} - 1.0\text{ m}} = 4.9\,\text{N}.$$

(b) The force points in the $+x$ direction (but there is some uncertainty in reading the graph which makes the last digit not very significant).

(c) The total mechanical energy at $x = 2.0$ m is

$$E = \frac{1}{2}mv^2 + U \approx \frac{1}{2}(2.0)(-1.5)^2 - 7.7 = -5.5$$

in SI units (Joules). Again, there is some uncertainty in reading the graph which makes the last digit not very significant. At that level (–5.5 J) on the graph, we find two points where the potential energy curve has that value — at $x \approx 1.5$ m and $x \approx 13.5$ m. Therefore, the particle remains in the region $1.5 < x < 13.5$ m. The left boundary is at $x = 1.5$ m.

(d) From the above results, the right boundary is at $x = 13.5$ m.

(e) At $x = 7.0$ m, we read $U \approx -17.5$ J. Thus, if its total energy (calculated in the previous part) is $E \approx -5.5$ J, then we find

$$\frac{1}{2}mv^2 = E - U \approx 12\text{ J} \Rightarrow v = \sqrt{\frac{2}{m}(E - U)} \approx 3.5\ \text{m/s}$$

where there is certainly room for disagreement on that last digit for the reasons cited above.

122. The connection between angle θ (measured from vertical) and height h (measured from the lowest point, which is our choice of reference position in computing the gravitational potential energy) is given by $h = L(1 - \cos\theta)$ where L is the length of the pendulum.

(a) We use energy conservation in the form of Eq. 8-17.

$$K_1 + U_1 = K_2 + U_2$$
$$0 + mgL(1 - \cos\theta_1) = \frac{1}{2}mv_2^2 + mgL(1 - \cos\theta_2)$$

With L = 1.4 m, θ_1 = 30°, and θ_2 = 20°, we have

$$v_2 = \sqrt{2gL(\cos\theta_2 - \cos\theta_1)} = 1.4 \text{ m/s}.$$

(b) The maximum speed v_3 is at the lowest point. Our formula for h gives $h_3 = 0$ when θ_3 = 0°, as expected. From

$$K_1 + U_1 = K_3 + U_3$$
$$0 + mgL(1 - \cos\theta_1) = \frac{1}{2}mv_3^2 + 0$$

we obtain $v_3 = 1.9 \text{ m/s}$.

(c) We look for an angle θ_4 such that the speed there is $v_4 = v_3/3$. To be as accurate as possible, we proceed algebraically (substituting $v_3^2 = 2gL(1 - \cos\theta_1)$ at the appropriate place) and plug numbers in at the end. Energy conservation leads to

$$K_1 + U_1 = K_4 + U_4$$
$$0 + mgL(1 - \cos\theta_1) = \frac{1}{2}mv_4^2 + mgL(1 - \cos\theta_4)$$
$$mgL(1 - \cos\theta_1) = \frac{1}{2}m\frac{v_3^2}{9} + mgL(1 - \cos\theta_4)$$
$$-gL\cos\theta_1 = \frac{1}{2}\frac{2gL(1 - \cos\theta_1)}{9} - gL\cos\theta_4$$

where in the last step we have subtracted out mgL and then divided by m. Thus, we obtain

$$\theta_4 = \cos^{-1}\left(\frac{1}{9} + \frac{8}{9}\cos\theta_1\right) = 28.2° \approx 28°.$$

123. Converting to SI units, $v_0 = 8.3\ \text{m/s}$ and $v = 11.1\ \text{m/s}$. The incline angle is $\theta = 5.0°$. The height difference between the car's highest and lowest points is (50 m) sin θ= 4.4 m. We take the lowest point (the car's final reported location) to correspond to the y = 0 reference level.

(a) Using Eq. 8-31 and Eq. 8-33, we find

$$f_k d = -\Delta K - \Delta U \Rightarrow f_k d = \frac{1}{2} m \left(v_0^2 - v^2\right) + mgy_0 \ .$$

Therefore, the mechanical energy reduction (due to friction) is $f_k\, d = 2.4 \times 10^4$ J.

(b) With d = 50 m, we solve for f_k and obtain 4.7×10^2 N.

124. Equating the mechanical energy at his initial position (as he emerges from the canon, where we set the reference level for computing potential energy) to his energy as he lands, we obtain

$$K_i = K_f + U_f$$

$$\frac{1}{2}(60\,\mathrm{kg})(16\,\mathrm{m/s})^2 = K_f + (60\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(3.9\,\mathrm{m})$$

which leads to $K_f = 5.4 \times 10^3$ J.

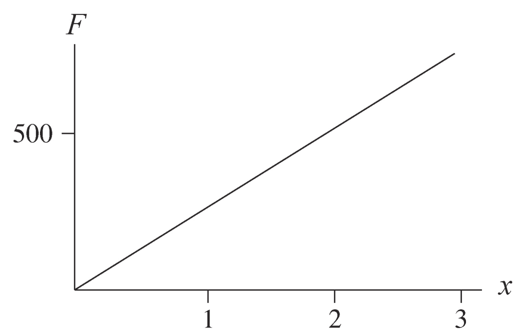
125. (a) The compression is "spring-like" so the maximum force relates to the distance x by Hooke's law:

$$F_x = kx \Rightarrow x = \frac{750}{2.5\times10^5} = 0.0030\,\text{m}.$$

(b) The work is what produces the "spring-like" potential energy associated with the compression. Thus, using Eq. 8-11,

$$W = \frac{1}{2}kx^2 = \frac{1}{2}(2.5\times10^5)(0.0030)^2 = 1.1\,\text{J}.$$

(c) By Newton's third law, the force F exerted by the tooth is equal and opposite to the "spring-like" force exerted by the licorice, so the graph of F is a straight line of slope k. We plot F (in newtons) versus x (in millimeters); both are taken as positive.



(d) As mentioned in part (b), the spring potential energy expression is relevant. Now, whether or not we can ignore dissipative processes is a deeper question. In other words, it seems unlikely that — if the tooth at any moment were to reverse its motion — that the licorice could "spring back" to its original shape. Still, to the extent that $U = \frac{1}{2}kx^2$ applies, the graph is a parabola (not shown here) which has its vertex at the origin and is either concave upward or concave downward depending on how one wishes to define the sign of F (the connection being $F = -dU/dx$).

(e) As a crude estimate, the area under the curve is roughly half the area of the entire plotting-area (8000 N by 12 mm). This leads to an approximate work of

$\frac{1}{2}$ (8000 N) (0.012 m) ≈ 50 J. Estimates in the range $40 \le W \le 50$ J are acceptable.

(f) Certainly dissipative effects dominate this process, and we cannot assign it a meaningful potential energy.

126. (a) This part is essentially a free-fall problem, which can be easily done with Chapter 2 methods. Instead, choosing energy methods, we take $y = 0$ to be the ground level.

$$K_i + U_i = K + U \Rightarrow 0 + mgy_i = \frac{1}{2}mv^2 + 0$$

Therefore $v = \sqrt{2gy_i} = 9.2\,\text{m/s}$, where $y_i = 4.3$ m.

(b) Eq. 8-29 provides $\Delta E_{\text{th}} = f_k d$ for thermal energy generated by the kinetic friction force. We apply Eq. 8-31:

$$K_i + U_i = K + U \Rightarrow 0 + mgy_i = \frac{1}{2}mv^2 + 0 + f_k d\ .$$

With $d = y_i$, $m = 70$ kg and $f_k = 500$ N, this yields $v = 4.8$ m/s.

127. (a) When there is no change in potential energy, Eq. 8-24 leads to

$$W_{\text{app}} = \Delta K = \frac{1}{2} m \left(v^2 - v_0^2\right).$$

Therefore, $\Delta E = 6.0 \times 10^3$ J.

(b) From the above manipulation, we see $W_{\text{app}} = 6.0 \times 10^3$ J. Also, from Chapter 2, we know that $\Delta t = \Delta v / a = 10$ s. Thus, using Eq. 7-42,

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{6.0 \times 10^3}{10} = 600 \text{ W}.$$

(c) and (d) The constant applied force is ma = 30 N and clearly in the direction of motion, so Eq. 7-48 provides the results for instantaneous power

$$P = \vec{F} \cdot \vec{v} = \begin{cases} 300 \text{ W} & \text{for } v = 10 \text{ m/s} \\ 900 \text{ W} & \text{for } v = 30 \text{ m/s} \end{cases}$$

We note that the average of these two values agrees with the result in part (b).

128. The distance traveled up the incline can be figured with Chapter 2 techniques: $v^2 = v_0^2 + 2a\Delta x \;\; \rightarrow \Delta x = 200 \text{ m}$. This corresponds to an increase in height equal to y = (200 m) sin θ = 17 m, where $\theta = 5.0°$. We take its initial height to be y = 0.

(a) Eq. 8-24 leads to

$$W_{\text{app}} = \Delta E = \frac{1}{2} m \left(v^2 - v_0^2\right) + mgy \; .$$

Therefore, $\Delta E = 8.6 \times 10^3 \text{ J}$.

(b) From the above manipulation, we see $W_{\text{app}} = 8.6 \times 10^3$ J. Also, from Chapter 2, we know that $\Delta t = \Delta v / a = 10 \text{ s}$. Thus, using Eq. 7-42,

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{8.6 \times 10^3}{10} = 860 \text{ W}$$

where the answer has been rounded off (from the 856 value that is provided by the calculator).

(c) and (d) Taking into account the component of gravity along the incline surface, the applied force is $ma + mg \sin \theta$ = 43 N and clearly in the direction of motion, so Eq. 7-48 provides the results for instantaneous power

$$P = \vec{F} \cdot \vec{v} = \begin{cases} 430 \text{ W} & \text{for } v = 10 \text{ m/s} \\ 1300 \text{ W} & \text{for } v = 30 \text{ m/s} \end{cases}$$

where these answers have been rounded off (from 428 and 1284, respectively). We note that the average of these two values agrees with the result in part (b).

129. We want to convert (at least in theory) the water that falls through h = 500 m into electrical energy. The problem indicates that in one year, a volume of water equal to $A\Delta z$ lands in the form of rain on the country, where $A = 8 \times 10^{12}$ m^2 and Δz = 0.75 m. Multiplying this volume by the density ρ = 1000 kg/m^3 leads to

$$m_{\text{total}} = \rho A\Delta z = (1000)(8\times10^{12})(0.75) = 6\times10^{15}\,\text{kg}$$

for the mass of rainwater. One-third of this "falls" to the ocean, so it is $m = 2 \times 10^{15}$ kg that we want to use in computing the gravitational potential energy mgh (which will turn into electrical energy during the year). Since a year is equivalent to 3.2×10^7 s, we obtain

$$P_{\text{avg}} = \frac{(2\times10^{15})(9.8)(500)}{3.2\times10^7} = 3.1\times10^{11}\,\text{W}.$$

130. The spring is relaxed at $y = 0$, so the elastic potential energy (Eq. 8-11) is $U_{\text{el}} = \frac{1}{2}ky^2$. The total energy is conserved, and is zero (determined by evaluating it at its initial position). We note that U is the same as ΔU in these manipulations. Thus, we have

$$0 = K + U_g + U_e \quad \Rightarrow \quad K = -U_g - U_e$$

where $U_g = mgy$ = (20 N)y with y in meters (so that the energies are in Joules). We arrange the results in a table:

position y	–0.05	–0.10	–0.15	–0.20
K	(a) 0.75	(d) 1.0	(g) 0.75	(j) 0
U_g	(b) –1.0	(e) –2.0	(h) –3.0	(k) –4.0
U_e	(c) 0.25	(f) 1.0	(i) 2.25	(l) 4.0

131. The power generation (assumed constant, so average power is the same as instantaneous power) is

$$P = \frac{mgh}{t} = \frac{(3/4)(1200\,\mathrm{m}^3)(10^3\,\mathrm{kg/m}^3)(9.8\,\mathrm{m/s}^2)(100\,\mathrm{m})}{1.0\,\mathrm{s}} = 8.80\times 10^8\ \mathrm{W}.$$

132. The style of reasoning used here is presented in §8-5.

(a) The horizontal line representing E_1 intersects the potential energy curve at a value of $r \approx 0.07$ nm and seems not to intersect the curve at larger r (though this is somewhat unclear since $U(r)$ is graphed only up to $r = 0.4$ nm). Thus, if m were propelled towards M from large r with energy E_1 it would "turn around" at 0.07 nm and head back in the direction from which it came.

(b) The line representing E_2 has two intersection points $r_1 \approx 0.16$ nm and $r_2 \approx 0.28$ nm with the $U(r)$ plot. Thus, if m starts in the region $r_1 < r < r_2$ with energy E_2 it will bounce back and forth between these two points, presumably forever.

(c) At $r = 0.3$ nm, the potential energy is roughly $U = -1.1 \times 10^{-19}$ J.

(d) With $M >> m$, the kinetic energy is essentially just that of m. Since $E = 1 \times 10^{-19}$ J, its kinetic energy is $K = E - U \approx 2.1 \times 10^{-19}$ J.

(e) Since force is related to the slope of the curve, we must (crudely) estimate $|F| \approx 1 \times 10^{-9}\,\text{N}$ at this point. The sign of the slope is positive, so by Eq. 8-20, the force is negative-valued. This is interpreted to mean that the atoms are attracted to each other.

(f) Recalling our remarks in the previous part, we see that the sign of F is positive (meaning it's repulsive) for $r < 0.2$ nm.

(g) And the sign of F is negative (attractive) for $r > 0.2$ nm.

(h) At $r = 0.2$ nm, the slope (hence, F) vanishes.

133. (a) Sample Problem 8-3 illustrates simple energy conservation in a similar situation, and derives the frequently encountered relationship: v = $\sqrt{2gh}$. In our present problem, the height change is equal to the rod length L. Thus, using the suggested notation for the speed, we have $v_o = \sqrt{2gL}$.

(b) At B the speed is (from Eq. 8-17)

$$v = \sqrt{v_0^2 + 2gL} = \sqrt{4gL} \ .$$

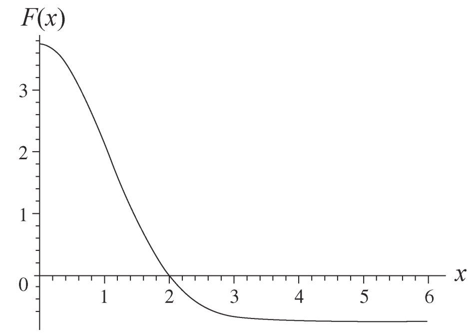
The direction of the centripetal acceleration (v^2/r = $4gL/L$ = $4g$) is upward (at that moment), as is the tension force. Thus, Newton's second law gives

$$T - mg = m(4g) \ \Rightarrow \ T = 5mg.$$

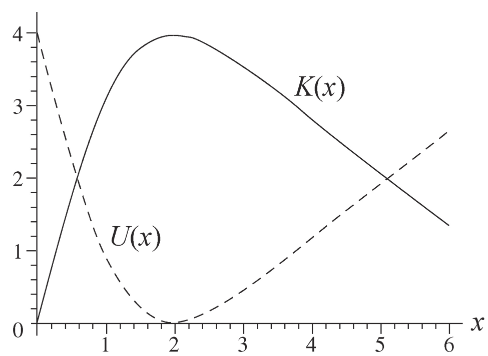
(c) The difference in height between C and D is L, so the "loss" of mechanical energy (which goes into thermal energy) is $-mgL$.

(d) The difference in height between B and D is $2L$, so the total "loss" of mechanical energy (which all goes into thermal energy) is $-2mgL$.

134. (a) The force (SI units understood) from Eq. 8-20 is plotted in the graph below.



(b) The potential energy U(x) and the kinetic energy $K(x)$ are shown in the next. The potential energy curve begins at 4 and drops (until about $x = 2$); the kinetic energy curve is the one that starts at zero and rises (until about $x = 2$).



135. Let the amount of stretch of the spring be x. For the object to be in equilibrium

$$kx - mg = 0 \Rightarrow x = mg/k.$$

Thus the gain in elastic potential energy for the spring is

$$\Delta U_e = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = \frac{m^2g^2}{2k}$$

while the loss in the gravitational potential energy of the system is

$$-\Delta U_g = mgx = mg\left(\frac{mg}{k}\right) = \frac{m^2g^2}{k}$$

which we see (by comparing with the previous expression) is equal to $2\Delta U_e$. The reason why $|\Delta U_g| \neq \Delta U_e$ is that, since the object is slowly lowered, an upward external force (e.g., due to the hand) must have been exerted on the object during the lowering process, preventing it from accelerating downward. This force does *negative* work on the object, reducing the total mechanical energy of the system.

Chapter 9

1. We use Eq. 9-5 to solve for (x_3, y_3).

(a) The *x* coordinates of the system's center of mass is:

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(2.00\text{ kg})(-1.20\text{ m}) + (4.00\text{ kg})(0.600\text{ m}) + (3.00\text{ kg})x_3}{2.00\text{ kg} + 4.00\text{ kg} + 3.00\text{ kg}}$$
$$= -0.500\text{ m}.$$

Solving the equation yields $x_3 = -1.50$ m.

(b) The *y* coordinates of the system's center of mass is:

$$y_{\text{com}} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(2.00\text{ kg})(0.500\text{ m}) + (4.00\text{ kg})(-0.750\text{ m}) + (3.00\text{ kg})y_3}{2.00\text{ kg} + 4.00\text{ kg} + 3.00\text{ kg}}$$
$$= -0.700\text{ m}.$$

Solving the equation yields $y_3 = -1.43$ m.

2. Our notation is as follows: $x_1 = 0$ and $y_1 = 0$ are the coordinates of the $m_1 = 3.0$ kg particle; $x_2 = 2.0$ m and $y_2 = 1.0$ m are the coordinates of the $m_2 = 4.0$ kg particle; and, $x_3 = 1.0$ m and $y_3 = 2.0$ m are the coordinates of the $m_3 = 8.0$ kg particle.

(a) The x coordinate of the center of mass is

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{0 + (4.0\text{ kg})(2.0\text{ m}) + (8.0\text{ kg})(1.0\text{ m})}{3.0\text{ kg} + 4.0\text{ kg} + 8.0\text{ kg}} = 1.1\text{ m}.$$

(b) The y coordinate of the center of mass is

$$y_{\text{com}} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{0 + (4.0\text{ kg})(1.0\text{ m}) + (8.0\text{ kg})(2.0\text{ m})}{3.0\text{ kg} + 4.0\text{ kg} + 8.0\text{ kg}} = 1.3\text{ m}.$$

(c) As the mass of m_3, the topmost particle, is increased, the center of mass shifts toward that particle. As we approach the limit where m_3 is infinitely more massive than the others, the center of mass becomes infinitesimally close to the position of m_3.

3. Since the plate is uniform, we can split it up into three rectangular pieces, with the mass of each piece being proportional to its area and its center of mass being at its geometric center. We'll refer to the large 35 cm × 10 cm piece (shown to the left of the y axis in Fig. 9-38) as section 1; it has 63.6% of the total area and its center of mass is at $(x_1, y_1) = (-5.0 \text{ cm}, -2.5 \text{ cm})$. The top 20 cm × 5 cm piece (section 2, in the first quadrant) has 18.2% of the total area; its center of mass is at $(x_2, y_2) = (10 \text{ cm}, 12.5 \text{ cm})$. The bottom 10 cm x 10 cm piece (section 3) also has 18.2% of the total area; its center of mass is at $(x_3, y_3) = (5 \text{ cm}, -15 \text{ cm})$.

(a) The x coordinate of the center of mass for the plate is

$$x_{\text{com}} = (0.636)x_1 + (0.182)x_2 + (0.182)x_3 = -0.45 \text{ cm} .$$

(b) The y coordinate of the center of mass for the plate is

$$y_{\text{com}} = (0.636)y_1 + (0.182)y_2 + (0.182)y_3 = -2.0 \text{ cm} .$$

4. We will refer to the arrangement as a "table." We locate the coordinate origin at the left end of the tabletop (as shown in Fig. 9-37). With $+x$ rightward and $+y$ upward, then the center of mass of the right leg is at $(x,y) = (+L, -L/2)$, the center of mass of the left leg is at $(x,y) = (0, -L/2)$, and the center of mass of the tabletop is at $(x,y) = (L/2, 0)$.

(a) The x coordinate of the (whole table) center of mass is

$$x_{\text{com}} = \frac{M(+L) + M(0) + 3M(+L/2)}{M + M + 3M} = 0.5L\,.$$

With $L = 22$ cm, we have $x_{\text{com}} = 11$ cm.

(b) The y coordinate of the (whole table) center of mass is

$$y_{\text{com}} = \frac{M(-L/2) + M(-L/2) + 3M(0)}{M + M + 3M} = -\frac{L}{5},$$

or $y_{\text{com}} = -4.4$ cm.

From the coordinates, we see that the whole table center of mass is a small distance 4.4 cm directly below the middle of the tabletop.

5. (a) By symmetry the center of mass is located on the axis of symmetry of the molecule – the y axis. Therefore $x_{com} = 0$.

(b) To find y_{com}, we note that $3m_H y_{com} = m_N(y_N - y_{com})$, where y_N is the distance from the nitrogen atom to the plane containing the three hydrogen atoms:

$$y_N = \sqrt{\left(10.14\times10^{-11}\text{ m}\right)^2 - \left(9.4\times10^{-11}\text{ m}\right)^2} = 3.803\times10^{-11}\text{ m}.$$

Thus,

$$y_{com} = \frac{m_N y_N}{m_N + 3m_H} = \frac{(14.0067)\left(3.803\times10^{-11}\text{ m}\right)}{14.0067 + 3(1.00797)} = 3.13\times10^{-11}\text{ m}$$

where Appendix F has been used to find the masses.

6. The centers of mass (with centimeters understood) for each of the five sides are as follows:

$$(x_1, y_1, z_1) = (0, 20, 20) \qquad \text{for the side in the } yz \text{ plane}$$
$$(x_2, y_2, z_2) = (20, 0, 20) \qquad \text{for the side in the } xz \text{ plane}$$
$$(x_3, y_3, z_3) = (20, 20, 0) \qquad \text{for the side in the } xy \text{ plane}$$
$$(x_4, y_4, z_4) = (40, 20, 20) \qquad \text{for the remaining side parallel to side 1}$$
$$(x_5, y_5, z_5) = (20, 40, 20) \qquad \text{for the remaining side parallel to side 2}$$

Recognizing that all sides have the same mass m, we plug these into Eq. 9-5 to obtain the results (the first two being expected based on the symmetry of the problem).

(a) The x coordinate of the center of mass is

$$x_{\text{com}} = \frac{mx_1 + mx_2 + mx_3 + mx_4 + mx_5}{5m} = \frac{0 + 20 + 20 + 40 + 20}{5} = 20 \text{ cm}$$

(b) The y coordinate of the center of mass is

$$y_{\text{com}} = \frac{my_1 + my_2 + my_3 + my_4 + my_5}{5m} = \frac{20 + 0 + 20 + 20 + 40}{5} = 20 \text{ cm}$$

(c) The z coordinate of the center of mass is

$$z_{\text{com}} = \frac{mz_1 + mz_2 + mz_3 + mz_4 + mz_5}{5m} = \frac{20 + 20 + 0 + 20 + 20}{5} = 16 \text{ cm}$$

7. We use Eq. 9-5 to locate the coordinates.

(a) By symmetry $x_{com} = -d_1/2 = -(13 \text{ cm})/2 = -6.5$ cm. The negative value is due to our choice of the origin.

(b) We find y_{com} as

$$
\begin{aligned}
y_{com} &= \frac{m_i y_{com,i} + m_a y_{com,a}}{m_i + m_a} = \frac{\rho_i V_i y_{com,i} + \rho_a V_a y_{cm,a}}{\rho_i V_i + \rho_a V_a} \\
&= \frac{(11 \text{ cm}/2)(7.85 \text{ g/cm}^3) + 3(11 \text{ cm}/2)(2.7 \text{ g/cm}^3)}{7.85 \text{ g/cm}^3 + 2.7 \text{ g/cm}^3} = 8.3 \text{ cm.}
\end{aligned}
$$

(c) Again by symmetry, we have $z_{com} = (2.8 \text{ cm})/2 = 1.4$ cm.

8. (a) Since the can is uniform, its center of mass is at its geometrical center, a distance $H/2$ above its base. The center of mass of the soda alone is at its geometrical center, a distance $x/2$ above the base of the can. When the can is full this is $H/2$. Thus the center of mass of the can and the soda it contains is a distance

$$h = \frac{M(H/2) + m(H/2)}{M+m} = \frac{H}{2}$$

above the base, on the cylinder axis. With $H = 12$ cm, we obtain $h = 6.0$ cm.

(b) We now consider the can alone. The center of mass is $H/2 = 6.0$ cm above the base, on the cylinder axis.

(c) As x decreases the center of mass of the soda in the can at first drops, then rises to $H/2 = 6.0$ cm again.

(d) When the top surface of the soda is a distance x above the base of the can, the mass of the soda in the can is $m_p = m(x/H)$, where m is the mass when the can is full ($x = H$). The center of mass of the soda alone is a distance $x/2$ above the base of the can. Hence

$$h = \frac{M(H/2) + m_p(x/2)}{M + m_p} = \frac{M(H/2) + m(x/H)(x/2)}{M + (mx/H)} = \frac{MH^2 + mx^2}{2(MH + mx)}.$$

We find the lowest position of the center of mass of the can and soda by setting the derivative of h with respect to x equal to 0 and solving for x. The derivative is

$$\frac{dh}{dx} = \frac{2mx}{2(MH+mx)} - \frac{(MH^2 + mx^2)m}{2(MH+mx)^2} = \frac{m^2x^2 + 2MmHx - MmH^2}{2(MH+mx)^2}.$$

The solution to $m^2x^2 + 2MmHx - MmH^2 = 0$ is

$$x = \frac{MH}{m}\left(-1 + \sqrt{1 + \frac{m}{M}}\right).$$

The positive root is used since x must be positive. Next, we substitute the expression found for x into $h = (MH^2 + mx^2)/2(MH + mx)$. After some algebraic manipulation we obtain

$$h = \frac{HM}{m}\left(\sqrt{1 + \frac{m}{M}} - 1\right) = \frac{(12\text{ cm})(0.14\text{ kg})}{1.31\text{ kg}}\left(\sqrt{1 + \frac{1.31\text{ kg}}{0.14\text{ kg}}} - 1\right) = 2.8\text{ cm}.$$

9. The implication in the problem regarding $\vec{v}_0$ is that the olive and the nut start at rest. Although we could proceed by analyzing the forces on each object, we prefer to approach this using Eq. 9-14. The total force on the nut-olive system is $\vec{F}_o + \vec{F}_n = (-\hat{\mathrm{i}} + \hat{\mathrm{j}})$ N. Thus, Eq. 9-14 becomes

$$(-\hat{\mathrm{i}} + \hat{\mathrm{j}})\ \mathrm{N} = M\vec{a}_{\mathrm{com}}$$

where M = 2.0 kg. Thus, $\vec{a}_{\mathrm{com}} = (-\tfrac{1}{2}\hat{\mathrm{i}} + \tfrac{1}{2}\hat{\mathrm{j}})\ \mathrm{m/s^2}$. Each component is constant, so we apply the equations discussed in Chapters 2 and 4 and obtain

$$\Delta\vec{r}_{\mathrm{com}} = \frac{1}{2}\vec{a}_{\mathrm{com}}t^2 = (-4.0\ \mathrm{m})\hat{\mathrm{i}} + (4.0\ \mathrm{m})\hat{\mathrm{j}}$$

when t = 4.0 s. It is perhaps instructive to work through this problem the *long way* (separate analysis for the olive and the nut and then application of Eq. 9-5) since it helps to point out the computational advantage of Eq. 9-14.

10. Since the center of mass of the two-skater system does not move, both skaters will end up at the center of mass of the system. Let the center of mass be a distance x from the 40-kg skater, then

$$(65\text{ kg})(10\text{ m}-x)=(40\text{ kg})x\Rightarrow x=6.2\text{ m}.$$

Thus the 40-kg skater will move by 6.2 m.

11. We use the constant-acceleration equations of Table 2-1 (with $+y$ downward and the origin at the release point), Eq. 9-5 for y_{com} and Eq. 9-17 for $\vec{v}_{com}$.

(a) The location of the first stone (of mass m_1) at $t = 300 \times 10^{-3}$ s is

$$y_1 = (1/2)gt^2 = (1/2)(9.8 \text{ m/s}^2)\,(300 \times 10^{-3} \text{ s})^2 = 0.44 \text{ m},$$

and the location of the second stone (of mass $m_2 = 2m_1$) at $t = 300 \times 10^{-3}$ s is

$$y_2 = (1/2)gt^2 = (1/2)(9.8 \text{ m/s}^2)(300 \times 10^{-3} \text{ s} - 100 \times 10^{-3} \text{ s})^2 = 0.20 \text{ m}.$$

Thus, the center of mass is at

$$y_{com} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m_1(0.44 \text{ m}) + 2m_1(0.20 \text{ m})}{m_1 + 2m_2} = 0.28 \text{ m}.$$

(b) The speed of the first stone at time t is $v_1 = gt$, while that of the second stone is

$$v_2 = g(t - 100 \times 10^{-3} \text{ s}).$$

Thus, the center-of-mass speed at $t = 300 \times 10^{-3}$ s is

$$\begin{aligned} v_{com} &= \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{m_1\left(9.8 \text{ m/s}^2\right)\left(300\times10^{-3} \text{ s}\right) + 2m_1\left(9.8 \text{ m/s}^2\right)\left(300\times10^{-3} \text{ s} - 100\times10^{-3} \text{ s}\right)}{m_1 + 2m_1} \\ &= 2.3 \text{ m/s}. \end{aligned}$$

12. We use the constant-acceleration equations of Table 2-1 (with the origin at the traffic light), Eq. 9-5 for x_{com} and Eq. 9-17 for $\vec{v}_{com}$. At t = 3.0 s, the location of the automobile (of mass m_1) is

$$x_1 = \tfrac{1}{2}at^2 = \tfrac{1}{2}\left(4.0 \text{ m/s}^2\right)\left(3.0 \text{ s}\right)^2 = 18 \text{ m},$$

while that of the truck (of mass m_2) is $x_2 = vt$ = (8.0 m/s)(3.0s) = 24 m. The speed of the automobile then is $v_1 = at = \left(4.0 \text{ m/s}^2\right)\left(3.0 \text{ s}\right) = 12 \text{ m/s}$, while the speed of the truck remains v_2 = 8.0 m/s.

(a) The location of their center of mass is

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(1000 \text{ kg})(18 \text{ m}) + (2000 \text{ kg})(24 \text{ m})}{1000 \text{ kg} + 2000 \text{ kg}} = 22 \text{ m}.$$

(b) The speed of the center of mass is

$$v_{com} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(1000 \text{ kg})(12 \text{ m/s}) + (2000 \text{ kg})(8.0 \text{ m/s})}{1000 \text{ kg} + 2000 \text{ kg}} = 9.3 \text{ m/s}.$$

13. (a) The net force on the *system* (of total mass $m_1 + m_2$) is $m_2 g$. Thus, Newton's second law leads to $a = g(m_2/(m_1 + m_2)) = 0.4g$. For block1, this acceleration is to the right (the $\hat{\mathrm{i}}$ direction), and for block 2 this is an acceleration downward (the $-\hat{\mathrm{j}}$ direction). Therefore, Eq. 9-18 gives

$$\vec{a}_{\text{com}} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2}{m_1 + m_2} = \frac{(0.6)(0.4g\hat{\mathrm{i}}) + (0.4)(-0.4g\hat{\mathrm{j}})}{0.6 + 0.4} = (2.35\,\hat{\mathrm{i}} - 1.57\,\hat{\mathrm{j}})\ \text{m/s}^2 .$$

(b) Integrating Eq. 4-16, we obtain

$$\vec{\mathrm{v}}_{\text{com}} = (2.35\,\hat{\mathrm{i}} - 1.57\hat{\mathrm{j}})\, t$$

(with SI units understood), since it started at rest. We note that the *ratio* of the *y*-component to the *x*-component (for the velocity vector) does not change with time, and it is that ratio which determines the angle of the velocity vector (by Eq. 3-6), and thus the direction of motion for the center of mass of the system.

(c) The last sentence of our answer for part (b) implies that the path of the center-of-mass is a straight line.

(d) Eq. 3-6 leads to $\theta = -34^\circ$. The path of the center of mass is therefore straight, at downward angle 34°.

14. (a) The phrase (in the problem statement) "such that it [particle 2] always stays directly above particle 1 during the flight" means that the shadow (as if a light were directly above the particles shining down on them) of particle 2 coincides with the position of particle 1, at each moment. We say, in this case, that they are vertically aligned. Because of that alignment, $v_{2x} = v_1 = 10.0$ m/s. Because the initial value of v_2 is given as 20.0 m/s, then (using the Pythagorean theorem) we must have

$$v_{2y} = \sqrt{v_2^2 - v_{2x}^2} = \sqrt{300} \text{ m/s}$$

for the initial value of the y component of particle 2's velocity. Eq. 2-16 (or conservation of energy) readily yields $y_{\max} = 300/19.6 = 15.3$ m. Thus, we obtain

$$H_{\max} = m_2 y_{\max} / m_{\text{total}} = (3.00 \text{ g})(15.3 \text{ m})/(8.00 \text{ g}) = 5.74 \text{ m}.$$

(b) Since both particles have the same horizontal velocity, and particle 2's vertical component of velocity vanishes at that highest point, then the center of mass velocity then is simply $(10.0 \text{ m/s})\hat{\mathrm{i}}$ (as one can verify using Eq. 9-17).

(c) Only particle 2 experiences any acceleration (the free fall acceleration downward), so Eq. 9-18 (or Eq. 9-19) leads to

$$a_{\text{com}} = m_2 g / m_{\text{total}} = (3.00 \text{ g})(9.8 \text{ m/s}^2)/(8.00 \text{ g}) = 3.68 \text{ m/s}^2$$

for the magnitude of the downward acceleration of the center of mass of this system. Thus, $\vec{a}_{\text{com}} = (-3.68 \text{ m/s}^2)\hat{\mathrm{j}}$.

15. We need to find the coordinates of the point where the shell explodes and the velocity of the fragment that does not fall straight down. The coordinate origin is at the firing point, the $+x$ axis is rightward, and the $+y$ direction is upward. The y component of the velocity is given by $v = v_{0\,y} - gt$ and this is zero at time $t = v_{0\,y}/g = (v_0/g)\sin\theta_0$, where v_0 is the initial speed and θ_0 is the firing angle. The coordinates of the highest point on the trajectory are

$$x = v_{0x}t = v_0 t\cos\theta_0 = \frac{v_0^2}{g}\sin\theta_0\cos\theta_0 = \frac{(20\text{ m/s})^2}{9.8\text{ m/s}^2}\sin 60°\cos 60° = 17.7\text{ m}$$

and

$$y = v_{0y}t - \frac{1}{2}gt^2 = \frac{1}{2}\frac{v_0^2}{g}\sin^2\theta_0 = \frac{1}{2}\frac{(20\text{ m/s})^2}{9.8\text{ m/s}^2}\sin^2 60^\circ = 15.3\text{ m.}$$

Since no horizontal forces act, the horizontal component of the momentum is conserved. Since one fragment has a velocity of zero after the explosion, the momentum of the other equals the momentum of the shell before the explosion. At the highest point the velocity of the shell is $v_0\cos\theta_0$, in the positive x direction. Let M be the mass of the shell and let V_0 be the velocity of the fragment. Then $Mv_0\cos\theta_0 = MV_0/2$, since the mass of the fragment is $M/2$. This means

$$V_0 = 2v_0\cos\theta_0 = 2(20\text{ m/s})\cos 60^\circ = 20\text{ m/s.}$$

This information is used in the form of initial conditions for a projectile motion problem to determine where the fragment lands. Resetting our clock, we now analyze a projectile launched horizontally at time $t = 0$ with a speed of 20 m/s from a location having coordinates $x_0 = 17.7$ m, $y_0 = 15.3$ m. Its y coordinate is given by $y = y_0 - \frac{1}{2}gt^2$, and when it lands this is zero. The time of landing is $t = \sqrt{2y_0/g}$ and the x coordinate of the landing point is

$$x = x_0 + V_0 t = x_0 + V_0\sqrt{\frac{2y_0}{g}} = 17.7\text{ m} + (20\text{ m/s})\sqrt{\frac{2(15.3\text{ m})}{9.8\text{ m/s}^2}} = 53\text{ m.}$$

16. We denote the mass of Ricardo as M_R and that of Carmelita as M_C. Let the center of mass of the two-person system (assumed to be closer to Ricardo) be a distance x from the middle of the canoe of length L and mass m. Then

$$M_R(L/2 - x) = mx + M_C(L/2 + x).$$

Now, after they switch positions, the center of the canoe has moved a distance $2x$ from its initial position. Therefore, x = 40 cm/2 = 0.20 m, which we substitute into the above equation to solve for M_C:

$$M_C = \frac{M_R(L/2 - x) - mx}{L/2 + x} = \frac{(80)(\frac{3.0}{2} - 0.20) - (30)(0.20)}{(3.0/2) + 0.20} = 58 \text{ kg}.$$

17. There is no net horizontal force on the dog-boat system, so their center of mass does not move. Therefore by Eq. 9-16, $M\Delta x_{\text{com}} = 0 = m_b \Delta x_b + m_d \Delta x_d$, which implies

$$\left|\Delta x_b\right| = \frac{m_d}{m_b}\left|\Delta x_d\right|.$$

Now we express the geometrical condition that *relative to the boat* the dog has moved a distance $d = 2.4$ m:

$$\left|\Delta x_b\right| + \left|\Delta x_d\right| = d$$

which accounts for the fact that the dog moves one way and the boat moves the other. We substitute for $|\Delta x_b|$ from above:

$$\frac{m_d}{m_b}\left|\left(\Delta x_d\right)\right| + \left|\Delta x_d\right| = d$$

which leads to $\left|\Delta x_d\right| = \dfrac{d}{1 + m_d / m_b} = \dfrac{2.4 \text{ m}}{1 + (4.5/18)} = 1.92$ m.

The dog is therefore 1.9 m closer to the shore than initially (where it was $D = 6.1$ m from it). Thus, it is now $D - |\Delta x_d| = 4.2$ m from the shore.

18. The magnitude of the ball's momentum change is

$$\Delta p = \left| mv_i - mv_f \right| = (0.70 \text{ kg}) \left| 5.0 \text{ m/s} - (-2.0 \text{ m/s}) \right| = 4.9 \text{ kg} \cdot \text{m/s}.$$

19. (a) The change in kinetic energy is

$$\begin{aligned}\Delta K &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(2100\text{ kg})\left((51\text{ km/h})^2 - (41\text{ km/h})^2\right)\\ &= 9.66\times10^4\text{ kg}\cdot(\text{km/h})^2\left((10^3\text{ m/km})(1\text{ h/3600 s})\right)^2\\ &= 7.5\times10^4\text{ J.}\end{aligned}$$

(b) The magnitude of the change in velocity is

$$|\Delta\vec{v}| = \sqrt{(-v_i)^2 + (v_f)^2} = \sqrt{(-41\text{ km/h})^2 + (51\text{ km/h})^2} = 65.4\text{ km/h}$$

so the magnitude of the change in momentum is

$$|\Delta\vec{p}| = m|\Delta\vec{v}| = (2100\text{ kg})(65.4\text{ km/h})\left(\frac{1000\text{ m/km}}{3600\text{ s/h}}\right) = 3.8\times10^4\text{ kg}\cdot\text{m/s.}$$

(c) The vector $\Delta\vec{p}$ points at an angle θ south of east, where

$$\theta = \tan^{-1}\left(\frac{v_i}{v_f}\right) = \tan^{-1}\left(\frac{41\text{ km/h}}{51\text{ km/h}}\right) = 39°.$$

20. (a) Since the force of impact on the ball is in the y direction, p_x is conserved:

$$p_{x\,i} = mv_i \sin\theta_1 = p_{x\,f} = mv_i \sin\theta_2.$$

With $\theta_1 = 30.0°$, we find $\theta_2 = 30.0°$.

(b) The momentum change is

$$\begin{aligned}\Delta\vec{p} &= mv_i \cos\theta_2 \left(-\hat{\mathrm{j}}\right) - mv_i \cos\theta_2 \left(+\hat{\mathrm{j}}\right) = -2\,(0.165 \text{ kg})\,(2.00 \text{ m/s})\,(\cos 30°)\,\hat{\mathrm{j}} \\ &= (-0.572 \text{ kg}\cdot\text{m/s})\hat{\mathrm{j}}.\end{aligned}$$

21. We use coordinates with $+x$ horizontally toward the pitcher and $+y$ upward. Angles are measured counterclockwise from the $+x$ axis. Mass, velocity and momentum units are SI. Thus, the initial momentum can be written $\vec{p}_0 = (4.5\angle 215°)$ in magnitude-angle notation.

(a) In magnitude-angle notation, the momentum change is

$$(6.0 \angle -90°) - (4.5 \angle 215°) = (5.0 \angle -43°)$$

(efficiently done with a vector-capable calculator in polar mode). The magnitude of the momentum change is therefore 5.0 kg·m/s.

(b) The momentum change is $(6.0 \angle 0°) - (4.5 \angle 215°) = (10 \angle 15°)$. Thus, the magnitude of the momentum change is 10 kg·m/s.

22. We infer from the graph that the horizontal component of momentum p_x is 4.0 kg·m/s. Also, its initial magnitude of momentum p_o is 6.0 kg·m/s. Thus,

$$\cos\theta_o = \frac{p_x}{p_o} \quad \Rightarrow \quad \theta_o = 48° .$$

23. The initial direction of motion is in the +x direction. The magnitude of the average force F_{avg} is given by

$$F_{avg} = \frac{J}{\Delta t} = \frac{32.4\ \text{N}\cdot\text{s}}{2.70\times10^{-2}\ \text{s}} = 1.20\times10^{3}\ \text{N}$$

The force is in the negative direction. Using the linear momentum-impulse theorem stated in Eq. 9-31, we have

$$-F_{\text{avg}}\Delta t = mv_f - mv_i.$$

where m is the mass, v_i the initial velocity, and v_f the final velocity of the ball. Thus,

$$v_f = \frac{mv_i - F_{\text{avg}}\Delta t}{m} = \frac{(0.40\,\text{kg})(14\,\text{m/s}) - (1200\,\text{N})\left(27\times10^{-3}\text{s}\right)}{0.40\,\text{kg}} = -67\,\text{m/s}.$$

(a) The final speed of the ball is $|v_f| = 67$ m/s.

(b) The negative sign indicates that the velocity is in the $-x$ direction, which is opposite to the initial direction of travel.

(c) From the above, the average magnitude of the force is $F_{\text{avg}} = 1.20\times10^{3}\ \text{N}$.

(d) The direction of the impulse on the ball is $-x$, same as the applied force.

24. (a) By energy conservation, the speed of the victim when he falls to the floor is

$$\frac{1}{2}mv^2 = mgh \quad \Rightarrow \quad v = \sqrt{2gh} = \sqrt{2(9.8\ \text{m/s}^2)(0.50\ \text{m})} = 3.1\ \text{m/s}.$$

Thus, the magnitude of the impulse is

$$J = |\Delta p| = m\,|\Delta v| = mv = (70\ \text{kg})(3.1\ \text{m/s}) \approx 2.2\times10^2\ \text{N}\cdot\text{s}.$$

(b) With duration of $\Delta t = 0.082\ \text{s}$ for the collision, the average force is

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{2.2\times10^2\ \text{N}\cdot\text{s}}{0.082\ \text{s}} \approx 2.7\times10^3\ \text{N}.$$

25. We estimate his mass in the neighborhood of 70 kg and compute the upward force F of the water from Newton's second law: $F - mg = ma$, where we have chosen $+y$ upward, so that $a > 0$ (the acceleration is upward since it represents a deceleration of his downward motion through the water). His speed when he arrives at the surface of the water is found either from Eq. 2-16 or from energy conservation: $v = \sqrt{2gh}$, where $h = 12$ m , and since the deceleration a reduces the speed to zero over a distance d = 0.30 m we also obtain $v = \sqrt{2ad}.$ We use these observations in the following.

Equating our two expressions for v leads to $a = gh/d$. Our force equation, then, leads to

$$F = mg + m\left(g\frac{h}{d}\right) = mg\left(1 + \frac{h}{d}\right)$$

which yields $F \approx 2.8 \times 10^4$ kg. Since we are not at all certain of his mass, we express this as a guessed-at range (in kN) $25 < F < 30$.

Since $F \gg mg$, the impulse $\vec{J}$ due to the net force (while he is in contact with the water) is overwhelmingly caused by the upward force of the water: $\int F\,dt = \vec{J}$ to a good approximation. Thus, by Eq. 9-29,

$$\int Fdt = \vec{p}_f - \vec{p}_i = 0 - m\left(-\sqrt{2gh}\right)$$

(the minus sign with the initial velocity is due to the fact that downward is the negative direction) which yields $(70\text{ kg})\sqrt{2\left(9.8\text{ m/s}^2\right)(12\text{ m})} = 1.1\times10^3\,\text{kg}\cdot\text{m/s}$. Expressing this as a range we estimate

$$1.0\times10^3\,\text{kg}\cdot\text{m/s} < \int F\,dt < 1.2\times10^3\,\text{kg}\cdot\text{m/s}.$$

26. We choose $+y$ upward, which implies $a > 0$ (the acceleration is upward since it represents a deceleration of his downward motion through the snow).

(a) The maximum deceleration a_{max} of the paratrooper (of mass m and initial speed $v = 56$ m/s) is found from Newton's second law

$$F_{snow} - mg = ma_{max}$$

where we require $F_{snow} = 1.2 \times 10^5$ N. Using Eq. 2-15 $v^2 = 2a_{max}d$, we find the minimum depth of snow for the man to survive:

$$d = \frac{v^2}{2a_{max}} = \frac{mv^2}{2(F_{snow} - mg)} \approx \frac{(85\,\text{kg})(56\,\text{m/s})^2}{2(1.2 \times 10^5\,\text{N})} = 1.1 \text{ m.}$$

(b) His short trip through the snow involves a change in momentum

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = 0 - (85\,\text{kg})(-56\,\text{m/s}) = -4.8 \times 10^3\ \text{kg} \cdot \text{m/s},$$

or $|\Delta\vec{p}| = 4.8 \times 10^3\ \text{kg} \cdot \text{m/s}$. The negative value of the initial velocity is due to the fact that downward is the negative direction. By the impulse-momentum theorem, this equals the impulse due to the net force $F_{snow} - mg$, but since $F_{snow} \gg mg$ we can approximate this as the impulse on him just from the snow.

27. We choose $+y$ upward, which means $\vec{v}_i = -25\,\text{m/s}$ and $\vec{v}_f = +10\,\text{m/s}$. During the collision, we make the reasonable approximation that the net force on the ball is equal to F_{avg} – the average force exerted by the floor up on the ball.

(a) Using the impulse momentum theorem (Eq. 9-31) we find

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = (1.2)(10) - (1.2)(-25) = 42\,\text{kg}\cdot\text{m/s}.$$

(b) From Eq. 9-35, we obtain

$$\vec{F}_{\text{avg}} = \frac{\vec{J}}{\Delta t} = \frac{42}{0.020} = 2.1\times10^{3}\,\text{N}.$$

28. (a) The magnitude of the impulse is

$$J = |\Delta p| = m\,|\Delta v| = mv = (0.70\ \text{kg})(13\ \text{m/s}) \approx 9.1\ \text{kg}\cdot\text{m/s} = 9.1\ \text{N}\cdot\text{s}.$$

(b) With duration of $\Delta t = 5.0\times10^{-3}$ s for the collision, the average force is

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{9.1\ \text{N}\cdot\text{s}}{5.0\times10^{-3}\ \text{s}} \approx 1.8\times10^{3}\ \text{N}.$$

29. We choose the positive direction in the direction of rebound so that $\vec{v}_f > 0$ and $\vec{v}_i < 0$. Since they have the same speed v, we write this as $\vec{v}_f = v$ and $\vec{v}_i = -v$. Therefore, the change in momentum for each bullet of mass m is $\Delta\vec{p} = m\Delta v = 2mv$. Consequently, the total change in momentum for the 100 bullets (each minute) $\Delta\vec{P} = 100\Delta\vec{p} = 200mv$. The average force is then

$$\vec{F}_{\text{avg}} = \frac{\Delta\vec{P}}{\Delta t} = \frac{(200)(3\times10^{-3}\,\text{kg})(500\,\text{m/s})}{(1\,\text{min})(60\,\text{s/min})} \approx 5\,\text{N}.$$

30. (a) By the impulse-momentum theorem (Eq. 9-31) the change in momentum must equal the "area" under the $F(t)$ curve. Using the facts that the area of a triangle is $\frac{1}{2}$ (base)(height), and that of a rectangle is (height)(width), we find the momentum at $t = 4$ s to be (30 kg·m/s)$\hat{\text{i}}$.

(b) Similarly (but keeping in mind that areas beneath the axis are counted negatively) we find the momentum at $t = 7$ s is (38 kg·m/s)$\hat{\text{i}}$.

(c) At $t = 9$ s, we obtain $\vec{p} = (6.0 \text{ m/s})\hat{\text{i}}$.

31. We use coordinates with $+x$ rightward and $+y$ upward, with the usual conventions for measuring the angles (so that the initial angle becomes 180 + 35 = 215°). Using SI units and magnitude-angle notation (efficient to work with when using a vector-capable calculator), the change in momentum is

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = (3.00\angle 90°) - (3.60\angle 215°) = (5.86\angle 59.8°).$$

(a) The magnitude of the impulse is $J = \Delta p = 5.86\ \text{kg}\cdot\text{m/s} = 5.86\ \text{N}\cdot\text{s}$.

(b) The direction of $\vec{J}$ is 59.8° measured counterclockwise from the $+x$ axis.

(c) Eq. 9-35 leads to

$$J = F_{\text{avg}}\Delta t = 5.86\ \text{N}\cdot\text{s} \quad\Rightarrow\quad F_{\text{avg}} = \frac{5.86\ \text{N}\cdot\text{s}}{2.00\times10^{-3}\ \text{s}} \approx 2.93\times10^{3}\,\text{N}.$$

We note that this force is very much larger than the weight of the ball, which justifies our (implicit) assumption that gravity played no significant role in the collision.

(d) The direction of $\vec{F}_{\text{avg}}$ is the same as $\vec{J}$, 59.8° measured counterclockwise from the $+x$ axis.

32. (a) Choosing upward as the positive direction, the momentum change of the foot is

$$\Delta\vec{p} = 0 - m_{\text{foot}}\vec{v}_i = -(0.003\text{ kg})\ (-1.50\ \text{ m/s}) = 4.50\times10^{-3}\ \text{N}\cdot\text{s}.$$

(b) Using Eq. 9-35 and now treating *downward* as the positive direction, we have

$$\vec{J} = \vec{F}_{\text{avg}}\Delta t = m_{\text{lizard}}g\ \Delta t = (0.090\text{ kg})(9.80\text{ m/s}^2)(0.60\text{ s}) = 0.529\ \text{N}\cdot\text{s}.$$

(c) Push is what provides the primary support.

33. (a) By energy conservation, the speed of the passenger when the elevator hits the floor is

$$\frac{1}{2}mv^2 = mgh \quad \Rightarrow \quad v = \sqrt{2gh} = \sqrt{2(9.8 \text{ m/s}^2)(36 \text{ m})} = 26.6 \text{ m/s}.$$

Thus, the magnitude of the impulse is

$$J = |\Delta p| = m\,|\Delta v| = mv = (90 \text{ kg})(26.6 \text{ m/s}) \approx 2.39\times10^3 \text{ N}\cdot\text{s}.$$

(b) With duration of $\Delta t = 5.0\times10^{-3}$ s for the collision, the average force is

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{2.39\times10^3 \text{ N}\cdot\text{s}}{5.0\times10^{-3} \text{ s}} \approx 4.78\times10^5 \text{ N}.$$

(c) If the passenger were to jump upward with a speed of $v' = 7.0$ m/s, then the resulting downward velocity would be

$$v'' = v - v' = 26.6 \text{ m/s} - 7.0 \text{ m/s} = 19.6 \text{ m/s},$$

and the magnitude of the impulse becomes

$$J'' = |\Delta p''| = m\,|\Delta v''| = mv'' = (90 \text{ kg})(19.6 \text{ m/s}) \approx 1.76\times10^3 \text{ N}\cdot\text{s}.$$

(d) The corresponding average force would be

$$F''_{\text{avg}} = \frac{J''}{\Delta t} = \frac{1.76\times10^3 \text{ N}\cdot\text{s}}{5.0\times10^{-3} \text{ s}} \approx 3.52\times10^5 \text{ N}.$$

34. (a) By Eq. 9-30, impulse can be determined from the "area" under the $F(t)$ curve. Keeping in mind that the area of a triangle is $\frac{1}{2}$(base)(height), we find the impulse in this case is 1.00 N·s.

(b) By definition (of the average of function, in the calculus sense) the average force must be the result of part (a) divided by the time (0.010 s). Thus, the average force is found to be 100 N.

(c) Consider ten hits. Thinking of ten hits as 10 $F(t)$ triangles, our total time interval is 10(0.050 s) = 0.50 s, and the total area is 10(1.0 N·s). We thus obtain an average force of 10/0.50 = 20.0 N. One could consider 15 hits, 17 hits, and so on, and still arrive at this same answer.

35. (a) We take the force to be in the positive direction, at least for earlier times. Then the impulse is

$$
\begin{aligned}
J &= \int_0^{3.0\times10^{-3}} F\,dt = \int_0^{3.0\times10^{-3}} \left[(6.0\times10^6)\,t-(2.0\times10^9)t^2\right]dt \\
&= \left[\frac{1}{2}(6.0\times10^6)t^2-\frac{1}{3}(2.0\times10^9)t^3\right]\Bigg|_0^{3.0\times10^{-3}} \\
&= 9.0\,\mathrm{N\cdot s}.
\end{aligned}
$$

(b) Since $J = F_{\text{avg}}\,\Delta t$, we find

$$F_{\text{avg}}\,\frac{J}{\Delta t}=\frac{9.0\ \mathrm{N\cdot s}}{3.0\times10^{-3}\ \mathrm{s}}=3.0\times10^{3}\ \mathrm{N}.$$

(c) To find the time at which the maximum force occurs, we set the derivative of F with respect to time equal to zero – and solve for t. The result is $t = 1.5 \times 10^{-3}$ s. At that time the force is

$$F_{\max}=\left(6.0\times10^{6}\right)\left(1.5\times10^{-3}\right)-\left(2.0\times10^{9}\right)\left(1.5\times10^{-3}\right)^{2}=4.5\times10^{3}\ \mathrm{N}.$$

(d) Since it starts from rest, the ball acquires momentum equal to the impulse from the kick. Let m be the mass of the ball and v its speed as it leaves the foot. Then,

$$v=\frac{p}{m}=\frac{J}{m}=\frac{9.0\ \mathrm{N\cdot s}}{0.45\ \mathrm{kg}}=20\ \mathrm{m/s}.$$

36. From Fig. 9-55, $+y$ corresponds to the direction of the rebound (directly away from the wall) and $+x$ towards the right. Using unit-vector notation, the ball's initial and final velocities are

$$\vec{v}_i = v\cos\theta\,\hat{\mathrm{i}} - v\sin\theta\,\hat{\mathrm{j}} = 5.2\,\hat{\mathrm{i}} - 3.0\,\hat{\mathrm{j}}$$
$$\vec{v}_f = v\cos\theta\,\hat{\mathrm{i}} + v\sin\theta\,\hat{\mathrm{j}} = 5.2\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}}$$

respectively (with SI units understood).

(a) With m = 0.30 kg, the impulse-momentum theorem (Eq. 9-31) yields

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = 2\left(0.30\ \text{kg}\right)(3.0\ \text{m/s}\ \hat{\mathrm{j}}) = (1.8\ \text{N}\cdot\text{s})\hat{\mathrm{j}}$$

(b) Using Eq. 9-35, the force on the ball by the wall is $\vec{J}/\Delta t = (1.8/0.010)\hat{\mathrm{j}} = (180\,\text{N})\,\hat{\mathrm{j}}$. By Newton's third law, the force on the wall by the ball is $(-180\ \text{N})\hat{\mathrm{j}}$ (that is, its magnitude is 180 N and its direction is directly into the wall, or "down" in the view provided by Fig. 9-55).

37. We choose our positive direction in the direction of the rebound (so the ball's initial velocity is negative-valued). We evaluate the integral $J = \int F\,dt$ by adding the appropriate areas (of a triangle, a rectangle, and another triangle) shown in the graph (but with the t converted to seconds). With m = 0.058 kg and v = 34 m/s, we apply the impulse-momentum theorem:

$$\int F_{\text{wall}}\,dt = m\vec{v}_f - m\vec{v}_i \quad \Rightarrow \quad \int_0^{0.002} F\,dt + \int_{0.002}^{0.004} F\,dt + \int_{0.004}^{0.006} F\,dt = m(+v) - m(-v)$$

$$\Rightarrow \quad \frac{1}{2}F_{\text{max}}(0.002\,\text{s}) + F_{\text{max}}(0.002\,\text{s}) + \frac{1}{2}F_{\text{max}}(0.002\,\text{s}) = 2mv$$

which yields $F_{\text{max}}(0.004\,\text{s}) = 2(0.058\,\text{kg})(34\,\text{m/s}) = 9.9 \times 10^2$ N.

38. (a) Performing the integral (from time a to time b) indicated in Eq. 9-30, we obtain

$$\int_a^b (12 - 3t^2)\,dt = 12(b-a) - (b^3 - a^3)$$

in SI units. If b = 1.25 s and a = 0.50 s, this gives 7.17 N·s.

(b) This integral (the impulse) relates to the change of momentum in Eq. 9-31. We note that the force is zero at t = 2.00 s. Evaluating the above expression for a = 0 and b = 2.00 gives an answer of 16.0 kg·m/s.

39. No external forces with horizontal components act on the man-stone system and the vertical forces sum to zero, so the total momentum of the system is conserved. Since the man and the stone are initially at rest, the total momentum is zero both before and after the stone is kicked. Let m_s be the mass of the stone and v_s be its velocity after it is kicked; let m_m be the mass of the man and v_m be his velocity after he kicks the stone. Then

$$m_s v_s + m_m v_m = 0 \rightarrow \ v_m = -m_s v_s / m_m.$$

We take the axis to be positive in the direction of motion of the stone. Then

$$v_m = -\frac{(0.068 \text{ kg})(4.0 \text{ m/s})}{91 \text{ kg}} = -3.0 \times 10^{-3} \text{ m/s},$$

or $|v_m| = 3.0 \times 10^{-3}$ m/s. The negative sign indicates that the man moves in the direction opposite to the direction of motion of the stone.

40. Our notation is as follows: the mass of the motor is M; the mass of the module is m; the initial speed of the system is v_0; the relative speed between the motor and the module is v_r; and, the speed of the module relative to the Earth is v after the separation. Conservation of linear momentum requires

$$(M + m)v_0 = mv + M(v - v_r).$$

Therefore,

$$v = v_0 + \frac{Mv_r}{M+m} = 4300 \text{ km/h} + \frac{(4m)(82 \text{ km/h})}{4m+m} = 4.4 \times 10^3 \text{ km/h}.$$

41. With $\vec{v}_0 = (9.5\,\hat{\text{i}} + 4.0\,\hat{\text{j}})$ m/s, the initial speed is

$$v_0 = \sqrt{v_{x0}^2 + v_{y0}^2} = \sqrt{(9.5\text{ m/s})^2 + (4.0\text{ m/s})^2} = 10.31\text{ m/s}$$

and the takeoff angle of the athlete is

$$\theta_0 = \tan^{-1}\left(\frac{v_{y0}}{v_{x0}}\right) = \tan^{-1}\left(\frac{4.0}{9.5}\right) = 22.8°.$$

Using Equation 4-26, the range of the athlete without using halteres is

$$R_0 = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{(10.31\text{ m/s})^2 \sin 2(22.8°)}{9.8\text{ m/s}^2} = 7.75\text{ m.}$$

On the other hand, if two halteres of mass m = 5.50 kg were thrown at the maximum height, then, by momentum conservation, the subsequent speed of the athlete would be

$$(M + 2m)v_{x0} = Mv_x' \quad \Rightarrow v_x' = \frac{M + 2m}{M} v_{x0}$$

Thus, the change in the x-component of the velocity is

$$\Delta v_x = v_x' - v_{x0} = \frac{M + 2m}{M} v_{x0} - v_{x0} = \frac{2m}{M} v_{x0} = \frac{2(5.5\text{ kg})}{78\text{ kg}}(9.5\text{ m/s}) = 1.34\text{ m/s.}$$

The maximum height is attained when $v_y = v_{y0} - gt = 0$, or

$$t = \frac{v_{y0}}{g} = \frac{4.0\text{ m/s}}{9.8\text{ m/s}^2} = 0.41\text{ s.}$$

Therefore, the increase in range with use of halteres is

$$\Delta R = (\Delta v_x')t = (1.34\text{ m/s})(0.41\text{ s}) = 0.55\text{ m.}$$

42. Our $+x$ direction is east and $+y$ direction is north. The linear momenta for the two m = 2.0 kg parts are then

$$\vec{p}_1 = m\vec{v}_1 = mv_1\,\hat{\mathrm{j}}$$

where v_1 = 3.0 m/s, and

$$\vec{p}_2 = m\vec{v}_2 = m\left(v_{2x}\,\hat{\mathrm{i}} + v_{2y}\,\hat{\mathrm{j}}\right) = mv_2\left(\cos\theta\,\hat{\mathrm{i}} + \sin\theta\,\hat{\mathrm{j}}\right)$$

where v_2 = 5.0 m/s and θ = 30°. The combined linear momentum of both parts is then

$$\begin{aligned}\vec{P} &= \vec{p}_1 + \vec{p}_2 = mv_1\,\hat{\mathrm{j}} + mv_2\left(\cos\theta\,\hat{\mathrm{i}} + \sin\theta\,\hat{\mathrm{j}}\right) = \left(mv_2\cos\theta\right)\hat{\mathrm{i}} + \left(mv_1 + mv_2\sin\theta\right)\hat{\mathrm{j}}\\ &= (2.0\text{ kg})(5.0\text{ m/s})(\cos 30°)\,\hat{\mathrm{i}} + (2.0\text{ kg})\left(3.0\text{ m/s} + (5.0\text{ m/s})(\sin 30°)\right)\hat{\mathrm{j}}\\ &= \left(8.66\,\hat{\mathrm{i}} + 11\,\hat{\mathrm{j}}\right)\text{ kg}\cdot\text{m/s}.\end{aligned}$$

From conservation of linear momentum we know that this is also the linear momentum of the whole kit before it splits. Thus the speed of the 4.0-kg kit is

$$v = \frac{P}{M} = \frac{\sqrt{P_x^2 + P_y^2}}{M} = \frac{\sqrt{(8.66\text{ kg}\cdot\text{m/s})^2 + (11\text{ kg}\cdot\text{m/s})^2}}{4.0\text{ kg}} = 3.5\text{ m/s}.$$

43. (a) With SI units understood, the velocity of block L (in the frame of reference indicated in the figure that goes with the problem) is $(v_1 - 3)\hat{\mathrm{i}}$. Thus, momentum conservation (for the explosion at $t = 0$) gives

$$m_L(v_1 - 3) + (m_C + m_R)v_1 = 0$$

which leads to

$$v_1 = \frac{3\,m_L}{m_L + m_C + m_R} = \frac{3(2\text{ kg})}{10\text{ kg}} = 0.60\text{ m/s}.$$

Next, at $t = 0.80$ s, momentum conservation (for the second explosion) gives

$$m_C\,v_2 + m_R(v_2 + 3) = (m_C + m_R)v_1 = (8\text{ kg})(0.60\text{ m/s}) = 4.8\text{ kg}\cdot\text{m/s}.$$

This yields $v_2 = -0.15$. Thus, the velocity of block C after the second explosion is

$$v_2 = -(0.15\text{ m/s})\hat{\mathrm{i}}.$$

(b) Between $t = 0$ and $t = 0.80$ s, the block moves $v_1\Delta t = (0.60\text{ m/s})(0.80\text{ s}) = 0.48$ m. Between $t = 0.80$ s and $t = 2.80$ s, it moves an additional

$$v_2\Delta t = (-0.15\text{ m/s})(2.00\text{ s}) = -0.30\text{ m}.$$

Its net displacement since $t = 0$ is therefore 0.48 m – 0.30 m = 0.18 m.

44. Our notation (and, implicitly, our choice of coordinate system) is as follows: the mass of the original body is m; its initial velocity is $\vec{v}_0 = v\,\hat{\mathrm{i}}$; the mass of the less massive piece is m_1; its velocity is $\vec{v}_1 = 0$; and, the mass of the more massive piece is m_2. We note that the conditions $m_2 = 3m_1$ (specified in the problem) and $m_1 + m_2 = m$ generally assumed in classical physics (before Einstein) lead us to conclude

$$m_1 = \frac{1}{4}m \;\text{ and }\; m_2 = \frac{3}{4}m.$$

Conservation of linear momentum requires

$$m\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 \quad\Rightarrow\quad mv\,\hat{\mathrm{i}} = 0 + \frac{3}{4}m\vec{v}_2$$

which leads to $\vec{v}_2 = \frac{4}{3}v\,\hat{\mathrm{i}}$. The increase in the system's kinetic energy is therefore

$$\Delta K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{1}{2}mv_0^2 = 0 + \frac{1}{2}\left(\frac{3}{4}m\right)\left(\frac{4}{3}v\right)^2 - \frac{1}{2}mv^2 = \frac{1}{6}mv^2.$$

45. Our notation (and, implicitly, our choice of coordinate system) is as follows: the mass of one piece is $m_1 = m$; its velocity is $\vec{v}_1 = (-30 \text{ m/s})\hat{\text{i}}$; the mass of the second piece is $m_2 = m$; its velocity is $\vec{v}_2 = (-30 \text{ m/s})\hat{\text{j}}$; and, the mass of the third piece is $m_3 = 3m$.

(a) Conservation of linear momentum requires

$$m\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 \quad \Rightarrow \quad 0 = m\left(-30\hat{\text{i}}\right) + m\left(-30\hat{\text{j}}\right) + 3m\vec{v}_3$$

which leads to $\vec{v}_3 = (10\hat{\text{i}} + 10\hat{\text{j}})$ m/s. Its magnitude is $v_3 = 10\sqrt{2} \approx 14 \text{ m/s}$.

(b) The direction is 45° *counterclockwise* from $+x$ (in this system where we have m_1 flying off in the $-x$ direction and m_2 flying off in the $-y$ direction).

46. We can think of the sliding-until-stopping as an example of kinetic energy converting into thermal energy (see Eq. 8-29 and Eq. 6-2, with $F_N = mg$). This leads to $v^2 = 2\mu gd$ being true separately for each piece. Thus we can set up a ratio:

$$\left(\frac{v_L}{v_R}\right)^2 = \frac{2\mu_L g d_L}{2\mu_R g d_R} = \frac{12}{25} .$$

But (by the conservation of momentum) the ratio of speeds must be inversely proportional to the ratio of masses (since the initial momentum – before the explosion – was zero). Consequently,

$$\left(\frac{m_R}{m_L}\right)^2 = \frac{12}{25} \;\Rightarrow\; m_R = \tfrac{2}{5}\sqrt{3}\, m_L = 1.39 \text{ kg}.$$

Therefore, the total mass is $m_R + m_L \approx 3.4$ kg.

47. Our notation is as follows: the mass of the original body is $M = 20.0$ kg; its initial velocity is $\vec{v}_0 = (200\text{ m/s})\hat{\text{i}}$; the mass of one fragment is $m_1 = 10.0$ kg; its velocity is $\vec{v}_1 = (-100\text{ m/s})\hat{\text{j}}$; the mass of the second fragment is $m_2 = 4.0$ kg; its velocity is $\vec{v}_2 = (-500\text{ m/s})\hat{\text{i}}$; and, the mass of the third fragment is $m_3 = 6.00$ kg.

(a) Conservation of linear momentum requires $M\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3$, which (using the above information) leads to

$$\vec{v}_3 = (1.00\times10^3\ \hat{\text{i}} - 0.167\times10^3\ \hat{\text{j}})\text{ m/s}.$$

The magnitude of $\vec{v}_3$ is $v_3 = \sqrt{(1000\text{ m/s})^2 + (-167\text{ m/s})^2} = 1.01\times10^3\text{ m/s}$. It points at $\theta = \tan^{-1}(-167/1000) = -9.48°$ (that is, at 9.5° measured clockwise from the $+x$ axis).

(b) We are asked to calculate ΔK or

$$\left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2\right) - \frac{1}{2}Mv_0^2 = 3.23\times10^6\text{ J}.$$

48. This problem involves both mechanical energy conservation $U_i = K_1 + K_2$, where U_i = 60 J, and momentum conservation

$$0 = m_1\vec{v}_1 + m_2\vec{v}_2$$

where $m_2 = 2m_1$. From the second equation, we find $|\vec{v}_1| = 2|\vec{v}_2|$ which in turn implies (since $v_1 = |\vec{v}_1|$ and likewise for v_2)

$$K_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}\left(\frac{1}{2}m_2\right)(2v_2)^2 = 2\left(\frac{1}{2}m_2v_2^2\right) = 2K_2.$$

(a) We substitute $K_1 = 2K_2$ into the energy conservation relation and find

$$U_i = 2K_2 + K_2 \Rightarrow K_2 = \frac{1}{3}U_i = 20 \text{ J}.$$

(b) And we obtain $K_1 = 2(20) = 40$ J.

49. We refer to the discussion in the textbook (see Sample Problem 9-8, which uses the same notation that we use here) for many of the important details in the reasoning. Here we only present the primary computational step (using SI units):

$$v = \frac{m+M}{m}\sqrt{2gh} = \frac{2.010}{0.010}\sqrt{2(9.8)(0.12)} = 3.1\times 10^2 \text{ m/s}.$$

50. (a) We choose $+x$ along the initial direction of motion and apply momentum conservation:

$$m_{\text{bullet}}\vec{v}_i = m_{\text{bullet}}\vec{v}_1 + m_{\text{block}}\vec{v}_2$$
$$(5.2\text{ g})(672\text{ m/s}) = (5.2\text{ g})(428\text{ m/s}) + (700\text{ g})\vec{v}_2$$

which yields $v_2 = 1.81$ m/s.

(b) It is a consequence of momentum conservation that the velocity of the center of mass is unchanged by the collision. We choose to evaluate it before the collision:

$$\vec{v}_{\text{com}} = \frac{m_{\text{bullet}}\vec{v}_i}{m_{\text{bullet}} + m_{\text{block}}} = \frac{(5.2\text{ g})(672\text{ m/s})}{5.2\text{ g} + 700\text{ g}} = 4.96\text{ m/s}.$$

51. With an initial speed of v_i, the initial kinetic energy of the car is $K_i = m_c v_i^2/2$. After a totally inelastic collision with a moose of mass m_m, by momentum conservation, the speed of the combined system is

$$m_c v_i = (m_c + m_m)v_f \quad \Rightarrow \quad v_f = \frac{m_c v_i}{m_c + m_m},$$

with final kinetic energy

$$K_f = \frac{1}{2}(m_c + m_m)v_f^2 = \frac{1}{2}(m_c + m_m)\left(\frac{m_c v_i}{m_c + m_m}\right)^2 = \frac{1}{2}\frac{m_c^2}{m_c + m_m}v_i^2.$$

(a) The percentage loss of kinetic energy due to collision is

$$\frac{\Delta K}{K_i} = \frac{K_i - K_f}{K_i} = 1 - \frac{K_f}{K_i} = 1 - \frac{m_c}{m_c + m_m} = \frac{m_m}{m_c + m_m} = \frac{500\text{ kg}}{1000\text{ kg} + 500\text{ kg}} = \frac{1}{3} = 33.3\%.$$

(b) If the collision were with a camel of mass $m_{\text{camel}} = 300\text{ kg}$, then the percentage loss of kinetic energy would be

$$\frac{\Delta K}{K_i} = \frac{m_{\text{camel}}}{m_c + m_{\text{camel}}} = \frac{300\text{ kg}}{1000\text{ kg} + 300\text{ kg}} = \frac{3}{13} = 23\%.$$

(c) As the animal mass decreases, the percentage loss of kinetic energy also decreases.

52. (a) The magnitude of the deceleration of each of the cars is $a = f/m = \mu_k mg/m = \mu_k g$. If a car stops in distance d, then its speed v just after impact is obtained from Eq. 2-16:

$$v^2 = v_0^2 + 2ad \Rightarrow v = \sqrt{2ad} = \sqrt{2\mu_k gd}$$

since $v_0 = 0$ (this could alternatively have been derived using Eq. 8-31). Thus,

$$v_A = \sqrt{2\mu_k gd_A} = \sqrt{2(0.13)(9.8 \text{ m/s}^2)(8.2 \text{ m})} = 4.6 \text{ m/s}.$$

(b) Similarly, $v_B = \sqrt{2\mu_k gd_B} = \sqrt{2(0.13)(9.8 \text{ m/s}^2)(6.1 \text{ m})} = 3.9 \text{ m/s}.$

(c) Let the speed of car B be v just before the impact. Conservation of linear momentum gives $m_B v = m_A v_A + m_B v_B$, or

$$v = \frac{(m_A v_A + m_B v_B)}{m_B} = \frac{(1100)(4.6) + (1400)(3.9)}{1400} = 7.5 \text{ m/s}.$$

(d) The conservation of linear momentum during the impact depends on the fact that the only significant force (during impact of duration Δt) is the force of contact between the bodies. In this case, that implies that the force of friction exerted by the road on the cars is neglected during the brief Δt. This neglect would introduce some error in the analysis. Related to this is the assumption we are making that the transfer of momentum occurs at one location – that the cars do not slide appreciably during Δt – which is certainly an approximation (though probably a good one). Another source of error is the application of the friction relation Eq. 6-2 for the sliding portion of the problem (after the impact); friction is a complex force that Eq. 6-2 only partially describes.

53. In solving this problem, our $+x$ direction is to the right (so all velocities are positive-valued).

(a) We apply momentum conservation to relate the situation just before the bullet strikes the second block to the situation where the bullet is embedded within the block.

$$(0.0035 \text{ kg})v = (1.8035 \text{ kg})(1.4 \text{ m/s}) \ \Rightarrow \ v = 721 \text{ m/s}.$$

(b) We apply momentum conservation to relate the situation just before the bullet strikes the first block to the instant it has passed through it (having speed v found in part (a)).

$$(0.0035 \text{ kg})v_0 = (1.20 \text{ kg})(0.630 \text{ m/s}) + (0.00350 \text{ kg})(721 \text{ m/s})$$

which yields $v_0 = 937$ m/s.

54. We think of this as having two parts: the first is the collision itself – where the bullet passes through the block so quickly that the block has not had time to move through any distance yet – and then the subsequent "leap" of the block into the air (up to height h measured from its initial position). The first part involves momentum conservation (with $+y$ upward):

$$(0.01\,\text{kg})(1000\,\text{m/s}) = (5.0\,\text{kg})\vec{v} + (0.01\,\text{kg})(400\,\text{m/s})$$

which yields $\vec{v} = 1.2\,\text{m/s}$. The second part involves either the free-fall equations from Ch. 2 (since we are ignoring air friction) or simple energy conservation from Ch. 8. Choosing the latter approach, we have

$$\frac{1}{2}(5.0\,\text{kg})(1.2\,\text{m/s})^2 = (5.0\,\text{kg})\left(9.8\,\text{m/s}^2\right)h$$

which gives the result h = 0.073 m.

55. (a) Let v be the final velocity of the ball-gun system. Since the total momentum of the system is conserved $mv_i = (m + M)v$. Therefore,

$$v = \frac{mv_i}{m+M} = \frac{(60\text{ g})(22\text{ m/s})}{60\text{ g} + 240\text{ g}} = 4.4\text{ m/s}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}mv_i^2$ and the final kinetic energy is $K_f = \frac{1}{2}(m+M)v^2 = \frac{1}{2}m^2v_i^2/(m+M)$. The problem indicates $\Delta E_{\text{th}} = 0$, so the difference $K_i - K_f$ must equal the energy U_s stored in the spring:

$$U_s = \frac{1}{2}mv_i^2 - \frac{1}{2}\frac{m^2v_i^2}{(m+M)} = \frac{1}{2}mv_i^2\left(1 - \frac{m}{m+M}\right) = \frac{1}{2}mv_i^2\frac{M}{m+M}.$$

Consequently, the fraction of the initial kinetic energy that becomes stored in the spring is

$$\frac{U_s}{K_i} = \frac{M}{m+M} = \frac{240}{60+240} = 0.80.$$

56. The total momentum immediately before the collision (with $+x$ upward) is

$$p_i = (3.0\text{ kg})(20\text{ m/s}) + (2.0\text{ kg})(-12\text{ m/s}) = 36\text{ kg}\cdot\text{m/s}.$$

Their momentum immediately after, when they constitute a combined mass of $M = 5.0$ kg, is $p_f = (5.0\text{ kg})\vec{v}$. By conservation of momentum, then, we obtain $\vec{v} = 7.2$ m/s, which becomes their "initial" velocity for their subsequent free-fall motion. We can use Ch. 2 methods or energy methods to analyze this subsequent motion; we choose the latter. The level of their collision provides the reference ($y = 0$) position for the gravitational potential energy, and we obtain

$$K_0 + U_0 = K + U \quad \Rightarrow \quad \frac{1}{2}Mv_0^2 + 0 = 0 + Mgy_{\max}.$$

Thus, with $v_0 = 7.2$ m/s, we find $y_{\max} = 2.6$ m.

57. We choose $+x$ in the direction of (initial) motion of the blocks, which have masses m_1 = 5 kg and m_2 = 10 kg. Where units are not shown in the following, SI units are to be understood.

(a) Momentum conservation leads to

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$
$$(5\text{ kg})(3.0\text{ m/s}) + (10\text{ kg})(2.0\text{ m/s}) = (5\text{ kg})\vec{v}_{1f} + (10\text{ kg})(2.5\text{ m/s})$$

which yields $\vec{v}_{1f} = 2.0\text{ m/s}$. Thus, the speed of the 5.0 kg block immediately after the collision is 2.0 m/s.

(b) We find the reduction in total kinetic energy:

$$\begin{aligned} K_i - K_f &= \frac{1}{2}(5\text{ kg})(3\text{ m/s})^2 + \frac{1}{2}(10\text{ kg})(2\text{ m/s})^2 - \frac{1}{2}(5\text{ kg})(2\text{ m/s})^2 - \frac{1}{2}(10\text{ kg})(2.5\text{ m/s})^2 \\ &= -1.25\text{ J} \approx -1.3\text{ J}. \end{aligned}$$

(c) In this new scenario where $\vec{v}_{2f} = 4.0\text{ m/s}$, momentum conservation leads to $\vec{v}_{1f} = -1.0\text{ m/s}$ and we obtain $\Delta K = +40\text{ J}$.

(d) The creation of additional kinetic energy is possible if, say, some gunpowder were on the surface where the impact occurred (initially stored chemical energy would then be contributing to the result).

58. We think of this as having two parts: the first is the collision itself – where the blocks "join" so quickly that the 1.0-kg block has not had time to move through any distance yet – and then the subsequent motion of the 3.0 kg system as it compresses the spring to the maximum amount x_m. The first part involves momentum conservation (with $+x$ rightward):

$$m_1v_1 = (m_1+m_2)v \quad \Rightarrow \quad (2.0 \text{ kg})(4.0 \text{ m/s}) = (3.0 \text{ kg})\vec{v}$$

which yields $\vec{v} = 2.7 \text{ m/s}$. The second part involves mechanical energy conservation:

$$\frac{1}{2}(3.0 \text{ kg})\,(2.7 \text{ m/s})^2 = \frac{1}{2}\,(200 \text{ N/m})x_m^2$$

which gives the result $x_m = 0.33$ m.

59. As hinted in the problem statement, the velocity v of the system as a whole – when the spring reaches the maximum compression x_m – satisfies

$$m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v.$$

The change in kinetic energy of the system is therefore

$$\Delta K = \frac{1}{2}(m_1 + m_2)v^2 - \frac{1}{2}m_1v_{1i}^2 - \frac{1}{2}m_2v_{2i}^2 = \frac{(m_1v_{1i} + m_2v_{2i})^2}{2(m_1 + m_2)} - \frac{1}{2}m_1v_{1i}^2 - \frac{1}{2}m_2v_{2i}^2$$

which yields $\Delta K = -35$ J. (Although it is not necessary to do so, still it is worth noting that algebraic manipulation of the above expression leads to $|\Delta K| = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)\, v_{\text{rel}}^2$ where $v_{\text{rel}} = v_1 - v_2$). Conservation of energy then requires

$$\frac{1}{2}kx_{\text{m}}^2 = -\Delta K \Rightarrow x_{\text{m}} = \sqrt{\frac{-2\Delta K}{k}} = \sqrt{\frac{-2(-35\text{ J})}{1120\text{ N/m}}} = 0.25\text{ m}.$$

60. (a) Let m_1 be the mass of one sphere, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the other sphere, v_{2i} be its velocity before the collision, and v_{2f} be its velocity after the collision. Then, according to Eq. 9-75,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} .$$

Suppose sphere 1 is originally traveling in the positive direction and is at rest after the collision. Sphere 2 is originally traveling in the negative direction. Replace v_{1i} with v, v_{2i} with $-v$, and v_{1f} with zero to obtain $0 = m_1 - 3m_2$. Thus,

$$m_2 = m_1 / 3 = (300 \text{ g}) / 3 = 100 \text{ g} .$$

(b) We use the velocities before the collision to compute the velocity of the center of mass:

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(300 \text{ g})\,(2.00 \text{ m/s}) + (100 \text{ g})\,(-2.00 \text{ m/s})}{300 \text{ g} + 100 \text{ g}} = 1.00 \text{ m/s}.$$

61. (a) Let m_1 be the mass of the cart that is originally moving, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the cart that is originally at rest and v_{2f} be its velocity after the collision. Then, according to Eq. 9-67,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}.$$

Using SI units (so $m_1 = 0.34$ kg), we obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1i} + v_{1f}} m_1 = \left(\frac{1.2\text{ m/s} - 0.66\text{ m/s}}{1.2\text{ m/s} + 0.66\text{ m/s}} \right)(0.34\text{ kg}) = 0.099\text{ kg}.$$

(b) The velocity of the second cart is given by Eq. 9-68:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \left(\frac{2(0.34\text{ kg})}{0.34\text{ kg} + 0.099\text{ kg}} \right)(1.2\text{ m/s}) = 1.9\text{ m/s}.$$

(c) The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(0.34)(1.2) + 0}{0.34 + 0.099} = 0.93\ \text{m/s}.$$

Values for the initial velocities were used but the same result is obtained if values for the final velocities are used.

62. (a) Let m_A be the mass of the block on the left, v_{Ai} be its initial velocity, and v_{Af} be its final velocity. Let m_B be the mass of the block on the right, v_{Bi} be its initial velocity, and v_{Bf} be its final velocity. The momentum of the two-block system is conserved, so

$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

and

$$v_{Af} = \frac{m_A v_{Ai} + m_B v_{Bi} - m_B v_{Bf}}{m_A} = \frac{(1.6\text{ kg})(5.5\text{ m/s}) + (2.4\text{ kg})(2.5\text{ m/s}) - (2.4\text{ kg})(4.9\text{ m/s})}{1.6\text{ kg}}$$
$$= 1.9\text{ m/s}.$$

(b) The block continues going to the right after the collision.

(c) To see if the collision is elastic, we compare the total kinetic energy before the collision with the total kinetic energy after the collision. The total kinetic energy before is

$$K_i = \frac{1}{2} m_A v_{Ai}^2 + \frac{1}{2} m_B v_{Bi}^2 = \frac{1}{2}(1.6\text{ kg})(5.5\text{ m/s})^2 + \frac{1}{2}(2.4\text{ kg})(2.5\text{ m/s})^2 = 31.7\text{ J}.$$

The total kinetic energy after is

$$K_f = \frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_{Bf}^2 = \frac{1}{2}(1.6\text{ kg})(1.9\text{ m/s})^2 + \frac{1}{2}(2.4\text{ kg})(4.9\text{ m/s})^2 = 31.7\text{ J}.$$

Since $K_i = K_f$ the collision is found to be elastic.

63. (a) Let m_1 be the mass of the body that is originally moving, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the body that is originally at rest and v_{2f} be its velocity after the collision. Then, according to Eq. 9-67,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \ .$$

We solve for m_2 to obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1f} + v_{1i}} m_1 \ .$$

We combine this with $v_{1f} = v_{1i}/4$ to obtain $m_2 = 3m_1/5 = 3(2.0 \text{ kg})/5 = 1.2 \text{ kg}$.

(b) The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(2.0 \text{ kg})(4.0 \text{ m/s})}{2.0 \text{ kg} + 1.2 \text{ kg}} = 2.5 \text{ m/s} .$$

64. This is a completely inelastic collision, but Eq. 9-53 ($V = \frac{m_1}{m_1 + m_2} v_{1i}$) is not easily applied since that equation is designed for use when the struck particle is initially stationary. To deal with this case (where particle 2 is already in motion), we return to the principle of momentum conservation:

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2)\vec{V} \quad \Rightarrow \quad \vec{V} = \frac{2(4\hat{i}-5\hat{j}) + 4(6\hat{i}-2\hat{j})}{2+4} .$$

(a) In unit-vector notation, then,

$$\vec{V} = (2.67 \text{ m/s})\hat{i} + (-3.00 \text{ m/s})\hat{j} .$$

(b) The magnitude of $\vec{V}$ is $|\vec{V}| = 4.01$ m/s

(c) The direction of $\vec{V}$ is 48.4° (measured *clockwise* from the $+x$ axis).

65. We use Eq 9-67 and 9-68 to find the velocities of the particles after their first collision (at $x = 0$ and $t = 0$):

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{-0.1 \text{ kg}}{0.7 \text{ kg}} (2.0 \text{ m/s}) = \frac{-2}{7} \text{ m/s}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{0.6 \text{ kg}}{0.7 \text{ kg}} (2.0 \text{ m/s}) = \frac{12}{7} \text{ m/s} \approx 1.7 \text{ m/s.}$$

At a rate of motion of 1.7 m/s, $2x_w = 140$ cm (the distance to the wall and back to $x = 0$) will be traversed by particle 2 in 0.82 s. At $t = 0.82$ s, particle 1 is located at

$$x = (-2/7)(0.82) = -23 \text{ cm},$$

and particle 2 is "gaining" at a rate of (10/7) m/s leftward; this is their relative velocity at that time. Thus, this "gap" of 23 cm between them will be closed after an additional time of (0.23 m)/(10/7 m/s) = 0.16 s has passed. At this time ($t = 0.82 + 0.16 = 0.98$ s) the two particles are at $x = (-2/7)(0.98) = -28$ cm.

66. First, we find the speed v of the ball of mass m_1 right before the collision (just as it reaches its lowest point of swing). Mechanical energy conservation (with h = 0.700 m) leads to

$$m_1 gh = \frac{1}{2} m_1 v^2 \;\Rightarrow\; v = \sqrt{2gh} = 3.7 \text{ m/s}.$$

(a) We now treat the elastic collision using Eq. 9-67:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v = \frac{0.5 \text{ kg} - 2.5 \text{ kg}}{0.5 \text{ kg} + 2.5 \text{ kg}} (3.7 \text{ m/s}) = -2.47 \text{ m/s}$$

which means the final speed of the ball is 2.47 m/s.

(b) Finally, we use Eq. 9-68 to find the final speed of the block:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v = \frac{2(0.5 \text{ kg})}{0.5 \text{ kg} + 2.5 \text{ kg}} (3.7 \text{ m/s}) = 1.23 \text{ m/s}.$$

67. (a) The center of mass velocity does not change in the absence of external forces. In this collision, only forces of one block on the other (both being part of the same system) are exerted, so the center of mass velocity is 3.00 m/s before and after the collision.

(b) We can find the velocity v_{1i} of block 1 before the collision (when the velocity of block 2 is known to be zero) using Eq. 9-17:

$$(m_1 + m_2)v_{\text{com}} = m_1\, v_{1i} + 0 \qquad \Rightarrow \qquad v_{1i} = 12.0 \text{ m/s} .$$

Now we use Eq. 9-68 to find v_{2f}:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = 6.00 \text{ m/s} .$$

68. (a) If the collision is perfectly elastic, then Eq. 9-68 applies

$$v_2 = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{m_1 + (2.00)m_1} \sqrt{2gh} = \frac{2}{3}\sqrt{2gh}$$

where we have used the fact (found most easily from energy conservation) that the speed of block 1 at the bottom of the frictionless ramp is $\sqrt{2gh}$ (where h = 2.50 m). Next, for block 2's "rough slide" we use Eq. 8-37:

$$\frac{1}{2} m_2 v_2^2 = \Delta E_{\text{th}} = f_k d = \mu_k m_2 g\, d\,.$$

where μ_k = 0.500. Solving for the sliding distance d, we find that m_2 cancels out and we obtain d = 2.22 m.

(b) In a completely inelastic collision, we apply Eq. 9-53: $v_2 = \dfrac{m_1}{m_1 + m_2} v_{1i}$ (where, as above, $v_{1i} = \sqrt{2gh}$). Thus, in this case we have $v_2 = \sqrt{2gh}/3$. Now, Eq. 8-37 (using the total mass since the blocks are now joined together) leads to a sliding distance of $d = 0.556$ m (one-fourth of the part (a) answer).

69. (a) We use conservation of mechanical energy to find the speed of either ball after it has fallen a distance h. The initial kinetic energy is zero, the initial gravitational potential energy is $M\,gh$, the final kinetic energy is $\frac{1}{2}Mv^2$, and the final potential energy is zero. Thus $Mgh = \frac{1}{2}Mv^2$ and $v = \sqrt{2gh}$. The collision of the ball of M with the floor is an elastic collision of a light object with a stationary massive object. The velocity of the light object reverses direction without change in magnitude. After the collision, the ball is traveling upward with a speed of $\sqrt{2gh}$. The ball of mass m is traveling downward with the same speed. We use Eq. 9-75 to find an expression for the velocity of the ball of mass M after the collision:

$$v_{Mf} = \frac{M-m}{M+m}v_{Mi} + \frac{2m}{M+m}v_{mi} = \frac{M-m}{M+m}\sqrt{2gh} - \frac{2m}{M+m}\sqrt{2gh} = \frac{M-3m}{M+m}\sqrt{2gh}\ .$$

For this to be zero, $m = M/3$. With $M = 0.63$ kg, we have $m = 0.21$ kg.

(b) We use the same equation to find the velocity of the ball of mass m after the collision:

$$v_{mf} = -\frac{m-M}{M+m}\sqrt{2gh} + \frac{2M}{M+m}\sqrt{2gh} = \frac{3M-m}{M+m}\sqrt{2gh}$$

which becomes (upon substituting $M = 3m$) $v_{mf} = 2\sqrt{2gh}$. We next use conservation of mechanical energy to find the height h' to which the ball rises. The initial kinetic energy is $\frac{1}{2}mv_{mf}^2$, the initial potential energy is zero, the final kinetic energy is zero, and the final potential energy is mgh'. Thus,

$$\frac{1}{2}mv_{mf}^2 = mgh' \Rightarrow h' = \frac{v_{mf}^2}{2g} = 4h\ .$$

With $h = 1.8$ m, we have $h' = 7.2$ m.

70. We use Eqs. 9-67, 9-68 and 4-21 for the elastic collision and the subsequent projectile motion. We note that both pucks have the same time-of-fall t (during their projectile motions). Thus, we have

$$\Delta x_2 = v_2 t \quad \text{where } \Delta x_2 = d \text{ and } v_2 = \frac{2m_1}{m_1 + m_2} v_{1i}$$

$$\Delta x_1 = v_1 t \quad \text{where } \Delta x_1 = -2d \text{ and } v_1 = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \; .$$

Dividing the first equation by the second, we arrive at

$$\frac{d}{-2d} = \frac{\frac{2m_1}{m_1 + m_2} v_{1i}\, t}{\frac{m_1 - m_2}{m_1 + m_2} v_{1i}\, t} \; .$$

After canceling v_{1i}, t and d, and solving, we obtain $m_2 = 1.0$ kg.

71. We orient our $+x$ axis along the initial direction of motion, and specify angles in the "standard" way — so $\theta = +60°$ for the proton (1) which is assumed to scatter into the first quadrant and $\phi = -30°$ for the target proton (2) which scatters into the fourth quadrant (recall that the problem has told us that this is perpendicular to θ). We apply the conservation of linear momentum to the x and y axes respectively.

$$\begin{aligned} m_1 v_1 &= m_1 v'_1 \cos\theta + m_2 v'_2 \cos\phi \\ 0 &= m_1 v'_1 \sin\theta + m_2 v'_2 \sin\phi \end{aligned}$$

We are given v_1 = 500 m/s, which provides us with two unknowns and two equations, which is sufficient for solving. Since $m_1 = m_2$ we can cancel the mass out of the equations entirely.

(a) Combining the above equations and solving for v_2' we obtain

$$v_2' = \frac{v_1 \sin\theta}{\sin(\theta - \phi)} = \frac{(500\ \text{m/s})\sin(60°)}{\sin(90°)} = 433\ \text{m/s}.$$

We used the identity $\sin\theta\cos\phi - \cos\theta\sin\phi = \sin(\theta - \phi)$ in simplifying our final expression.

(b) In a similar manner, we find

$$v_1' = \frac{v_1 \sin\theta}{\sin(\phi - \theta)} = \frac{(500\ \text{m/s})\sin(-30°)}{\sin(-90°)} = 250\ \text{m/s}\ .$$

72. (a) Conservation of linear momentum implies

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B\,.$$

Since $m_A = m_B = m = 2.0$ kg, the masses divide out and we obtain

$$\begin{aligned}\vec{v}\,'_B &= \vec{v}_A + \vec{v}_B - \vec{v}\,'_A = (15\hat{\mathrm{i}} + 30\hat{\mathrm{j}})\ \mathrm{m/s} + (-10\,\hat{\mathrm{i}} + 5\hat{\mathrm{j}})\ \mathrm{m/s} - (-5\,\hat{\mathrm{i}} + 20\,\hat{\mathrm{j}})\ \mathrm{m/s} \\ &= (10\,\hat{\mathrm{i}} + 15\,\hat{\mathrm{j}})\ \mathrm{m/s}\,.\end{aligned}$$

(b) The final and initial kinetic energies are

$$\begin{aligned}K_f &= \frac{1}{2}mv'^2_A + \frac{1}{2}mv'^2_B = \frac{1}{2}(2.0)\left((-5)^2 + 20^2 + 10^2 + 15^2\right) = 8.0\times10^2\ \mathrm{J} \\ K_i &= \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 = \frac{1}{2}(2.0)\left(15^2 + 30^2 + (-10)^2 + 5^2\right) = 1.3\times10^3\ \mathrm{J}\,.\end{aligned}$$

The change kinetic energy is then $\Delta K = -5.0 \times 10^2$ J (that is, 500 J of the initial kinetic energy is lost).

73. We apply the conservation of linear momentum to the *x* and *y* axes respectively.

$$\begin{aligned} m_1 v_{1i} &= m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2 \\ 0 &= m_1 v_{1f} \sin\theta_1 - m_2 v_{2f} \sin\theta_2 \end{aligned}$$

We are given $v_{2f} = 1.20\times10^5$ m/s, $\theta_1 = 64.0°$ and $\theta_2 = 51.0°$. Thus, we are left with two unknowns and two equations, which can be readily solved.

(a) We solve for the final alpha particle speed using the *y*-momentum equation:

$$v_{1f} = \frac{m_2 v_{2f} \sin\theta_2}{m_1 \sin\theta_1} = \frac{(16.0)\left(1.20\times10^5\right)\sin(51.0°)}{(4.00)\sin(64.0°)} = 4.15\times10^5 \text{ m/s}.$$

(b) Plugging our result from part (a) into the *x*-momentum equation produces the initial alpha particle speed:

$$\begin{aligned} v_{1i} &= \frac{m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2}{m_{1i}} \\ &= \frac{(4.00)\left(4.15\times10^5\right)\cos(64.0°) + (16.0)\left(1.2\times10^5\right)\cos(51.0°)}{4.00} \\ &= 4.84\times10^5 \text{ m/s}. \end{aligned}$$

74. We orient our $+x$ axis along the initial direction of motion, and specify angles in the "standard" way — so $\theta = -90°$ for the particle B which is assumed to scatter "downward" and $\phi > 0$ for particle A which presumably goes into the first quadrant. We apply the conservation of linear momentum to the x and y axes respectively.

$$m_B v_B = m_B v_B' \cos\theta + m_A v_A' \cos\phi$$
$$0 = m_B v_B' \sin\theta + m_A v_A' \sin\phi$$

(a) Setting $v_B = v$ and $v_B' = v/2$, the y-momentum equation yields

$$m_A v_A' \sin\phi = m_B \frac{v}{2}$$

and the x-momentum equation yields $m_A v_A' \cos\phi = m_B v$.

Dividing these two equations, we find $\tan\phi = \frac{1}{2}$ which yields $\phi = 27°$.

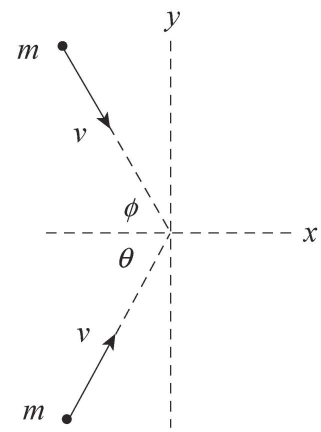
(b) We can *formally* solve for v_A' (using the y-momentum equation and the fact that $\phi = 1/\sqrt{5}$)

$$v_A' = \frac{\sqrt{5}}{2}\frac{m_B}{m_A} v$$

but lacking numerical values for v and the mass ratio, we cannot fully determine the final speed of A. Note: substituting $\cos\phi = 2/\sqrt{5}$, into the x-momentum equation leads to exactly this same relation (that is, no new information is obtained which might help us determine an answer).

75. Suppose the objects enter the collision along lines that make the angles $\theta > 0$ and $\phi > 0$ with the x axis, as shown in the diagram that follows. Both have the same mass m and the same initial speed v. We suppose that after the collision the combined object moves in the positive x direction with speed V. Since the y component of the total momentum of the two-object system is conserved,

$$mv \sin \theta - mv \sin \phi = 0.$$



This means $\phi = \theta$. Since the x component is conserved,

$$2mv \cos \theta = 2mV.$$

We now use $V = v/2$ to find that $\cos\theta = 1/2$. This means $\theta = 60°$. The angle between the initial velocities is 120°.

76. We use Eq. 9-88 and simplify with $v_i = 0$, $v_f = v$, and $v_{\text{rel}} = u$.

$$v_f - v_i = v_{\text{rel}} \ln\frac{M_i}{M_f} \Rightarrow \frac{M_i}{M_f} = e^{v/u}$$

(a) If $v = u$ we obtain $\frac{M_i}{M_f} = e^1 \approx 2.7$.

(b) If $v = 2u$ we obtain $\frac{M_i}{M_f} = e^2 \approx 7.4$.

77. (a) The thrust of the rocket is given by $T = Rv_{rel}$ where R is the rate of fuel consumption and v_{rel} is the speed of the exhaust gas relative to the rocket. For this problem $R = 480$ kg/s and $v_{rel} = 3.27 \times 10^3$ m/s, so

$$T = (480\,\text{kg/s})(3.27\times 10^3\ \text{m/s}) = 1.57\times 10^6\ \text{N}.$$

(b) The mass of fuel ejected is given by $M_{fuel} = R\Delta t$, where Δt is the time interval of the burn. Thus, M_{fuel} = (480 kg/s)(250 s) = 1.20×10^5 kg. The mass of the rocket after the burn is

$$M_f = M_i - M_{fuel} = (2.55 \times 10^5\ \text{kg}) - (1.20 \times 10^5\ \text{kg}) = 1.35 \times 10^5\ \text{kg}.$$

(c) Since the initial speed is zero, the final speed is given by

$$v_f = v_{rel} \ln \frac{M_i}{M_f} = (3.27\times 10^3)\ln\left(\frac{2.55\times 10^5}{1.35\times 10^5}\right) = 2.08\times 10^3\ \text{m/s}.$$

78. We use Eq. 9-88. Then

$$v_f = v_i + v_{\text{rel}} \ln\left(\frac{M_i}{M_f}\right) = 105 \text{ m/s} + (253 \text{ m/s}) \ln\left(\frac{6090 \text{ kg}}{6010 \text{ kg}}\right) = 108 \text{ m/s}.$$

79. (a) We consider what must happen to the coal that lands on the faster barge during one minute (Δt = 60s). In that time, a total of m = 1000 kg of coal must experience a change of velocity

$$\Delta v = 20\,\text{km/h} - 10\,\text{km/h} = 10\,\text{km/h} = 2.8\,\text{m/s},$$

where rightwards is considered the positive direction. The rate of change in momentum for the coal is therefore

$$\frac{\Delta \vec{p}}{\Delta t} = \frac{m\Delta \vec{v}}{\Delta t} = \frac{(1000\text{ kg})(2.8\text{ m/s})}{60\text{ s}} = 46\,\text{N}$$

which, by Eq. 9-23, must equal the force exerted by the (faster) barge on the coal. The processes (the shoveling, the barge motions) are constant, so there is no ambiguity in equating $\frac{\Delta p}{\Delta t}$ with $\frac{dp}{dt}$.

(b) The problem states that the frictional forces acting on the barges does not depend on mass, so the loss of mass from the slower barge does not affect its motion (so no extra force is required as a result of the shoveling).

80. (a) We use Eq. 9-68 twice:

$$v_2 = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{1.5m_1}(4.00 \text{ m/s}) = \frac{16}{3} \text{ m/s}$$

$$v_3 = \frac{2m_2}{m_2 + m_3} v_2 = \frac{2m_2}{1.5m_2}(16/3 \text{ m/s}) = \frac{64}{9} \text{ m/s} = 7.11 \text{ m/s} .$$

(b) Clearly, the speed of block 3 is greater than the (initial) speed of block 1.

(c) The kinetic energy of block 3 is

$$K_{3f} = \tfrac{1}{2} m_3 v_3^{\,2} = \left(\tfrac{1}{2}\right)^3 m_1 \left(\tfrac{16}{9}\right)^2 v_{1i}^{\,2} = \frac{64}{81} K_{1i} .$$

We see the kinetic energy of block 3 is less than the (initial) K of block 1. In the final situation, the initial K is being shared among the three blocks (which are all in motion), so this is not a surprising conclusion.

(d) The momentum of block 3 is

$$p_{3f} = m_3 v_3 = \left(\tfrac{1}{2}\right)^2 m_1 \left(\tfrac{16}{9}\right) v_{1i} = \frac{4}{9} p_{1i}$$

and is therefore less than the initial momentum (both of these being considered in magnitude, so questions about ± sign do not enter the discussion).

81. Using Eq. 9-67 and Eq. 9-68, we have after the first collision

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{-m_1}{3m_1} v_{1i} = -\tfrac{1}{3} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{3m_1} v_{1i} = \tfrac{2}{3} v_{1i} \ .$$

After the second collision, the velocities are

$$v_{2ff} = \frac{m_2 - m_3}{m_2 + m_3} v_{2f} = \frac{-m_2}{3m_2} \frac{2}{3} v_{1i} = -\tfrac{2}{9} v_{1i}$$

$$v_{3ff} = \frac{2m_2}{m_2 + m_3} v_{2f} = \frac{2m_2}{3m_2} \frac{2}{3} v_{1i} = \tfrac{4}{9} v_{1i} \ .$$

(a) Setting $v_{1i} = 4$ m/s, we find $v_{3ff} \approx 1.78$ m/s.

(b) We see that v_{3ff} is less than v_{1i} .

(c) The final kinetic energy of block 3 (expressed in terms of the initial kinetic energy of block 1) is

$$K_{3ff} = \tfrac{1}{2} m_3 v_3^{\,2} = \tfrac{1}{2}(4m_1)\left(\tfrac{16}{9}\right)^2 v_{1i}^{\,2} = \frac{64}{81} K_{1i} \ .$$

We see that this is less than K_{1i} .

(d) The final momentum of block 3 is $p_{3ff} = m_3 v_{3ff} = (4m_1)\left(\frac{16}{9}\right) v_1 > m_1 v_1$.

82. (a) This is a highly symmetric collision, and when we analyze the y-components of momentum we find their net value is zero. Thus, the stuck-together particles travel along the x axis.

(b) Since it is an elastic collision with identical particles, the final speeds are the same as the initial values. Conservation of momentum along each axis then assures that the angles of approach are the same as the angles of scattering. Therefore, one particle travels along line 2, the other along line 3.

(c) Here the final speeds are less than they were initially. The total x-component cannot be less, however, by momentum conservation, so the loss of speed shows up as a decrease in their y-velocity-components. This leads to smaller angles of scattering. Consequently, one particle travels through region B, the other through region C; the paths are symmetric about the x-axis. We note that this is intermediate between the final states described in parts (b) and (a).

(d) Conservation of momentum along the x-axis leads (because these are identical particles) to the simple observation that the x-component of each particle remains constant:

$$v_{fx} = v\cos\theta = 3.06 \text{ m/s}.$$

(e) As noted above, in this case the speeds are unchanged; both particles are moving at 4.00 m/s in the final state.

83. (a) Momentum conservation gives

$$m_R v_R + m_L v_L = 0 \;\Rightarrow\; (0.500\ \text{kg})\, v_R + (1.00\ \text{kg})(-1.20\ \text{m/s}) = 0$$

which yields $v_R = 2.40$ m/s. Thus, $\Delta x = v_R\, t = (2.40\ \text{m/s})(0.800\ \text{s}) = 1.92$ m.

(b) Now we have $m_R v_R + m_L (v_R - 1.20\ \text{m/s}) = 0$, which yields

$$v_R = \frac{(1.2\ \text{m/s})m_L}{m_L + m_R} = \frac{(1.20\ \text{m/s})(1.00\ \text{kg})}{1.00\ \text{kg} + 0.500\ \text{kg}} = 0.800\ \text{m/s}.$$

Consequently, $\Delta x = v_R\, t = 0.640$ m.

84. Let m be the mass of the higher floors. By energy conservation, the speed of the higher floors just before impact is

$$mgd = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2gd}.$$

The magnitude of the impulse during the impact is

$$J = |\Delta p| = m|\Delta v| = mv = m\sqrt{2gd} = mg\sqrt{\frac{2d}{g}} = W\sqrt{\frac{2d}{g}}$$

where $W = mg$ represents the weight of the higher floors. Thus, the average force exerted on the lower floor is

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{W}{\Delta t}\sqrt{\frac{2d}{g}}$$

With $F_{\text{avg}} = sW$, where s is the safety factor, we have

$$s = \frac{1}{\Delta t}\sqrt{\frac{2d}{g}} = \frac{1}{1.5\times10^{-3}\ \text{s}}\sqrt{\frac{2(4.0\ \text{m})}{9.8\ \text{m/s}^2}} = 6.0\times10^2.$$

85. We convert mass rate to SI units: R = (540 kg/min)/(60 s/min) = 9.00 kg/s. In the absence of the asked-for additional force, the car would decelerate with a magnitude given by Eq. 9-87:

$$R\,v_{\text{rel}} = M\,|a|$$

so that if $a = 0$ is desired then the additional force must have a magnitude equal to $R\,v_{\text{rel}}$ (so as to cancel that effect).

$$F = Rv_{\text{rel}} = (9.00\ \text{kg/s})(3.20\ \text{m/s}) = 28.8\ \text{N}.$$

86. From mechanical energy conservation (or simply using Eq. 2-16 with $\vec{a} = g$ downward) we obtain

$$v = \sqrt{2gh} = \sqrt{2(9.8 \text{ m/s}^2)(1.5 \text{ m})} = 5.4 \text{ m/s}$$

for the speed just as the body makes contact with the ground.

(a) During the compression of the body, the center of mass must decelerate over a distance d = 0.30 m. Choosing $+y$ downward, the deceleration a is found using Eq. 2-16.

$$0 = v^2 + 2ad \;\;\Rightarrow\;\; a = -\frac{v^2}{2d} = -\frac{5.4^2}{2(0.30)}$$

which yields $a = -49 \text{ m/s}^2$. Thus, the magnitude of the net (vertical) force is $m|a|$ = 49m in SI units, which (since 49 m/s^2 = 5(9.8 m/s^2) = 5g) can be expressed as 5mg.

(b) During the deceleration process, the forces on the dinosaur are (in the vertical direction) $\vec{F}_N$ and $m\vec{g}$. If we choose $+y$ upward, and use the final result from part (a), we therefore have

$$F_N - mg = 5mg \;\;\Rightarrow\;\; F_N = 6mg.$$

In the horizontal direction, there is also a deceleration (from v_0 = 19 m/s to zero), in this case due to kinetic friction $f_k = \mu_k F_N = \mu_k(6mg)$. Thus, the net force exerted by the ground on the dinosaur is

$$F_{\text{ground}} = \sqrt{f_k^2 + F_N^2} \approx 7mg.$$

(c) We can applying Newton's second law in the horizontal direction (with the sliding distance denoted as Δx) and then use Eq. 2-16, or we can apply the general notions of energy conservation. The latter approach is shown:

$$\frac{1}{2}mv_\text{o}^2 = \mu_k(6mg)\Delta x \;\;\Rightarrow\;\; \Delta x = \frac{(19 \text{ m/s})^2}{2(6)(0.6)(9.8 \text{ m/s}^2)} \approx 5 \text{ m}.$$

87. Denoting the new speed of the car as v, then the new speed of the man relative to the ground is $v - v_{rel}$. Conservation of momentum requires

$$\left(\frac{W}{g}+\frac{w}{g}\right)v_0=\left(\frac{W}{g}\right)v+\left(\frac{w}{g}\right)(v-v_{rel}).$$

Consequently, the change of velocity is

$$\Delta\vec{v}=v-v_0=\frac{w\,v_{rel}}{W+w}=\frac{(915\text{ N})(4.00\text{ m/s})}{(2415\text{ N})+(915\text{ N})}=1.10\text{ m/s}.$$

88. First, we imagine that the small square piece (of mass *m*) that was cut from the large plate is returned to it so that the large plate is again a complete 6 m × 6 m (d =1.0 m) square plate (which has its center of mass at the origin). Then we "add" a square piece of "negative mass" (–*m*) at the appropriate location to obtain what is shown in Fig. 9-75. If the mass of the whole plate is *M*, then the mass of the small square piece cut from it is obtained from a simple ratio of areas:

$$m = \left(\frac{2.0\,\text{m}}{6.0\,\text{m}}\right)^2 M \Rightarrow M = 9m.$$

(a) The x coordinate of the small square piece is x = 2.0 m (the middle of that square "gap" in the figure). Thus the x coordinate of the center of mass of the remaining piece is

$$x_{\text{com}} = \frac{(-m)x}{M+(-m)} = \frac{-m(2.0\ \text{m})}{9m-m} = -0.25\ \text{m}.$$

(b) Since the y coordinate of the small square piece is zero, we have $y_{\text{com}} = 0$.

89. We assume no external forces act on the system composed of the two parts of the last stage. Hence, the total momentum of the system is conserved. Let m_c be the mass of the rocket case and m_p the mass of the payload. At first they are traveling together with velocity v. After the clamp is released m_c has velocity v_c and m_p has velocity v_p. Conservation of momentum yields

$$(m_c + m_p)v = m_c v_c + m_p v_p.$$

(a) After the clamp is released the payload, having the lesser mass, will be traveling at the greater speed. We write $v_p = v_c + v_{\text{rel}}$, where v_{rel} is the relative velocity. When this expression is substituted into the conservation of momentum condition, the result is

$$\left(m_c + m_p\right)v = m_c v_c + m_p v_c + m_p v_{\text{rel}}.$$

Therefore,

$$v_c = \frac{\left(m_c + m_p\right)v - m_p v_{\text{rel}}}{m_c + m_p} = \frac{(290.0\text{ kg} + 150.0\text{ kg})(7600\text{ m/s}) - (150.0\text{ kg})(910.0\text{ m/s})}{290.0\text{ kg} + 150.0\text{ kg}}$$
$$= 7290\text{ m/s}.$$

(b) The final speed of the payload is $v_p = v_c + v_{\text{rel}} = 7290\text{ m/s} + 910.0\text{ m/s} = 8200\text{ m/s}$.

(c) The total kinetic energy before the clamp is released is

$$K_i = \frac{1}{2}\left(m_c + m_p\right)v^2 = \frac{1}{2}(290.0\text{ kg} + 150.0\text{ kg})(7600\text{ m/s})^2 = 1.271 \times 10^{10}\text{ J}.$$

(d) The total kinetic energy after the clamp is released is

$$K_f = \frac{1}{2}m_c v_c^2 + \frac{1}{2}m_p v_p^2 = \frac{1}{2}(290.0\text{ kg})(7290\text{ m/s})^2 + \frac{1}{2}(150.0\text{ kg})(8200\text{ m/s})^2$$
$$= 1.275 \times 10^{10}\text{ J}.$$

The total kinetic energy increased slightly. Energy originally stored in the spring is converted to kinetic energy of the rocket parts.

90. The velocity of the object is

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left((3500 - 160t)\,\hat{\text{i}} + 2700\,\hat{\text{j}} + 300\,\hat{\text{k}}\right) = -(160 \text{ m/s})\hat{\text{i}}.$$

(a) The linear momentum is $\vec{p} = m\vec{v} = (250 \text{ kg})\left(-160 \text{ m/s}\,\hat{\text{i}}\right) = (-4.0\times10^4 \text{ kg}\cdot\text{m/s})\,\hat{\text{i}}.$

(b) The object is moving west (our $-\hat{\text{i}}$ direction).

(c) Since the value of $\vec{p}$ does not change with time, the net force exerted on the object is zero, by Eq. 9-23.

91. (a) If m is the mass of a pellet and v is its velocity as it hits the wall, then its momentum is $p = mv = (2.0 \times 10^{-3}$ kg)(500 m/s) = 1.0 kg · m/s, toward the wall.

(b) The kinetic energy of a pellet is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(2.0\times10^{-3}\,\text{kg}\right)\left(500\,\text{m/s}\right)^2 = 2.5\times10^2\,\text{J} .$$

(c) The force on the wall is given by the rate at which momentum is transferred from the pellets to the wall. Since the pellets do not rebound, each pellet that hits transfers p = 1.0 kg · m/s. If ΔN pellets hit in time Δt, then the average rate at which momentum is transferred is

$$F_{\text{avg}} = \frac{p\Delta N}{\Delta t} = \left(1.0\,\text{kg}\cdot\text{m/s}\right)\left(10\,\text{s}^{-1}\right) = 10\,\text{N}.$$

The force on the wall is in the direction of the initial velocity of the pellets.

(d) If Δt is the time interval for a pellet to be brought to rest by the wall, then the average force exerted on the wall by a pellet is

$$F_{\text{avg}} = \frac{p}{\Delta t} = \frac{1.0\,\text{kg}\cdot\text{m/s}}{0.6\times10^{-3}\,\text{s}} = 1.7\times10^3\,\text{N}.$$

The force is in the direction of the initial velocity of the pellet.

(e) In part (d) the force is averaged over the time a pellet is in contact with the wall, while in part (c) it is averaged over the time for many pellets to hit the wall. During the majority of this time, no pellet is in contact with the wall, so the average force in part (c) is much less than the average force in part (d).

92. One approach is to choose a *moving* coordinate system which travels the center of mass of the body, and another is to do a little extra algebra analyzing it in the original coordinate system (in which the speed of the m = 8.0 kg mass is v_0 = 2 m/s, as given). Our solution is in terms of the latter approach since we are assuming that this is the approach most students would take. Conservation of linear momentum (along the direction of motion) requires

$$mv_0 = m_1v_1 + m_2v_2 \quad \Rightarrow \quad (8.0)(2.0) = (4.0)v_1 + (4.0)v_2$$

which leads to $v_2 = 4 - v_1$ in SI units (m/s). We require

$$\Delta K = \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right) - \frac{1}{2}mv_0^2 \quad \Rightarrow \quad 16 = \left(\frac{1}{2}(4.0)v_1^2 + \frac{1}{2}(4.0)v_2^2\right) - \frac{1}{2}(8.0)(2.0)^2$$

which simplifies to $v_2^2 = 16 - v_1^2$ in SI units. If we substitute for v_2 from above, we find

$$(4 - v_1)^2 = 16 - v_1^2$$

which simplifies to $2v_1^2 - 8v_1 = 0$, and yields either v_1 = 0 or v_1 = 4 m/s. If v_1 = 0 then v_2 = 4 – v_1 = 4 m/s, and if v_1 = 4 m/s then v_2 = 0.

(a) Since the forward part continues to move in the original direction of motion, the speed of the rear part must be zero.

(b) The forward part has a velocity of 4.0 m/s along the original direction of motion.

93. (a) The initial momentum of the car is

$$\vec{p}_i = m\vec{v}_i = (1400\,\text{kg})(5.3\,\text{m/s})\hat{\text{j}} = (7400\,\text{kg}\cdot\text{m/s})\hat{\text{j}}$$

and the final momentum is $\vec{p}_f = (7400\,\text{kg}\cdot\text{m/s})\hat{\text{i}}$. The impulse on it equals the change in its momentum:

$$\vec{J} = \vec{p}_f - \vec{p}_i = \left(7.4\times10^3\ \text{N}\cdot\text{s}\right)\left(\hat{\text{i}} - \hat{\text{j}}\right).$$

(b) The initial momentum of the car is $\vec{p}_i = (7400\,\text{kg}\cdot\text{m/s})\hat{\text{i}}$ and the final momentum is $\vec{p}_f = 0$. The impulse acting on it is $\vec{J} = \vec{p}_f - \vec{p}_i = (-7.4\times10^3\,\text{N}\cdot\text{s})\hat{\text{i}}$.

(c) The average force on the car is

$$\vec{F}_{\text{avg}} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\vec{J}}{\Delta t} = \frac{(7400\,\text{kg}\cdot\text{m/s})\left(\hat{\text{i}} - \hat{\text{j}}\right)}{4.6\,\text{s}} = (1600\,\text{N})\left(\hat{\text{i}} - \hat{\text{j}}\right)$$

and its magnitude is $F_{\text{avg}} = (1600\,\text{N})\sqrt{2} = 2.3\times10^3\,\text{N}$.

(d) The average force is

$$\vec{F}_{\text{avg}} = \frac{\vec{J}}{\Delta t} = \frac{(-7400\,\text{kg}\cdot\text{m/s})\hat{\text{i}}}{350\times10^{-3}\,\text{s}} = \left(-2.1\times10^4\,\text{N}\right)\hat{\text{i}}$$

and its magnitude is $F_{\text{avg}} = 2.1 \times 10^4$ N.

(e) The average force is given above in unit vector notation. Its x and y components have equal magnitudes. The x component is positive and the y component is negative, so the force is 45° below the positive x axis.

94. We first consider the 1200 kg part. The impulse has magnitude J and is (by our choice of coordinates) in the positive direction. Let m_1 be the mass of the part and v_1 be its velocity after the bolts are exploded. We assume both parts are at rest before the explosion. Then $J = m_1 v_1$, so

$$v_1 = \frac{J}{m_1} = \frac{300 \text{ N}\cdot\text{s}}{1200 \text{ kg}} = 0.25 \text{ m/s}.$$

The impulse on the 1800 kg part has the same magnitude but is in the opposite direction, so $-J = m_2 v_2$, where m_2 is the mass and v_2 is the velocity of the part. Therefore,

$$v_2 = -\frac{J}{m_2} = -\frac{300\text{N}\cdot\text{s}}{1800\,\text{kg}} = -0.167\,\text{m/s}.$$

Consequently, the relative speed of the parts after the explosion is

$$u = 0.25 \text{ m/s} - (-0.167 \text{ m/s}) = 0.417 \text{ m/s}.$$

95. We choose our positive direction in the direction of the rebound (so the ball's initial velocity is negative-valued $\vec{v}_i = -5.2\ \text{m/s}$).

(a) The speed of the ball right after the collision is

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2\left(\frac{1}{2}K_i\right)}{m}} = \sqrt{\frac{\frac{1}{2}mv_i^2}{m}} = \frac{v_i}{\sqrt{2}} \approx 3.7\ \text{m/s}\,.$$

(b) With m = 0.15 kg, the impulse-momentum theorem (Eq. 9-31) yields

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = (0.15\ \text{kg})(3.7\ \text{m/s}) - (0.15\ \text{kg})(-5.2\ \text{m/s}) = 1.3\ \text{N}\cdot\text{s}.$$

(c) Eq. 9-35 leads to $F_{\text{avg}} = J/\Delta t = 1.3/0.0076 = 1.8 \times 10^2$ N.

96. Let m_c be the mass of the Chrysler and v_c be its velocity. Let m_f be the mass of the Ford and v_f be its velocity. Then the velocity of the center of mass is

$$v_{\text{com}} = \frac{m_c v_c + m_f v_f}{m_c + m_f} = \frac{(2400 \text{ kg})(80 \text{ km/h}) + (1600 \text{ kg})(60 \text{ km/h})}{2400 \text{ kg} + 1600 \text{ kg}} = 72 \text{ km/h}.$$

We note that the two velocities are in the same direction, so the two terms in the numerator have the same sign.

97. Let m_F be the mass of the freight car and v_F be its initial velocity. Let m_C be the mass of the caboose and v be the common final velocity of the two when they are coupled. Conservation of the total momentum of the two-car system leads to

$$m_F v_F = (m_F + m_C)v \;\Rightarrow\; v = v_F m_F / (m_F + m_C).$$

The initial kinetic energy of the system is

$$K_i = \frac{1}{2} m_F v_F^2$$

and the final kinetic energy is

$$K_f = \frac{1}{2}(m_F + m_C)v^2 = \frac{1}{2}(m_F + m_C)\frac{m_F^2 v_F^2}{(m_F + m_C)^2} = \frac{1}{2}\frac{m_F^2 v_F^2}{(m_F + m_C)}.$$

Since 27% of the original kinetic energy is lost, we have $K_f = 0.73 K_i$. Thus,

$$\frac{1}{2}\frac{m_F^2 v_F^2}{(m_F + m_C)} = (0.73)\left(\frac{1}{2} m_F v_F^2\right).$$

Simplifying, we obtain $m_F/(m_F + m_C) = 0.73,$ which we use in solving for the mass of the caboose:

$$m_C = \frac{0.27}{0.73} m_F = 0.37 m_F = (0.37)(3.18\times10^4 \text{ kg}) = 1.18\times10^4 \text{ kg}.$$

98. The fact that they are connected by a spring is not used in the solution. We use Eq. 9-17 for $\vec{v}_{\text{com}}$:

$$M\vec{v}_{\text{com}} = m_1\vec{v}_1 + m_2\vec{v}_2 = (1.0\text{ kg})(1.7\text{ m/s}) + (3.0\text{ kg})\vec{v}_2$$

which yields $|\vec{v}_2| = 0.57\text{ m/s}$. The direction of $\vec{v}_2$ is opposite that of $\vec{v}_1$ (that is, they are both headed towards the center of mass, but from opposite directions).

99. No external forces with horizontal components act on the cart-man system and the vertical forces sum to zero, so the total momentum of the system is conserved. Let m_c be the mass of the cart, v be its initial velocity, and v_c be its final velocity (after the man jumps off). Let m_m be the mass of the man. His initial velocity is the same as that of the cart and his final velocity is zero. Conservation of momentum yields $(m_m + m_c)v = m_c v_c$. Consequently, the final speed of the cart is

$$v_c = \frac{v(m_m + m_c)}{m_c} = \frac{(2.3 \text{ m/s})(75 \text{ kg} + 39 \text{ kg})}{39 \text{ kg}} = 6.7 \text{ m/s}.$$

The cart speeds up by 6.7 m/s – 2.3 m/s = + 4.4 m/s. In order to slow himself, the man gets the cart to push backward on him by pushing forward on it, so the cart speeds up.

100. (a) We find the momentum $\vec{p}_{n\,r}$ of the residual nucleus from momentum conservation.

$$\vec{p}_{n\,i} = \vec{p}_e + \vec{p}_\nu + \vec{p}_{n\,r} \;\;\Rightarrow\;\; 0 = (-1.2\times10^{-22}\text{ kg}\cdot\text{m/s})\,\hat{\text{i}} + (-6.4\times10^{-23}\text{ kg}\cdot\text{m/s})\,\hat{\text{j}} + \vec{p}_{n\,r}$$

Thus, $\vec{p}_{n\,r} = (1.2\times10^{-22}\,\text{kg}\cdot\text{m/s})\,\hat{\text{i}} + (6.4\times10^{-23}\,\text{kg}\cdot\text{m/s})\,\hat{\text{j}}$. Its magnitude is

$$|\,\vec{p}_{n\,r}\,| = \sqrt{\left(1.2\times10^{-22}\text{ kg}\cdot\text{m/s}\right)^2 + \left(6.4\times10^{-23}\text{ kg}\cdot\text{m/s}\right)^2} = 1.4\times10^{-22}\text{ kg}\cdot\text{m/s}.$$

(b) The angle measured from the $+x$ axis to $\vec{p}_{n\,r}$ is

$$\theta = \tan^{-1}\left(\frac{6.4\times10^{-23}\text{ kg}\cdot\text{m/s}}{1.2\times10^{-22}\text{ kg}\cdot\text{m/s}}\right) = 28°.$$

(c) Combining the two equations $p = mv$ and $K = \frac{1}{2}mv^2$, we obtain (with $p = p_{n\,r}$ and $m = m_{n\,r}$)

$$K = \frac{p^2}{2m} = \frac{\left(1.4\times10^{-22}\text{ kg}\cdot\text{m/s}\right)^2}{2\left(5.8\times10^{-26}\text{ kg}\right)} = 1.6\times10^{-19}\text{ J.}$$

101. The mass of each ball is *m*, and the initial speed of one of the balls is $v_{1i} = 2.2\,\text{m/s}$. We apply the conservation of linear momentum to the *x* and *y* axes respectively.

$$mv_{1i} = mv_{1f}\cos\theta_1 + mv_{2f}\cos\theta_2$$
$$0 = mv_{1f}\sin\theta_1 - mv_{2f}\sin\theta_2$$

The mass *m* cancels out of these equations, and we are left with two unknowns and two equations, which is sufficient to solve.

(a) The *y*-momentum equation can be rewritten as, using $\theta_2 = 60°$ and $v_{2f} = 1.1$ m/s ,

$$v_{1f}\sin\theta_1 = (1.1\text{ m/s})\sin 60° = 0.95\text{ m/s}.$$

and the *x*-momentum equation yields

$$v_{1f}\cos\theta_1 = (2.2\text{ m/s}) - (1.1\text{ m/s})\cos 60° = 1.65\text{ m/s}.$$

Dividing these two equations, we find $\tan\theta_1 = 0.576$ which yields $\theta_1 = 30°$. We plug the value into either equation and find $v_{1f} \approx 1.9$ m/s.

(b) From the above, we have $\theta_1 = 30°$, measured *clockwise* from the +*x*-axis, or equivalently, –30°, measured *counterclockwise* from the +*x*-axis.

(c) One can check to see if this an elastic collision by computing

$$\frac{2K_i}{m} = v_{1i}^2 \quad\text{and}\quad \frac{2K_f}{m} = v_{1f}^2 + v_{2f}^2$$

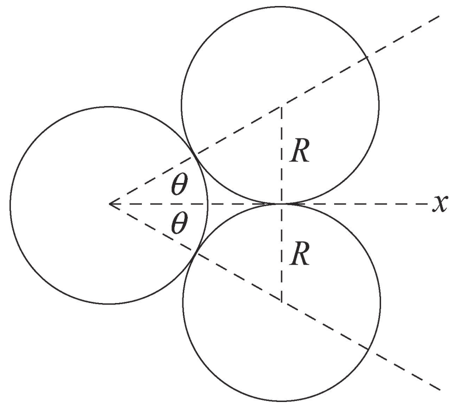
and seeing if they are equal (they are), but one must be careful not to use rounded-off values. Thus, it is useful to note that the answer in part (a) can be expressed "exactly" as $v_{1f} = \frac{1}{2}v_{1i}\sqrt{3}$ (and of course $v_{2f} = \frac{1}{2}v_{1i}$ "exactly" — which makes it clear that these two kinetic energy expressions are indeed equal).

102. (a) We use Eq. 9-87. The thrust is

$$Rv_{\text{rel}} = Ma = \left(4.0\times10^4\ \text{kg}\right)\left(2.0\,\text{m/s}^2\right) = 8.0\times10^4\ \text{N}.$$

(b) Since v_{rel} = 3000 m/s, we see from part (a) that $R \approx 27$ kg/s.

103. The diagram below shows the situation as the incident ball (the left-most ball) makes contact with the other two.



It exerts an impulse of the same magnitude on each ball, along the line that joins the centers of the incident ball and the target ball. The target balls leave the collision along those lines, while the incident ball leaves the collision along the x axis. The three dotted lines that join the centers of the balls in contact form an equilateral triangle, so both of the angles marked θ are 30°. Let v_0 be the velocity of the incident ball before the collision and V be its velocity afterward. The two target balls leave the collision with the same speed. Let v represent that speed. Each ball has mass m. Since the x component of the total momentum of the three-ball system is conserved,

$$mv_0 = mV + 2mv\cos\theta$$

and since the total kinetic energy is conserved,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mV^2 + 2\left(\frac{1}{2}mv^2\right).$$

We know the directions in which the target balls leave the collision so we first eliminate V and solve for v. The momentum equation gives $V = v_0 - 2v\cos\theta$, so

$$V^2 = v_0^2 - 4v_0v\cos\theta + 4v^2\cos^2\theta$$

and the energy equation becomes $v_0^2 = v_0^2 - 4v_0v\cos\theta + 4v^2\cos^2\theta + 2v^2$. Therefore,

$$v = \frac{2v_0\cos\theta}{1+2\cos^2\theta} = \frac{2(10\ \text{m/s})\cos 30°}{1+2\cos^2 30°} = 6.93\ \text{m/s}.$$

(a) The discussion and computation above determines the final speed of ball 2 (as labeled in Fig. 9-83) to be 6.9 m/s.

(b) The direction of ball 2 is at 30° counterclockwise from the $+x$ axis.

(c) Similarly, the final speed of ball 3 is 6.9 m/s.

(d) The direction of ball 3 is at –30° counterclockwise from the $+x$ axis.

(e) Now we use the momentum equation to find the final velocity of ball 1:

$$V = v_0 - 2v\cos\theta = 10\ \text{m/s} - 2(6.93\ \text{m/s})\ \cos 30° = -2.0\,\text{m/s}.$$

So the speed of ball 1 is $|V| = 2.0$ m/s.

(f) The minus sign indicates that it bounces back in the $-x$ direction. The angle is $-180°$.

104. (a) We use Fig. 9-22 of the text (which treats both angles as positive-valued, even though one of them is in the fourth quadrant; this is why there is an explicit minus sign in Eq. 9-80 as opposed to it being implicitly in the angle). We take the cue ball to be body 1 and the other ball to be body 2. Conservation of the x and the components of the total momentum of the two-ball system leads to:

$$mv_{1i} = mv_{1f}\cos\theta_1 + mv_{2f}\cos\theta_2$$

$$0 = -mv_{1f}\sin\theta_1 + mv_{2f}\sin\theta_2.$$

The masses are the same and cancel from the equations. We solve the second equation for $\sin\theta_2$:

$$\sin\theta_2 = \frac{v_{1f}}{v_{2f}}\sin\theta_1 = \left(\frac{3.50\text{ m/s}}{2.00\text{ m/s}}\right)\sin 22.0° = 0.656\ .$$

Consequently, the angle between the second ball and the initial direction of the first is θ_2 = 41.0°.

(b) We solve the first momentum conservation equation for the initial speed of the cue ball.

$$v_{1i} = v_{1f}\cos\theta_1 + v_{2f}\cos\theta_2 = (3.50\text{ m/s})\cos 22.0° + (2.00\text{ m/s})\cos 41.0° = 4.75\text{ m/s}\ .$$

(c) With SI units understood, the initial kinetic energy is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}m(4.75)^2 = 11.3m$$

and the final kinetic energy is

$$K_f = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2 = \frac{1}{2}m\left((3.50)^2 + (2.00)^2\right) = 8.1m.$$

Kinetic energy is not conserved.

105. (a) We place the origin of a coordinate system at the center of the pulley, with the x axis horizontal and to the right and with the y axis downward. The center of mass is halfway between the containers, at $x = 0$ and $y = \ell$, where ℓ is the vertical distance from the pulley center to either of the containers. Since the diameter of the pulley is 50 mm, the center of mass is at a horizontal distance of 25 mm from each container.

(b) Suppose 20 g is transferred from the container on the left to the container on the right. The container on the left has mass $m_1 = 480$ g and is at $x_1 = -25$ mm. The container on the right has mass $m_2 = 520$ g and is at $x_2 = +25$ mm. The x coordinate of the center of mass is then

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(480\text{ g})(-25\text{ mm}) + (520\text{ g})(25\text{ mm})}{480\text{ g} + 520\text{ g}} = 1.0\text{ mm}.$$

The y coordinate is still ℓ. The center of mass is 26 mm from the lighter container, along the line that joins the bodies.

(c) When they are released the heavier container moves downward and the lighter container moves upward, so the center of mass, which must remain closer to the heavier container, moves downward.

(d) Because the containers are connected by the string, which runs over the pulley, their accelerations have the same magnitude but are in opposite directions. If a is the acceleration of m_2, then $-a$ is the acceleration of m_1. The acceleration of the center of mass is

$$a_{\text{com}} = \frac{m_1(-a) + m_2 a}{m_1 + m_2} = a\frac{m_2 - m_1}{m_1 + m_2}.$$

We must resort to Newton's second law to find the acceleration of each container. The force of gravity $m_1 g$, down, and the tension force of the string T, up, act on the lighter container. The second law for it is $m_1 g - T = -m_1 a$. The negative sign appears because a is the acceleration of the heavier container. The same forces act on the heavier container and for it the second law is $m_2 g - T = m_2 a$. The first equation gives $T = m_1 g + m_1 a$. This is substituted into the second equation to obtain $m_2 g - m_1 g - m_1 a = m_2 a$, so

$$a = (m_2 - m_1)g/(m_1 + m_2).$$

Thus,

$$a_{\text{com}} = \frac{g(m_2 - m_1)^2}{(m_1 + m_2)^2} = \frac{(9.8\text{ m/s}^2)(520\text{ g} - 480\text{ g})^2}{(480\text{ g} + 520\text{ g})^2} = 1.6\times10^{-2}\text{ m/s}^2.$$

The acceleration is downward.

106. (a) The momentum change for the 0.15 kg object is

$$\Delta\vec{p} = (0.15)[2\,\hat{\mathrm{i}} + 3.5\,\hat{\mathrm{j}} - 3.2\,\hat{\mathrm{k}} - (5\,\hat{\mathrm{i}} + 6.5\,\hat{\mathrm{j}} + 4\,\hat{\mathrm{k}})] = (-0.450\hat{\mathrm{i}} - 0.450\hat{\mathrm{j}} - 1.08\hat{\mathrm{k}})\ \mathrm{kg\cdot m/s}.$$

(b) By the impulse-momentum theorem (Eq. 9-31), $\vec{J} = \Delta\vec{p}$, we have

$$\vec{J} = (-0.450\hat{\mathrm{i}} - 0.450\hat{\mathrm{j}} - 1.08\hat{\mathrm{k}})\ \mathrm{N\cdot s}.$$

(c) Newton's third law implies $\vec{J}_{\mathrm{wall}} = -\vec{J}_{\mathrm{ball}}$ (where $\vec{J}_{\mathrm{ball}}$ is the result of part (b)), so

$$\vec{J}_{\mathrm{wall}} = (0.450\hat{\mathrm{i}} + 0.450\hat{\mathrm{j}} + 1.08\hat{\mathrm{k}})\ \mathrm{N\cdot s}.$$

107. (a) Noting that the initial velocity of the system is zero, we use Eq. 9-19 and Eq. 2-15 (adapted to two dimensions) to obtain

$$\vec{d} = \frac{1}{2}\left(\frac{\vec{F}_1 + \vec{F}_2}{m_1 + m_2}\right)t^2 = \frac{1}{2}\left(\frac{-2\hat{\text{i}} + \hat{\text{j}}}{0.006}\right)(0.002)^2$$

which has a magnitude of 0.745 mm.

(b) The angle of $\vec{d}$ is 153° counterclockwise from $+x$-axis.

(c) A similar calculation using Eq. 2-11 (adapted to two dimensions) leads to a center of mass velocity of $\vec{\text{v}} = 0.7453$ m/s at 153°. Thus, the center of mass kinetic energy is

$$K_{\text{com}} = \tfrac{1}{2}(m_1 + m_2)v^2 = 0.00167 \text{ J}.$$

108. (a) The change in momentum (taking upwards to be the positive direction) is

$$\Delta\vec{p} = (0.550 \text{ kg})[\,(3 \text{ m/s})\hat{\text{j}} - (-12 \text{ m/s})\hat{\text{j}}\,] = (+8.25 \text{ kg·m/s})\,\hat{\text{j}}\,.$$

(b) By the impulse-momentum theorem (Eq. 9-31) $\vec{J} = \Delta\vec{p} = (+8.25 \text{ N·s})\,\hat{\text{j}}$.

(c) By Newton's third law, $\vec{J}_c = -\vec{J}_b = (-8.25 \text{ N·s})\,\hat{\text{j}}$.

109. Using Eq. 9-67 and Eq. 9-68, we have after the collision

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{0.6m_1}{1.4m_1} v_{1i} = -\tfrac{3}{7}(4 \text{ m/s})$$

$$v_2 = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{1.4m_1} v_{1i} = \tfrac{1}{7}(4 \text{ m/s}) \ .$$

(a) During the (subsequent) sliding, the kinetic energy of block 1 $K_{1f} = \frac{1}{2} m_1 v_1^2$ is converted into thermal form ($\Delta E_{\text{th}} = \mu_k\, m_1 g\, d_1$). Solving for the sliding distance d_1 we obtain $d_1 = 0.2999$ m ≈ 30 cm.

(b) A very similar computation (but with subscript 2 replacing subscript 1) leads to block 2's sliding distance $d_2 = 3.332$ m ≈ 3.3 m.

110. (a) Since the initial momentum is zero, then the final momenta must add (in the vector sense) to 0. Therefore, with SI units understood, we have

$$\begin{aligned}\vec{p}_3 &= -\vec{p}_1 - \vec{p}_2 = -m_1\vec{v}_1 - m_2\vec{v}_2 \\ &= -\left(16.7\times10^{-27}\right)\left(6.00\times10^{6}\,\hat{\mathrm{i}}\right) - \left(8.35\times10^{-27}\right)\left(-8.00\times10^{6}\,\hat{\mathrm{j}}\right) \\ &= \left(-1.00\times10^{-19}\,\hat{\mathrm{i}} + 0.67\times10^{-19}\,\hat{\mathrm{j}}\right)\mathrm{kg\cdot m/s}.\end{aligned}$$

(b) Dividing by $m_3 = 11.7 \times 10^{-27}$ kg and using the Pythagorean theorem we find the speed of the third particle to be $v_3 = 1.03 \times 10^7$ m/s. The total amount of kinetic energy is

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 = 1.19\times10^{-12}\ \mathrm{J}.$$

111. We use m_1 for the mass of the electron and $m_2 = 1840m_1$ for the mass of the hydrogen atom. Using Eq. 9-68,

$$v_{2f} = \frac{2m_1}{m_1 + 1840m_1} v_{1i} = \frac{2}{1841} v_{1i}$$

we compute the final kinetic energy of the hydrogen atom:

$$K_{2f} = \frac{1}{2}(1840m_1)\left(\frac{2v_{1i}}{1841}\right)^2 = \frac{(1840)(4)}{1841^2}\left(\frac{1}{2}(1840m_1)v_{1i}^2\right)$$

so we find the fraction to be $(1840)(4)/1841^2 \approx 2.2 \times 10^{-3}$, or 0.22%.

112. We treat the car (of mass m_1) as a "point-mass" (which is initially 1.5 m from the right end of the boat). The left end of the boat (of mass m_2) is initially at $x = 0$ (where the dock is), and its left end is at $x = 14$ m. The boat's center of mass (in the absence of the car) is initially at $x = 7.0$ m. We use Eq. 9-5 to calculate the center of mass of the system:

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{(1500\text{ kg})(14\text{ m} - 1.5\text{ m}) + (4000\text{ kg})(7\text{ m})}{1500\text{ kg} + 4000\text{ kg}} = 8.5\text{ m}.$$

In the absence of *external* forces, the center of mass of the system does not change. Later, when the car (about to make the jump) is near the left end of the boat (which has moved from the shore an amount δx), the value of the system center of mass is still 8.5 m. The car (at this moment) is thought of as a "point-mass" 1.5 m from the left end, so we must have

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{(1500\text{ kg})(\delta x + 1.5\text{ m}) + (4000\text{ kg})(7\text{ m} + \delta x)}{1500\text{ kg} + 4000\text{ kg}} = 8.5\text{ m}.$$

Solving this for δx, we find $\delta x = 3.0$ m.

113. By conservation of momentum, the final speed v of the sled satisfies

$$(2900 \text{ kg})(250 \text{ m/s}) = (2900 \text{ kg} + 920 \text{ kg})v$$

which gives v = 190 m/s.

114. (a) The magnitude of the impulse is equal to the change in momentum:

$$J = mv - m(-v) = 2mv = 2(0.140 \text{ kg})(7.80 \text{ m/s}) = 2.18 \text{ kg} \cdot \text{m/s}$$

(b) Since in the calculus sense the average of a function is the integral of it divided by the corresponding interval, then the average force is the impulse divided by the time Δt. Thus, our result for the magnitude of the average force is $2mv/\Delta t$. With the given values, we obtain

$$F_{avg} = \frac{2(0.140 \text{ kg})(7.80 \text{ m/s})}{0.00380 \text{ s}} = 575 \text{ N} .$$

115. (a) We locate the coordinate origin at the center of Earth. Then the distance r_{com} of the center of mass of the Earth-Moon system is given by

$$r_{\text{com}} = \frac{m_M r_M}{m_M + m_E}$$

where m_M is the mass of the Moon, m_E is the mass of Earth, and r_M is their separation. These values are given in Appendix C. The numerical result is

$$r_{\text{com}} = \frac{(7.36\times10^{22}\ \text{kg})(3.82\times10^{8}\ \text{m})}{7.36\times10^{22}\ \text{kg}+5.98\times10^{24}\ \text{kg}} = 4.64\times10^{6}\ \text{m} \approx 4.6\times10^{3}\ \text{km}.$$

(b) The radius of Earth is $R_E = 6.37 \times 10^6$ m, so $r_{\text{com}} / R_E = 0.73 = 73\%$.

116. Conservation of momentum leads to

$$(900\text{ kg})(1000\text{ m/s}) = (500\text{ kg})(v_{\text{shuttle}} - 100\text{ m/s}) + (400\text{ kg})(v_{\text{shuttle}})$$

which yields $v_{\text{shuttle}} = 1055.6$ m/s for the shuttle speed and $v_{\text{shuttle}} - 100\text{ m/s} = 955.6$ m/s for the module speed (all measured in the frame of reference of the stationary main spaceship). The fractional increase in the kinetic energy is

$$\frac{\Delta K}{K_i} = \frac{K_f}{K_i} - 1 = \frac{(500\text{ kg})(955.6\text{ m/s})^2/2 + (400\text{ kg})(1055.6\text{ m/s})^2/2}{(900\text{ kg})(1000\text{ m/s})^2/2} = 2.5\times10^{-3}.$$

117. (a) The thrust is Rv_{rel} where v_{rel} = 1200 m/s. For this to equal the weight Mg where M = 6100 kg, we must have R = (6100) (9.8)/1200 ≈ 50 kg/s.

(b) Using Eq. 9-42 with the additional effect due to gravity, we have

$$Rv_{\text{rel}} - Mg = Ma$$

so that requiring a = 21 m/s^2 leads to $R = (6100)(9.8 + 21)/1200 = 1.6 \times 10^2$ kg/s.

118. We denote the mass of the car as M and that of the sumo wrestler as m. Let the initial velocity of the sumo wrestler be $v_0 > 0$ and the final velocity of the car be v. We apply the momentum conservation law.

(a) From $mv_0 = (M + m)v$ we get

$$v = \frac{mv_0}{M+m} = \frac{(242\text{ kg})(5.3\text{ m/s})}{2140\text{ kg} + 242\text{ kg}} = 0.54\text{ m/s}.$$

(b) Since $v_{\text{rel}} = v_0$, we have

$$mv_0 = Mv + m(v + v_{\text{rel}}) = mv_0 + (M+m)v\,,$$

and obtain $v = 0$ for the final speed of the flatcar.

(c) Now $mv_0 = Mv + m\,(v - v_{\text{rel}})$, which leads to

$$v = \frac{m(v_0 + v_{\text{rel}})}{m + M} = \frac{(242\text{ kg})(5.3\text{ m/s} + 5.3\text{ m/s})}{242\text{ kg} + 2140\text{ kg}} = 1.1\text{ m/s}.$$

119. (a) Each block is assumed to have uniform density, so that the center of mass of each block is at its geometric center (the positions of which are given in the table [see problem statement] at $t = 0$). Plugging these positions (and the block masses) into Eq. 9-29 readily gives $x_{com} = -0.50$ m (at $t = 0$).

(b) Note that the left edge of block 2 (the middle of which is still at $x = 0$) is at $x = -2.5$ cm, so that at the moment they touch the right edge of block 1 is at $x = -2.5$ cm and thus the middle of block 1 is at $x = -5.5$ cm. Putting these positions (for the middles) and the block masses into Eq. 9-29 leads to $x_{com} = -1.83$ cm or -0.018 m (at $t = (1.445 \text{ m})/(0.75 \text{ m/s}) = 1.93$ s).

(c) We could figure where the blocks are at $t = 4.0$ s and use Eq. 9-29 again, but it is easier (and provides more insight) to note that in the absence of *external* forces on the system the center of mass should move at constant velocity:

$$\vec{v}_{com} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} = 0.25 \text{ m/s } \hat{i}$$

as can be easily verified by putting in the values at $t = 0$. Thus,

$$x_{com} = x_{com\ initial} + \vec{v}_{com}\, t = (-0.50 \text{ m}) + (0.25 \text{ m/s})(4.0 \text{ s}) = +0.50 \text{ m} .$$

120. (a) Since the center of mass of the man-balloon system does not move, the balloon will move downward with a certain speed u relative to the ground as the man climbs up the ladder.

(b) The speed of the man relative to the ground is $v_g = v - u$. Thus, the speed of the center of mass of the system is

$$v_{\text{com}} = \frac{mv_g - Mu}{M+m} = \frac{m(v-u) - Mu}{M+m} = 0.$$

This yields

$$u = \frac{mv}{M+m} = \frac{(80\text{ kg})(2.5\text{ m/s})}{320\text{ kg} + 80\text{ kg}} = 0.50\text{ m/s}.$$

(c) Now that there is no relative motion within the system, the speed of both the balloon and the man is equal to v_{com}, which is zero. So the balloon will again be stationary.

121. Using Eq. 9-67, we have after the elastic collision

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{-200\text{ g}}{600\text{ g}} v_{1i} = -\tfrac{1}{3}(3.00\text{ m/s}) = -\,1.00\text{ m/s}\,.$$

(a) The impulse is therefore

$$J = m_1 v_{1f} - m_1 v_{1i} = (0.200\text{ kg})(-1.00\text{ m/s}) - (0.200\text{ kg})(3.00\text{ m/s}) = -\,0.800\text{ N}\cdot\text{s}$$
$$= -\,0.800\text{ kg}\cdot\text{m/s},$$

or $|\,J\,| = -0.800$ kg·m/s.

(b) For the completely inelastic collision Eq. 9-75 applies

$$v_{1f} = V = \frac{m_1}{m_1 + m_2} v_{1i} = +\,1.00\text{ m/s}\,.$$

Now the impulse is

$$J = m_1 v_{1f} - m_1 v_{1i} = (0.200\text{ kg})(1.00\text{ m/s}) - (0.200\text{ kg})(3.00\text{ m/s}) = 0.400\text{ N}\cdot\text{s}$$
$$= 0.400\text{ kg}\cdot\text{m/s}.$$

122. We use Eq. 9-88 and simplify with $v_f - v_i = \Delta v$, and $v_{rel} = u$.

$$v_f - v_i = v_{rel} \ln\left(\frac{M_i}{M_f}\right) \Rightarrow \frac{M_f}{M_i} = e^{-\Delta v/u}$$

If $\Delta v = 2.2$ m/s and $u = 1000$ m/s, we obtain $\dfrac{M_i - M_f}{M_i} = 1 - e^{-0.0022} \approx 0.0022.$

123. This is a completely inelastic collision, followed by projectile motion. In the collision, we use momentum conservation.

$$\vec{p}_{\text{shoes}} = \vec{p}_{\text{together}} \quad \Rightarrow \quad (3.2 \text{ kg})(3.0 \text{ m/s}) = (5.2 \text{ kg})\vec{v}$$

Therefore, $\vec{v} = 1.8 \text{ m/s}$ toward the right as the combined system is projected from the edge of the table. Next, we can use the projectile motion material from Ch. 4 or the energy techniques of Ch. 8; we choose the latter.

$$K_{\text{edge}} + U_{\text{edge}} = K_{\text{floor}} + U_{\text{floor}}$$

$$\frac{1}{2}(5.2 \text{ kg})(1.8 \text{ m/s})^2 + (5.2 \text{ kg})(9.8 \text{ m/s}^2)(0.40 \text{ m}) = K_{\text{floor}} + 0$$

Therefore, the kinetic energy of the system right before hitting the floor is $K_{\text{floor}} = 29$ J.

124. We refer to the discussion in the textbook (Sample Problem 9-10, which uses the same notation that we use here) for some important details in the reasoning. We choose rightward in Fig. 9-21 as our $+x$ direction. We use the notation $\vec{v}$ when we refer to velocities and v when we refer to speeds (which are necessarily positive). Since the algebra is fairly involved, we find it convenient to introduce the notation $\Delta m = m_2 - m_1$ (which, we note for later reference, is a positive-valued quantity).

(a) Since $\vec{v}_{1i} = +\sqrt{2gh_1}$ where $h_1 = 9.0$ cm, we have

$$\vec{v}_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = -\frac{\Delta m}{m_1 + m_2}\sqrt{2gh_1}$$

which is to say that the *speed* of sphere 1 immediately after the collision is $v_{1f} = \left(\Delta m/(m_1 + m_2)\right)\sqrt{2gh_1}$ and that $\vec{v}_{1f}$ points in the $-x$ direction. This leads (by energy conservation $m_1 g h_{1f} = \frac{1}{2} m_1 v_{1f}^2$) to

$$h_{1f} = \frac{v_{1f}^2}{2g} = \left(\frac{\Delta m}{m_1 + m_2}\right)^2 h_1 \ .$$

With $m_1 = 50$ g and $m_2 = 85$ g, this becomes $h_{1f} \approx 0.60 \text{ cm}$.

(b) Eq. 9-68 gives

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{m_1 + m_2}\sqrt{2gh_1}$$

which leads (by energy conservation $m_2 g h_{2f} = \frac{1}{2} m_2 v_{2f}^2$) to

$$h_{2f} = \frac{v_{2f}^2}{2g} = \left(\frac{2m_1}{m_1 + m_2}\right)^2 h_1 \ .$$

With $m_1 = 50$ g and $m_2 = 85$ g, this becomes $h_{2f} \approx 4.9 \text{ cm}$.

(c) Fortunately, they hit again at the lowest point (as long as their amplitude of swing was "small" – this is further discussed in Chapter 16). At the risk of using cumbersome notation, we refer to the *next* set of heights as h_{1ff} and h_{2ff}. At the lowest point (before this second collision) sphere 1 has velocity $+\sqrt{2gh_{1f}}$ (rightward in Fig. 9-21) and sphere 2 has velocity $-\sqrt{2gh_{1f}}$ (that is, it points in the $-x$ direction). Thus, the velocity of sphere 1 immediately after the second collision is, using Eq. 9-75,

$$
\begin{aligned}
\vec{v}_{1ff} &= \frac{m_1 - m_2}{m_1 + m_2}\sqrt{2gh_{1f}} + \frac{2m_2}{m_1 + m_2}\left(-\sqrt{2gh_{2f}}\right) \\
&= \frac{-\Delta m}{m_1 + m_2}\left(\frac{\Delta m}{m_1 + m_2}\sqrt{2gh_1}\right) - \frac{2m_2}{m_1 + m_2}\left(\frac{2m_1}{m_1 + m_2}\sqrt{2gh_1}\right) \\
&= -\frac{(\Delta m)^2 + 4m_1 m_2}{(m_1 + m_2)^2}\sqrt{2gh_1} \ .
\end{aligned}
$$

This can be greatly simplified (by expanding $(\Delta m)^2$ and $(m_1 + m_2)^2$) to arrive at the conclusion that the speed of sphere 1 immediately after the second collision is simply $v_{1ff} = \sqrt{2gh_1}$ and that $\vec{v}_{1ff}$ points in the $-x$ direction. Energy conservation $\left(m_1 g h_{1ff} = \frac{1}{2} m_1 v_{1ff}^2\right)$ leads to

$$
h_{1ff} = \frac{v_{1ff}^2}{2g} = h_1 = 9.0 \text{ cm} \ .
$$

(d) One can reason (energy-wise) that $h_{1ff} = 0$ simply based on what we found in part (c). Still, it might be useful to see how this shakes out of the algebra. Eq. 9-76 gives the velocity of sphere 2 immediately after the second collision:

$$
\begin{aligned}
v_{2ff} &= \frac{2m_1}{m_1 + m_2}\sqrt{2gh_{1f}} + \frac{m_2 - m_1}{m_1 + m_2}\left(-\sqrt{2gh_{2f}}\right) \\
&= \frac{2m_1}{m_1 + m_2}\left(\frac{\Delta m}{m_1 + m_2}\sqrt{2gh_1}\right) + \frac{\Delta m}{m_1 + m_2}\left(\frac{-2m_1}{m_1 + m_2}\sqrt{2gh_1}\right)
\end{aligned}
$$

which vanishes since $(2m_1)(\Delta m) - (\Delta m)(2m_1) = 0$. Thus, the second sphere (after the second collision) stays at the lowest point, which basically recreates the conditions at the start of the problem (so all subsequent swings-and-impacts, neglecting friction, can be easily predicted – as they are just replays of the first two collisions).

125. From mechanical energy conservation (or simply using Eq. 2-16 with $\vec{a} = g$ downward) we obtain

$$v = \sqrt{2gh} = \sqrt{2(9.8\text{ m/s}^2)(6.0\text{ m})} = 10.8\text{ m/s}$$

for the speed just as the m = 3000-kg block makes contact with the pile. At the moment of "joining," they are a system of mass M = 3500 kg and speed V. With downward positive, momentum conservation leads to

$$mv = MV \Rightarrow V = \frac{(3000)(10.8)}{3500} = 9.3\text{ m/s}.$$

Now this block-pile "object" must be rapidly decelerated over the small distance d = 0.030 m. Using Eq. 2-16 and choosing $+y$ downward, we have

$$0 = V^2 + 2ad \Rightarrow a = -\frac{9.3^2}{2(0.030)} = -1440$$

in SI units (m/s^2). Thus, the net force during the decelerating process has magnitude

$$M\,|a| = 5.0 \times 10^6\text{ N}.$$

126. The momentum before the collision (with +*x* rightward) is

$$(6.0 \text{ kg})(8.0 \text{ m/s}) + (4.0 \text{ kg})(2.0 \text{ m/s}) = 56 \text{ kg}\cdot\text{m/s}.$$

(a) The total momentum at this instant is $(6.0 \text{ kg})(6.4 \text{ m/s}) + (4.0 \text{ kg})\vec{v}$. Since this must equal the initial total momentum (56, using SI units), then we find $\vec{v} = 4.4$ m/s.

(b) The initial kinetic energy was

$$\frac{1}{2}(6.0 \text{ kg})(8.0 \text{ m/s})^2 + \frac{1}{2}(4.0 \text{ kg})(2.0 \text{ m/s})^2 = 200 \text{ J}.$$

The kinetic energy at the instant described in part (a) is

$$\frac{1}{2}(6.0 \text{ kg})(6.4 \text{ m/s})^2 + \frac{1}{2}(4.0 \text{ kg})(4.4 \text{ m/s})^2 = 162 \text{ J}.$$

The "missing" 38 J is not dissipated since there is no friction; it is the energy stored in the spring at this instant when it is compressed. Thus, $U_e = 38$ J.

127. (a) The initial momentum of the system is zero, and it remains so as the electron and proton move toward each other. If p_e is the magnitude of the electron momentum at some instant (during their motion) and p_p is the magnitude of the proton momentum, then these must be equal (and their directions must be opposite) in order to maintain the zero total momentum requirement. Thus, the ratio of their momentum magnitudes is +1.

(b) With v_e and v_p being their respective speeds, we obtain (from the $p_e = p_p$ requirement)

$$m_e v_e = m_p v_p \quad \Rightarrow \quad v_e / v_p = m_p / m_e \;\approx\; 1830 \approx 1.83 \times 10^3.$$

(c) We can rewrite $K = \frac{1}{2} m v^2$ as $K = \frac{1}{2}\, p^2/m$ which immediately leads to

$$K_e / K_p = m_p / m_e \;\approx\; 1830 \approx 1.83 \times 10^3.$$

(d) Although the speeds (and kinetic energies) increase, they do so in the proportions indicated above. The answers stay the same.

128. In the momentum relationships, we could as easily work with weights as with masses, but because part (b) of this problem asks for kinetic energy—we will find the masses at the outset: $m_1 = 280 \times 10^3/9.8 = 2.86 \times 10^4$ kg and $m_2 = 210 \times 10^3/9.8 = 2.14 \times 10^4$ kg. Both cars are moving in the $+x$ direction: $v_{1i} = 1.52$ m/s and $v_{2i} = 0.914$ m/s.

(a) If the collision is completely elastic, momentum conservation leads to a final speed of

$$V = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = 1.26 \text{ m/s}.$$

(b) We compute the total initial kinetic energy and subtract from it the final kinetic energy.

$$K_i - K_f = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 - \frac{1}{2}(m_1 + m_2)V^2 = 2.25 \times 10^3 \text{ J}.$$

(c) Using Eq. 9-76, we find

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i} = 1.61 \text{ m/s}$$

(d) Using Eq. 9-75, we find

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} = 1.00 \text{ m/s}.$$

129. Using Eq. 9-68 with $m_1 = 3.0$ kg, $v_{1i} = 8.0$ m/s and $v_{2f} = 6.0$ m/s, then

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} \quad \Rightarrow \quad m_2 = m_1 \left(\frac{2v_{1i}}{v_{2f}} - 1 \right)$$

leads to $m_2 = M = 5.0$ kg.

130. (a) The center of mass does not move in the absence of external forces (since it was initially at rest).

(b) They collide at their center of mass. If the initial coordinate of P is $x = 0$ and the initial coordinate of Q is $x = 1.0$ m, then Eq. 9-5 gives

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{0 + (0.30 \text{ kg})(1.0 \text{ m})}{0.1 \text{ kg} + 0.3 \text{ kg}} = 0.75 \text{ m}.$$

Thus, they collide at a point 0.75 m from P's original position.

131. The velocities of m_1 and m_2 just after the collision with each other are given by Eq. 9-75 and Eq. 9-76 (setting $v_{1i} = 0$):

$$v_{1f} = \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

After bouncing off the wall, the velocity of m_2 becomes $-v_{2f}$. In these terms, the problem requires

$$v_{1f} = -v_{2f}$$

$$\frac{2m_2}{m_1 + m_2} v_{2i} = -\frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

which simplifies to

$$2m_2 = -(m_2 - m_1) \Rightarrow m_2 = \frac{m_1}{3} \ .$$

With $m_1 = 6.6$ kg, we have $m_2 = 2.2$ kg.

132. Momentum conservation (with SI units understood) gives

$$m_1(v_f - 20) + (M - m_1)v_f = Mv_i$$

which yields

$$v_f = \frac{M\mathrm{v}_i + 20\, m_1}{M} = v_i + 20\,\frac{m_1}{M} = 40 + 20\,(m_1/M).$$

(a) The minimum value of v_f is 40 m/s,

(b) The final speed v_f reaches a minimum as m_1 approaches zero.

(c) The maximum value of v_f is 60 m/s.

(d) The final speed v_f reaches a maximum as m_1 approaches M.

133. By the principle of momentum conservation, we must have

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 = 0,$$

which implies

$$\vec{v}_3 = -\frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_3}.$$

With

$$m_1\vec{v}_1 = (0.500)(10.0\,\hat{\mathrm{i}} + 12.0\,\hat{\mathrm{j}}) = 5.00\,\hat{\mathrm{i}} + 6.00\,\hat{\mathrm{j}}$$
$$m_2\vec{v}_2 = (0.750)(14.0)(\cos 110°\,\hat{\mathrm{i}} + \sin 110°\,\hat{\mathrm{j}}) = -3.59\,\hat{\mathrm{i}} + 9.87\,\hat{\mathrm{j}}$$

(in SI units) and $m_3 = m - m_1 - m_2 = (2.65 - 0.500 - 0.750)\text{kg} = 1.40\ \text{kg}$, we solve for $\vec{v}_3$ and obtain $\vec{v}_3 = (-1.01\ \text{m/s})\hat{\mathrm{i}} + (-11.3\ \text{m/s})\hat{\mathrm{j}}$.

(a) The magnitude of $\vec{v}_3$ is $|\vec{v}_3| = 11.4$ m/s.

(b) Its angle is 264.9°, which means it is 95.1° clockwise from the $+x$ axis.

134. Using Eq. 9-75 and Eq. 9-76, we find after the collision

(a) $v_{1f} = \dfrac{m_1 - m_2}{m_1 + m_2} v_{1i} + \dfrac{2m_2}{m_1 + m_2} v_{2i} = (-3.8 \text{ m/s})\hat{\text{i}}$, and

(b) $v_{2f} = \dfrac{2m_1}{m_1 + m_2} v_{1i} + \dfrac{m_2 - m_1}{m_1 + m_2} v_{2i} = (7.2 \text{ m/s})\hat{\text{i}}$.

135. We use Eq. 9-5.

(a) The x coordinate of the center of mass is

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4} = \frac{0 + (4)(3) + 0 + (12)(-1)}{m_1 + m_2 + m_3 + m_4} = 0.$$

(b) The $y$coordinate of the center of mass is

$$y_{\text{com}} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4} = \frac{(2)(3) + 0 + (3)(-2) + 0}{m_1 + m_2 + m_3 + m_4} = 0 .$$

(c) We now use Eq. 9-17:

$$\vec{v}_{\text{com}} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + m_4\vec{v}_4}{m_1 + m_2 + m_3 + m_4}$$
$$= \frac{(2)(-9\hat{j}) + (4)(6\hat{i}) + (3)(6\hat{j}) + (12)(-2\hat{i})}{m_1 + m_2 + m_3 + m_4} = 0 .$$

136. Let M = 22.7 kg and m = 3.63 be the mass of the sled and the cat, respectively. Using the principle of momentum conservation, the speed of the first sled after the cat's first jump with a speed of $v_i = 3.05$ m/s is

$$v_{1f} = \frac{mv_i}{M} = 0.488 \text{ m/s}.$$

On the other hand, as the cat lands on the second sled, it sticks to it and the system (sled plus cat) moves forward with a speed

$$v_{2f} = \frac{mv_i}{M+m} = 0.4205 \text{ m/s}.$$

When the cat makes the second jump back to the first sled with a speed v_i, momentum conservation implies

$$Mv_{2ff} = mv_i + (M+m)v_{2f} = mv_i + mv_i = 2mv_i$$

which yields

$$v_{2ff} = \frac{2mv_i}{M} = 0.975 \text{ m/s}.$$

After the cat lands on the first sled, the entire system (cat and the sled) again moves together. By momentum conservation, we have

$$(M+m)v_{1ff} = mv_i + Mv_{1f} = mv_i + mv_i = 2mv_i$$

or

$$v_{1ff} = \frac{2mv_i}{M+m} = 0.841 \text{ m/s}.$$

(a) From the above, we conclude that the first sled moves with a speed $v_{1ff} = 0.841$ m/s a after the cat's two jumps.

(b) Similarly, the speed of the second sled is $v_{2ff} = 0.975$ m/s.

Chapter 10

1. The problem asks us to assume v_{com} and ω are constant. For consistency of units, we write

$$v_{\text{com}} = (85\,\text{mi/h})\left(\frac{5280\,\text{ft/mi}}{60\,\text{min/h}}\right) = 7480\,\text{ft/min}\ .$$

Thus, with $\Delta x = 60\,\text{ft}$, the time of flight is

$$t = \Delta x / v_{\text{com}} = (60\ \text{ft})/(7480\ \text{ft/min}) = 0.00802\,\text{min}\ .$$

During that time, the angular displacement of a point on the ball's surface is

$$\theta = \omega t = (1800\,\text{rev/min})(0.00802\,\text{min}) \approx 14\,\text{rev}\ .$$

2. (a) The second hand of the smoothly running watch turns through 2π radians during 60 s . Thus,

$$\omega = \frac{2\pi}{60} = 0.105 \text{ rad/s.}$$

(b) The minute hand of the smoothly running watch turns through 2π radians during 3600 s . Thus,

$$\omega = \frac{2\pi}{3600} = 1.75 \times 10^{-3} \text{ rad / s.}$$

(c) The hour hand of the smoothly running 12-hour watch turns through 2π radians during 43200 s. Thus,

$$\omega = \frac{2\pi}{43200} = 1.45 \times 10^{-4} \text{ rad / s.}$$

3. Applying Eq. 2-15 to the vertical axis (with +y downward) we obtain the free-fall time:

$$\Delta y = v_{0y}t + \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2(10\text{ m})}{9.8\text{ m/s}^2}} = 1.4\text{ s}.$$

Thus, by Eq. 10-5, the magnitude of the average angular velocity is

$$\omega_{\text{avg}} = \frac{(2.5\text{ rev})(2\pi\text{ rad/rev})}{1.4\text{ s}} = 11\text{ rad/s}.$$

4. If we make the units explicit, the function is

$$\theta = (4.0 \text{ rad/s})t - (3.0 \text{ rad/s}^2)t^2 + (1.0 \text{ rad/s}^3)t^3$$

but generally we will proceed as shown in the problem—letting these units be understood. Also, in our manipulations we will generally not display the coefficients with their proper number of significant figures.

(a) Eq. 10-6 leads to

$$\omega = \frac{d}{dt}\left(4t - 3t^2 + t^3\right) = 4 - 6t + 3t^2.$$

Evaluating this at $t = 2$ s yields $\omega_2 = 4.0$ rad/s.

(b) Evaluating the expression in part (a) at $t = 4$ s gives $\omega_4 = 28$ rad/s.

(c) Consequently, Eq. 10-7 gives

$$\alpha_{\text{avg}} = \frac{\omega_4 - \omega_2}{4 - 2} = 12 \text{ rad/s}^2.$$

(d) And Eq. 10-8 gives

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(4 - 6t + 3t^2\right) = -6 + 6t.$$

Evaluating this at $t = 2$ s produces $\alpha_2 = 6.0$ rad/s^2.

(e) Evaluating the expression in part (d) at $t = 4$ s yields $\alpha_4 = 18$ rad/s^2. We note that our answer for α_{avg} does turn out to be the arithmetic average of α_2 and α_4 but point out that this will not always be the case.

5. The falling is the type of constant-acceleration motion you had in Chapter 2. The time it takes for the buttered toast to hit the floor is

$$\Delta t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(0.76\text{ m})}{9.8\text{ m/s}^2}} = 0.394\text{ s}.$$

(a) The smallest angle turned for the toast to land butter-side down is $\Delta\theta_{\text{min}} = 0.25\text{ rev} = \pi/2\text{ rad}.$ This corresponds to an angular speed of

$$\omega_{\text{min}} = \frac{\Delta\theta_{\text{min}}}{\Delta t} = \frac{\pi/2\text{ rad}}{0.394\text{ s}} = 4.0\text{ rad/s}.$$

(b) The largest angle (less than 1 revolution) turned for the toast to land butter-side down is $\Delta\theta_{\text{max}} = 0.75\text{ rev} = 3\pi/2\text{ rad}.$ This corresponds to an angular speed of

$$\omega_{\text{max}} = \frac{\Delta\theta_{\text{max}}}{\Delta t} = \frac{3\pi/2\text{ rad}}{0.394\text{ s}} = 12.0\text{ rad/s}.$$

6. If we make the units explicit, the function is

$$\theta = 2.0\text{ rad} + \left(4.0\text{ rad/s}^2\right)t^2 + \left(2.0\text{ rad/s}^3\right)t^3$$

but in some places we will proceed as indicated in the problem—by letting these units be understood.

(a) We evaluate the function θ at $t = 0$ to obtain $\theta_0 = 2.0$ rad.

(b) The angular velocity as a function of time is given by Eq. 10-6:

$$\omega = \frac{d\theta}{dt} = \left(8.0\text{ rad/s}^2\right)t + \left(6.0\text{ rad/s}^3\right)t^2$$

which we evaluate at $t = 0$ to obtain $\omega_0 = 0$.

(c) For $t = 4.0$ s, the function found in the previous part is

$$\omega_4 = (8.0)(4.0) + (6.0)(4.0)^2 = 128\text{ rad/s}.$$

If we round this to two figures, we obtain $\omega_4 \approx 1.3\times10^2$ rad/s.

(d) The angular acceleration as a function of time is given by Eq. 10-8:

$$\alpha = \frac{d\omega}{dt} = 8.0\text{ rad/s}^2 + \left(12\text{ rad/s}^3\right)t$$

which yields $\alpha_2 = 8.0 + (12)(2.0) = 32\text{ rad/s}^2$ at $t = 2.0$ s.

(e) The angular acceleration, given by the function obtained in the previous part, depends on time; it is not constant.

7. (a) To avoid touching the spokes, the arrow must go through the wheel in not more than

$$\Delta t = \frac{1/8 \text{ rev}}{2.5 \text{ rev}/\text{s}} = 0.050 \text{ s}.$$

The minimum speed of the arrow is then $v_{\min} = \dfrac{20 \text{ cm}}{0.050 \text{ s}} = 400 \text{ cm}/\text{s} = 4.0 \text{ m}/\text{s}.$

(b) No—there is no dependence on radial position in the above computation.

8. (a) We integrate (with respect to time) the $\alpha = 6.0t^4 - 4.0t^2$ expression, taking into account that the initial angular velocity is 2.0 rad/s. The result is

$$\omega = 1.2\,t^5 - 1.33\,t^3 + 2.0.$$

(b) Integrating again (and keeping in mind that $\theta_o = 1$) we get

$$\theta = 0.20t^6 - 0.33\,t^4 + 2.0\,t + 1.0\,.$$

9. We assume the sense of initial rotation is positive. Then, with $\omega_0 = +120$ rad/s and $\omega = 0$ (since it stops at time t), our angular acceleration (''deceleration'') will be negative-valued: $\alpha = -4.0\ \text{rad/s}^2$.

(a) We apply Eq. 10-12 to obtain t.

$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad t = \frac{0 - 120\ \text{rad/s}}{-4.0\ \text{rad/s}^2} = 30\ \text{s}.$$

(b) And Eq. 10-15 gives

$$\theta = \frac{1}{2}(\omega_0 + \omega)\,t = \frac{1}{2}(120\ \text{rad/s} + 0)(30\ \text{s}) = 1.8\times 10^3\ \text{rad}.$$

Alternatively, Eq. 10-14 could be used if it is desired to only use the given information (as opposed to using the result from part (a)) in obtaining θ. If using the result of part (a) is acceptable, then any angular equation in Table 10-1 (except Eq. 10-12) can be used to find θ.

10. (a) We assume the sense of rotation is positive. Applying Eq. 10-12, we obtain

$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad \alpha = \frac{(3000 - 1200)\ \text{rev/min}}{(12/60)\ \text{min}} = 9.0\times10^3\ \text{rev/min}^2.$$

(b) And Eq. 10-15 gives

$$\theta = \frac{1}{2}(\omega_0 + \omega)t = \frac{1}{2}(1200\ \text{rev/min} + 3000\ \text{rev/min})\left(\frac{12}{60}\ \text{min}\right) = 4.2\times10^2\ \text{rev}.$$

11. (a) With $\omega = 0$ and $\alpha = -4.2$ rad/s^2, Eq. 10-12 yields $t = -\omega_o/\alpha = 3.00$ s.

(b) Eq. 10-4 gives $\theta - \theta_o = -\ \omega_o^2 / 2\alpha = 18.9$ rad.

12. We assume the sense of rotation is positive, which (since it starts from rest) means all quantities (angular displacements, accelerations, etc.) are positive-valued.

(a) The angular acceleration satisfies Eq. 10-13:

$$25\text{ rad} = \frac{1}{2}\alpha(5.0\text{ s})^2 \;\Rightarrow\; \alpha = 2.0\text{ rad/s}^2.$$

(b) The average angular velocity is given by Eq. 10-5:

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} = \frac{25\text{ rad}}{5.0\text{ s}} = 5.0\text{ rad / s}.$$

(c) Using Eq. 10-12, the instantaneous angular velocity at t = 5.0 s is

$$\omega = \left(2.0\text{ rad/s}^2\right)(5.0\text{ s}) = 10\text{ rad/s}\,.$$

(d) According to Eq. 10-13, the angular displacement at t = 10 s is

$$\theta = \omega_0 + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}(2.0\text{ rad/s}^2)(10\text{ s})^2 = 100\text{ rad}.$$

Thus, the displacement between t = 5 s and t = 10 s is $\Delta\theta$ = 100 rad – 25 rad = 75 rad.

13. We take $t = 0$ at the start of the interval and take the sense of rotation as positive. Then at the end of the $t = 4.0$ s interval, the angular displacement is $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$. We solve for the angular velocity at the start of the interval:

$$\omega_0 = \frac{\theta - \frac{1}{2}\alpha t^2}{t} = \frac{120 \text{ rad } - \frac{1}{2}\left(3.0 \text{ rad/s}^2\right)\left(4.0 \text{ s}\right)^2}{4.0 \text{ s}} = 24 \text{ rad/s}.$$

We now use $\omega = \omega_0 + \alpha t$ (Eq. 10-12) to find the time when the wheel is at rest:

$$t = -\frac{\omega_0}{\alpha} = -\frac{24 \text{ rad / s}}{3.0 \text{ rad / s}^2} = -8.0 \text{ s}.$$

That is, the wheel started from rest 8.0 s before the start of the described 4.0 s interval.

14. (a) Eq. 10-13 gives

$$\theta - \theta_o = \omega_o t + \tfrac{1}{2}\alpha t^2 = 0 + \tfrac{1}{2}(1.5 \text{ rad/s}^2)\, t_1^{\,2}$$

where $\theta - \theta_o$ = (2 rev)(2π rad/rev). Therefore, t_1 = 4.09 s.

(b) We can find the time to go through a full 4 rev (using the same equation to solve for a new time t_2) and then subtract the result of part (a) for t_1 in order to find this answer.

$$(4 \text{ rev})(2\pi \text{ rad/rev}) = 0 + \tfrac{1}{2}(1.5 \text{ rad/s}^2)\, t_2^{\,2} \quad \Rightarrow \quad t_2 = 5.789 \text{ s.}$$

Thus, the answer is 5.789 s – 4.093 s ≈ 1.70 s.

15. The problem has (implicitly) specified the positive sense of rotation. The angular acceleration of magnitude 0.25 rad/s^2 in the negative direction is assumed to be constant over a large time interval, including negative values (for t).

(a) We specify θ_{max} with the condition $\omega = 0$ (this is when the wheel reverses from positive rotation to rotation in the negative direction). We obtain θ_{max} using Eq. 10-14:

$$\theta_{max} = -\frac{\omega_o^2}{2\alpha} = -\frac{(4.7 \text{ rad/s})^2}{2(-0.25 \text{ rad/s}^2)} = 44 \text{ rad.}$$

(b) We find values for t_1 when the angular displacement (relative to its orientation at $t = 0$) is $\theta_1 = 22$ rad (or 22.09 rad if we wish to keep track of accurate values in all intermediate steps and only round off on the final answers). Using Eq. 10-13 and the quadratic formula, we have

$$\theta_1 = \omega_o t_1 + \frac{1}{2}\alpha t_1^2 \Rightarrow t_1 = \frac{-\omega_o \pm \sqrt{\omega_o^2 + 2\theta_1\alpha}}{\alpha}$$

which yields the two roots 5.5 s and 32 s. Thus, the first time the reference line will be at $\theta_1 = 22$ rad is $t = 5.5$ s.

(c) The second time the reference line will be at $\theta_1 = 22$ rad is $t = 32$ s.

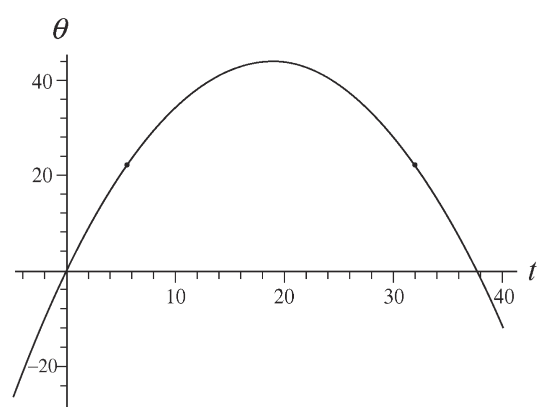
(d) We find values for t_2 when the angular displacement (relative to its orientation at $t = 0$) is $\theta_2 = -10.5$ rad. Using Eq. 10-13 and the quadratic formula, we have

$$\theta_2 = \omega_o t_2 + \frac{1}{2}\alpha t_2^2 \Rightarrow t_2 = \frac{-\omega_o \pm \sqrt{\omega_o^2 + 2\theta_2\alpha}}{\alpha}$$

which yields the two roots –2.1 s and 40 s. Thus, at $t = -2.1$ s the reference line will be at $\theta_2 = -10.5$ rad.

(e) At $t = 40$ s the reference line will be at $\theta_2 = -10.5$ rad.

(f) With radians and seconds understood, the graph of θ versus t is shown below (with the points found in the previous parts indicated as small circles).



16. The wheel starts turning from rest ($\omega_0 = 0$) at $t = 0$, and accelerates uniformly at $\alpha > 0$, which makes our choice for positive sense of rotation. At t_1 its angular velocity is $\omega_1 = +10$ rev/s, and at t_2 its angular velocity is $\omega_2 = +15$ rev/s. Between t_1 and t_2 it turns through $\Delta\theta = 60$ rev, where $t_2 - t_1 = \Delta t$.

(a) We find α using Eq. 10-14:

$$\omega_2^2 = \omega_1^2 + 2\alpha\Delta\theta \quad \Rightarrow \quad \alpha = \frac{(15 \text{ rev/s})^2 - (10 \text{ rev/s})^2}{2(60 \text{ rev})} = 1.04 \text{ rev/s}^2$$

which we round off to 1.0 rev/s^2.

(b) We find Δt using Eq. 10-15:

$$\Delta\theta = \frac{1}{2}(\omega_1 + \omega_2)\Delta t \Rightarrow \Delta t = \frac{2(60 \text{ rev})}{10 \text{ rev/s} + 15 \text{ rev/s}} = 4.8 \text{ s}.$$

(c) We obtain t_1 using Eq. 10-12: $\omega_1 = \omega_0 + \alpha t_1 \Rightarrow t_1 = \dfrac{10 \text{ rev/s}}{1.04 \text{ rev/s}^2} = 9.6 \text{ s}.$

(d) Any equation in Table 10-1 involving θ can be used to find θ_1 (the angular displacement during $0 \le t \le t_1$); we select Eq. 10-14.

$$\omega_1^2 = \omega_0^2 + 2\alpha\theta_1 \Rightarrow \theta_1 = \frac{(10 \text{ rev/s})^2}{2(1.04 \text{ rev/s}^2)} = 48 \text{ rev}.$$

17. The wheel has angular velocity ω_0 = +1.5 rad/s = +0.239 rev/s at t = 0, and has constant value of angular acceleration α < 0, which indicates our choice for positive sense of rotation. At t_1 its angular displacement (relative to its orientation at t = 0) is θ_1 = +20 rev, and at t_2 its angular displacement is θ_2 = +40 rev and its angular velocity is $\omega_2 = 0$.

(a) We obtain t_2 using Eq. 10-15:

$$\theta_2 = \frac{1}{2}(\omega_0 + \omega_2)t_2 \;\Rightarrow\; t_2 = \frac{2(40 \text{ rev})}{0.239 \text{ rev/s}} = 335 \text{ s}$$

which we round off to $t_2 \approx 3.4 \times 10^2$ s.

(b) Any equation in Table 10-1 involving α can be used to find the angular acceleration; we select Eq. 10-16.

$$\theta_2 = \omega_2 t_2 - \frac{1}{2}\alpha t_2^2 \;\Rightarrow\; \alpha = -\frac{2(40 \text{ rev})}{(335 \text{ s})^2} = -7.12 \times 10^{-4} \text{ rev/s}^2$$

which we convert to $\alpha = -4.5 \times 10^{-3}$ rad/s^2.

(c) Using $\theta_1 = \omega_0 t_1 + \frac{1}{2}\alpha t_1^2$ (Eq. 10-13) and the quadratic formula, we have

$$t_1 = \frac{-\omega_0 \pm \sqrt{\omega_0^2 + 2\theta_1\alpha}}{\alpha} = \frac{-(0.239 \text{ rev/s}) \pm \sqrt{(0.239 \text{ rev/s})^2 + 2(20 \text{ rev})(-7.12 \times 10^{-4} \text{ rev/s}^2)}}{-7.12 \times 10^{-4} \text{ rev/s}^2}$$

which yields two positive roots: 98 s and 572 s. Since the question makes sense only if t_1 < t_2 we conclude the correct result is t_1 = 98 s.

18. Converting $33\frac{1}{3}$ rev/min to radians-per-second, we get ω = 3.49 rad/s. Combining $v = \omega r$ (Eq. 10-18) with $\Delta t = d/v$ where Δt is the time between bumps (a distance d apart), we arrive at the rate of striking bumps:

$$\frac{1}{\Delta t} = \frac{\omega r}{d} \approx 199\,/\,\mathrm{s}\,.$$

19. We assume the given rate of 1.2×10^{-3} m/y is the linear speed of the top; it is also possible to interpret it as just the horizontal component of the linear speed but the difference between these interpretations is arguably negligible. Thus, Eq. 10-18 leads to

$$\omega = \frac{1.2 \times 10^{-3} \text{ m/y}}{55 \text{ m}} = 2.18 \times 10^{-5} \text{ rad/y}$$

which we convert (since there are about 3.16×10^7 s in a year) to $\omega = 6.9 \times 10^{-13}$ rad/s.

20. (a) Using Eq. 10-6, the angular velocity at t = 5.0s is

$$\omega = \left.\frac{d\theta}{dt}\right|_{t=5.0} = \left.\frac{d}{dt}\left(0.30t^2\right)\right|_{t=5.0} = 2(0.30)(5.0) = 3.0 \text{ rad / s.}$$

(b) Eq. 10-18 gives the linear speed at t = 5.0s: $v = \omega r = (3.0 \text{ rad/s})(10 \text{ m}) = 30 \text{ m/s}$.

(c) The angular acceleration is, from Eq. 10-8,

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(0.60t) = 0.60 \text{ rad / s}^2.$$

Then, the tangential acceleration at t = 5.0s is, using Eq. 10-22,

$$a_t = r\alpha = (10 \text{ m})\left(0.60 \text{ rad / s}^2\right) = 6.0 \text{ m / s}^2.$$

(d) The radial (centripetal) acceleration is given by Eq. 10-23:

$$a_r = \omega^2 r = \left(3.0 \text{ rad / s}\right)^2 \left(10 \text{ m}\right) = 90 \text{ m / s}^2.$$

21. (a) We obtain

$$\omega = \frac{(200 \text{ rev/min})(2\pi \text{ rad/rev})}{60 \text{ s/min}} = 20.9 \text{ rad/s}.$$

(b) With r = 1.20/2 = 0.60 m, Eq. 10-18 gives $v = r\omega = (0.60 \text{ m})(20.9 \text{ rad/s}) = 12.5 \text{ m/s}$.

(c) With t = 1 min, ω = 1000 rev/min and ω_0 = 200 rev/min, Eq. 10-12 gives

$$\alpha = \frac{\omega - \omega_o}{t} = 800 \text{ rev/min}^2.$$

(d) With the same values used in part (c), Eq. 10-15 becomes

$$\theta = \frac{1}{2}(\omega_o + \omega)t = \frac{1}{2}(200 \text{ rev/min} + 1000 \text{ rev/min})(1.0 \text{ min}) = 600 \text{ rev}.$$

22. First, we convert the angular velocity: ω = (2000 rev/min)(2π /60) = 209 rad/s. Also, we convert the plane's speed to SI units: (480)(1000/3600) = 133 m/s. We use Eq. 10-18 in part (a) and (implicitly) Eq. 4-39 in part (b).

(a) The speed of the tip as seen by the pilot is $v_t = \omega r = (209\,\text{rad/s})(1.5\,\text{m}) = 314\,\text{m/s}$, which (since the radius is given to only two significant figures) we write as $v_t = 3.1\times10^2\ \text{m/s}$.

(b) The plane's velocity $\vec{v}_p$ and the velocity of the tip $\vec{v}_t$ (found in the plane's frame of reference), in any of the tip's positions, must be perpendicular to each other. Thus, the speed as seen by an observer on the ground is

$$v = \sqrt{v_p^2 + v_t^2} = \sqrt{(133\,\text{m/s})^2 + (314\,\text{m/s})^2} = 3.4\times10^2\ \text{m/s}\,.$$

23. (a) Converting from hours to seconds, we find the angular velocity (assuming it is positive) from Eq. 10-18:

$$\omega = \frac{v}{r} = \frac{\left(2.90\times10^{4}\ \text{km/h}\right)\left(1.000\ \text{h}/3600\ \text{s}\right)}{3.22\times10^{3}\ \text{km}} = 2.50\times10^{-3}\ \text{rad/s}.$$

(b) The radial (or centripetal) acceleration is computed according to Eq. 10-23:

$$a_r = \omega^2 r = \left(2.50\times10^{-3}\ \text{rad/s}\right)^2\left(3.22\times10^{6}\ \text{m}\right) = 20.2\ \text{m/s}^2.$$

(c) Assuming the angular velocity is constant, then the angular acceleration and the tangential acceleration vanish, since

$$\alpha = \frac{d\omega}{dt} = 0 \ \text{ and }\ a_t = r\alpha = 0.$$

24. The function $\theta = \xi e^{\beta t}$ where ξ = 0.40 rad and β = 2 s^{-1} is describing the angular coordinate of a line (which is marked in such a way that all points on it have the same value of angle at a given time) on the object. Taking derivatives with respect to time leads to $\frac{d\theta}{dt} = \xi\beta e^{\beta t}$ and $\frac{d^2\theta}{dt^2} = \xi\beta^2 e^{\beta t}$.

(a) Using Eq. 10-22, we have $a_t = \alpha r = \dfrac{d^2\theta}{dt^2} r = 6.4\ \text{cm/s}^2$.

(b) Using Eq. 10-23, we get $a_r = \omega^2 r = \left(\dfrac{d\theta}{dt}\right)^2 r = 2.6\ \text{cm/s}^2$.

25. (a) The upper limit for centripetal acceleration (same as the radial acceleration – see Eq. 10-23) places an upper limit of the rate of spin (the angular velocity ω) by considering a point at the rim (r = 0.25 m). Thus, $\omega_{max} = \sqrt{a/r}$ = 40 rad/s. Now we apply Eq. 10-15 to first half of the motion (where $\omega_o = 0$):

$$\theta - \theta_o = \tfrac{1}{2}(\omega_o + \omega)t \;\Rightarrow\; 400 \text{ rad} = \tfrac{1}{2}(0 + 40 \text{ rad/s})t$$

which leads to t = 20 s. The second half of the motion takes the same amount of time (the process is essentially the reverse of the first); the total time is therefore 40 s.

(b) Considering the first half of the motion again, Eq. 10-11 leads to

$$\omega = \omega_o + \alpha t \;\Rightarrow\; \alpha = \frac{40 \text{ rad/s}}{20 \text{ s}} = 2.0 \text{ rad/s}^2 .$$

26. (a) The tangential acceleration, using Eq. 10-22, is

$$a_t = \alpha r = \left(14.2\ \text{rad/s}^2\right)(2.83\ \text{cm}) = 40.2\ \text{cm/s}^2.$$

(b) In rad/s, the angular velocity is ω = (2760)(2π/60) = 289 rad/s, so

$$a_r = \omega^2 r = (289\ \text{rad/s})^2(0.0283\ \text{m}) = 2.36\times10^3\ \text{m/s}^2.$$

(c) The angular displacement is, using Eq. 10-14,

$$\theta = \frac{\omega^2}{2\alpha} = \frac{(289\ \text{rad/s})^2}{2(14.2\ \text{rad/s}^2)} = 2.94\times10^3\ \text{rad}.$$

Then, using Eq. 10-1, the distance traveled is

$$s = r\theta = (0.0283\ \text{m})(2.94\times10^3\ \text{rad}) = 83.2\ \text{m}.$$

27. (a) In the time light takes to go from the wheel to the mirror and back again, the wheel turns through an angle of $\theta = 2\pi/500 = 1.26 \times 10^{-2}$ rad. That time is

$$t = \frac{2\ell}{c} = \frac{2(500\text{ m})}{2.998 \times 10^{8}\text{ m/s}} = 3.34 \times 10^{-6}\text{ s}$$

so the angular velocity of the wheel is

$$\omega = \frac{\theta}{t} = \frac{1.26 \times 10^{-2}\text{ rad}}{3.34 \times 10^{-6}\text{ s}} = 3.8 \times 10^{3}\text{ rad/s}.$$

(b) If r is the radius of the wheel, the linear speed of a point on its rim is

$$v = \omega r = \left(3.8 \times 10^{3}\text{ rad/s}\right)\left(0.050\text{ m}\right) = 1.9 \times 10^{2}\text{ m/s}.$$

28. (a) The angular acceleration is

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0 - 150 \text{ rev/min}}{(2.2 \text{ h})(60 \text{ min/1h})} = -1.14 \text{ rev/min}^2.$$

(b) Using Eq. 10-13 with $t = (2.2)\,(60) = 132$ min, the number of revolutions is

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (150 \text{ rev/min})(132 \text{ min}) + \frac{1}{2}\left(-1.14 \text{ rev/min}^2\right)\left(132 \text{ min}\right)^2 = 9.9\times 10^3 \text{ rev.}$$

(c) With $r = 500$ mm, the tangential acceleration is

$$a_t = \alpha r = \left(-1.14 \text{ rev/min}^2\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)^2 (500 \text{ mm})$$

which yields $a_t = -0.99$ mm/s^2.

(d) The angular speed of the flywheel is

$$\omega = (75 \text{ rev/min})(2\pi \text{ rad/rev})(1 \text{ min/ } 60 \text{ s}) = 7.85 \text{ rad/s.}$$

With $r = 0.50$ m, the radial (or centripetal) acceleration is given by Eq. 10-23:

$$a_r = \omega^2 r = (7.85 \text{ rad/s})^2\,(0.50 \text{ m}) \approx 31 \text{ m/s}^2$$

which is much bigger than a_t. Consequently, the magnitude of the acceleration is

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2} \approx a_r = 31 \text{ m/s}^2.$$

29. (a) Earth makes one rotation per day and 1 *d* is (24 h) (3600 s/h) = 8.64×10^4 s, so the angular speed of Earth is

$$\omega = \frac{2\pi \text{ rad}}{8.64\times10^4 \text{ s}} = 7.3\times10^{-5} \text{ rad/s}.$$

(b) We use $v = \omega r$, where r is the radius of its orbit. A point on Earth at a latitude of 40° moves along a circular path of radius $r = R \cos 40°$, where R is the radius of Earth (6.4×10^6 m). Therefore, its speed is

$$v = \omega(R \cos 40°) = (7.3\times10^{-5} \text{ rad/s})(6.4\times10^6 \text{ m})\cos 40° = 3.5\times10^2 \text{ m/s}.$$

(c) At the equator (and all other points on Earth) the value of ω is the same (7.3×10^{-5} rad/s).

(d) The latitude is 0° and the speed is

$$v = \omega R = (7.3\times10^{-5} \text{ rad/s})(6.4\times10^6 \text{ m}) = 4.6\times10^2 \text{ m/s}.$$

30. Since the belt does not slip, a point on the rim of wheel C has the same tangential acceleration as a point on the rim of wheel A. This means that $\alpha_A r_A = \alpha_C r_C$, where α_A is the angular acceleration of wheel A and α_C is the angular acceleration of wheel C. Thus,

$$\alpha_C = \left(\frac{r_A}{r_C}\right)\alpha_C = \left(\frac{10 \text{ cm}}{25 \text{ cm}}\right)(1.6 \text{ rad/s}^2) = 0.64 \text{ rad/s}^2.$$

Since the angular speed of wheel C is given by $\omega_C = \alpha_C t$, the time for it to reach an angular speed of $\omega = 100$ rev/min = 10.5 rad/s starting from rest is

$$t = \frac{\omega_C}{\alpha_C} = \frac{10.5 \text{ rad/s}}{0.64 \text{ rad/s}^2} = 16 \text{ s}.$$

31. (a) The angular speed in rad/s is

$$\omega = \left(33\frac{1}{3}\text{ rev/min}\right)\left(\frac{2\pi\text{ rad/rev}}{60\text{ s/min}}\right) = 3.49\text{ rad/s}.$$

Consequently, the radial (centripetal) acceleration is (using Eq. 10-23)

$$a = \omega^2 r = (3.49\text{ rad/s})^2(6.0\times10^{-2}\text{ m}) = 0.73\text{ m/s}^2.$$

(b) Using Ch. 6 methods, we have $ma = f_s \le f_{s,\max} = \mu_s mg$, which is used to obtain the (minimum allowable) coefficient of friction:

$$\mu_{s,\min} = \frac{a}{g} = \frac{0.73}{9.8} = 0.075.$$

(c) The radial acceleration of the object is $a_r = \omega^2 r$, while the tangential acceleration is $a_t = \alpha r$. Thus,

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2} = \sqrt{(\omega^2 r)^2 + (\alpha r)^2} = r\sqrt{\omega^4 + \alpha^2}.$$

If the object is not to slip at any time, we require

$$f_{s,\max} = \mu_s mg = ma_{\max} = mr\sqrt{\omega_{\max}^4 + \alpha^2}.$$

Thus, since $\alpha = \omega/t$ (from Eq. 10-12), we find

$$\mu_{s,\min} = \frac{r\sqrt{\omega_{\max}^4 + \alpha^2}}{g} = \frac{r\sqrt{\omega_{\max}^4 + (\omega_{\max}/t)^2}}{g} = \frac{(0.060)\sqrt{3.49^4 + (3.4/0.25)^2}}{9.8} = 0.11.$$

32. (a) A complete revolution is an angular displacement of $\Delta\theta = 2\pi$ rad, so the angular velocity in rad/s is given by $\omega = \Delta\theta/T = 2\pi/T$. The angular acceleration is given by

$$\alpha = \frac{d\omega}{dt} = -\frac{2\pi}{T^2}\frac{dT}{dt}.$$

For the pulsar described in the problem, we have

$$\frac{dT}{dt} = \frac{1.26\times10^{-5}\ \text{s/y}}{3.16\times10^{7}\ \text{s/y}} = 4.00\times10^{-13}.$$

Therefore,

$$\alpha = -\left(\frac{2\pi}{(0.033\ \text{s})^2}\right)(4.00\times10^{-13}) = -2.3\times10^{-9}\ \text{rad/s}^2.$$

The negative sign indicates that the angular acceleration is opposite the angular velocity and the pulsar is slowing down.

(b) We solve $\omega = \omega_0 + \alpha t$ for the time t when $\omega = 0$:

$$t = -\frac{\omega_0}{\alpha} = -\frac{2\pi}{\alpha T} = -\frac{2\pi}{(-2.3\times10^{-9}\ \text{rad/s}^2)(0.033\ \text{s})} = 8.3\times10^{10}\ \text{s} \approx 2.6\times10^{3}\ \text{ years}$$

(c) The pulsar was born 1992–1054 = 938 years ago. This is equivalent to (938 y)(3.16×10^7 s/y) = 2.96×10^{10} s. Its angular velocity at that time was

$$\omega = \omega_0 + \alpha t + \frac{2\pi}{T} + \alpha t = \frac{2\pi}{0.033\ \text{s}} + (-2.3\times10^{-9}\ \text{rad/s}^2)(-2.96\times10^{10}\ \text{s}) = 258\ \text{rad/s}.$$

Its period was

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{258\ \text{rad/s}} = 2.4\times10^{-2}\ \text{s}.$$

33. The kinetic energy (in J) is given by $K = \frac{1}{2} I\omega^2$, where I is the rotational inertia (in $\text{kg}\cdot\text{m}^2$) and ω is the angular velocity (in rad/s). We have

$$\omega = \frac{(602 \text{ rev/min})(2\pi \text{ rad/rev})}{60 \text{ s/min}} = 63.0 \text{ rad/s}.$$

Consequently, the rotational inertia is

$$I = \frac{2K}{\omega^2} = \frac{2(24400 \text{ J})}{(63.0 \text{ rad/s})^2} = 12.3 \text{ kg}\cdot\text{m}^2.$$

34. (a) Eq. 10-12 implies that the angular acceleration α should be the slope of the ω vs t graph. Thus, $\alpha = 9/6 = 1.5$ rad/s^2.

(b) By Eq. 10-34, K is proportional to ω^2. Since the angular velocity at $t = 0$ is –2 rad/s (and this value squared is 4) and the angular velocity at $t = 4$ s is 4 rad/s (and this value squared is 16), then the ratio of the corresponding kinetic energies must be

$$\frac{K_o}{K_4} = \frac{4}{16} \Rightarrow K_o = ¼\, K_4 = 0.40 \text{ J} .$$

35. We use the parallel axis theorem: $I = I_{com} + Mh^2$, where I_{com} is the rotational inertia about the center of mass (see Table 10-2(d)), M is the mass, and h is the distance between the center of mass and the chosen rotation axis. The center of mass is at the center of the meter stick, which implies $h = 0.50$ m – 0.20 m = 0.30 m. We find

$$I_{\text{com}} = \frac{1}{12}ML^2 = \frac{1}{12}(0.56\ \text{kg})(1.0\ \text{m})^2 = 4.67\times10^{-2}\ \text{kg}\cdot\text{m}^2.$$

Consequently, the parallel axis theorem yields

$$I = 4.67\times10^{-2}\ \text{kg}\cdot\text{m}^2 + (0.56\ \text{kg})(0.30\ \text{m})^2 = 9.7\times10^{-2}\ \text{kg}\cdot\text{m}^2.$$

36. (a) Eq. 10-33 gives

$$I_{\text{total}} = md^2 + m(2d)^2 + m(3d)^2 = 14\ md^2.$$

If the innermost one is removed then we would only obtain $m(2d)^2 + m(3d)^2 = 13\ md^2$. The percentage difference between these is $(13 - 14)/14 = 0.0714 \approx 7.1\%$.

(b) If, instead, the outermost particle is removed, we would have $md^2 + m(2d)^2 = 5\ md^2$. The percentage difference in this case is $0.643 \approx 64\%$.

37. Since the rotational inertia of a cylinder is $I = \frac{1}{2} MR^2$ (Table 10-2(c)), its rotational kinetic energy is

$$K = \frac{1}{2} I\omega^2 = \frac{1}{4} MR^2\omega^2.$$

(a) For the smaller cylinder, we have $K = \frac{1}{4}(1.25)(0.25)^2(235)^2 = 1.1 \times 10^3$ J.

(b) For the larger cylinder, we obtain $K = \frac{1}{4}(1.25)(0.75)^2(235)^2 = 9.7 \times 10^3$ J.

38. The parallel axis theorem (Eq. 10-36) shows that I increases with h. The phrase "out to the edge of the disk" (in the problem statement) implies that the maximum h in the graph is, in fact, the radius R of the disk. Thus, $R = 0.20$ m. Now we can examine, say, the $h = 0$ datum and use the formula for I_{com} (see Table 10-2(c)) for a solid disk, or (which might be a little better, since this is independent of whether it is really a solid disk) we can the difference between the $h = 0$ datum and the $h = h_{max} = R$ datum and relate that difference to the parallel axis theorem (thus the difference is $M(h_{max})^2 = 0.10\ \text{kg}{\cdot}\text{m}^2$). In either case, we arrive at $M = 2.5$ kg.

39. The particles are treated "point-like" in the sense that Eq. 10-33 yields their rotational inertia, and the rotational inertia for the rods is figured using Table 10-2(e) and the parallel-axis theorem (Eq. 10-36).

(a) With subscript 1 standing for the rod nearest the axis and 4 for the particle farthest from it, we have

$$I = I_1 + I_2 + I_3 + I_4 = \left(\frac{1}{12}Md^2 + M\left(\frac{1}{2}d\right)^2\right) + md^2 + \left(\frac{1}{12}Md^2 + M\left(\frac{3}{2}d\right)^2\right) + m(2d)^2$$

$$= \frac{8}{3}Md^2 + 5md^2 = \frac{8}{3}(1.2\text{ kg})(0.056\text{ m})^2 + 5(0.85\text{ kg})(0.056\text{ m})^2$$

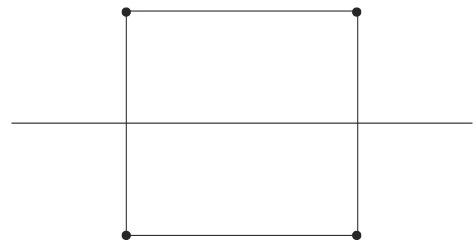
$$= 0.023\text{ kg}\cdot\text{m}^2.$$

(b) Using Eq. 10-34, we have

$$K = \frac{1}{2}I\omega^2 = \left(\frac{4}{3}M + \frac{5}{2}m\right)d^2\omega^2 = \left[\frac{4}{3}(1.2\text{ kg}) + \frac{5}{2}(0.85\text{ kg})\right](0.056\text{ m})^2(0.30\text{ rad/s})^2$$

$$= 1.1\times10^{-3}\text{ J}.$$

40. (a) We show the figure with its axis of rotation (the thin horizontal line).



We note that each mass is r = 1.0 m from the axis. Therefore, using Eq. 10-26, we obtain

$$I = \sum m_i r_i^2 = 4\ (0.50\ \text{kg})\ (1.0\ \text{m})^2 = 2.0\ \text{kg}\cdot\text{m}^2.$$

(b) In this case, the two masses nearest the axis are r = 1.0 m away from it, but the two furthest from the axis are $r = \sqrt{(1.0\ \text{m})^2 + (2.0\ \text{m})^2}$ from it. Here, then, Eq. 10-33 leads to

$$I = \sum m_i r_i^2 = 2(0.50\ \text{kg})\left(1.0\ \text{m}^2\right) + 2(0.50\ \text{kg})\left(5.0\ \text{m}^2\right) = 6.0\ \text{kg}\cdot\text{m}^2.$$

(c) Now, two masses are on the axis (with r = 0) and the other two are a distance $r = \sqrt{(1.0\ \text{m})^2 + (1.0\ \text{m})^2}$ away. Now we obtain $I = 2.0\ \text{kg}\cdot\text{m}^2$.

41. We use the parallel-axis theorem. According to Table 10-2(i), the rotational inertia of a uniform slab about an axis through the center and perpendicular to the large faces is given by

$$I_{\text{com}} = \frac{M}{12}\left(a^2 + b^2\right).$$

A parallel axis through the corner is a distance $h = \sqrt{(a/2)^2 + (b/2)^2}$ from the center. Therefore,

$$\begin{aligned} I &= I_{\text{com}} + Mh^2 = \frac{M}{12}\ \left(a^2 + b^2\right) + \frac{M}{4}\ \left(a^2 + b^2\right) = \frac{M}{3}\ \left(a^2 + b^2\right) \\ &= \frac{0.172\text{ kg}}{3}[(0.035\text{ m})^2 + (0.084\text{ m})^2] \\ &= 4.7\times 10^{-4}\ \text{kg}\cdot\text{m}^2. \end{aligned}$$

42. (a) Consider three of the disks (starting with the one at point O): ⊕OO . The first one (the one at point O – shown here with the plus sign inside) has rotational inertial (see item (c) in Table 10-2) $I = \frac{1}{2}mR^2$. The next one (using the parallel-axis theorem) has

$$I = \tfrac{1}{2}mR^2 + mh^2$$

where $h = 2R$. The third one has $I = \frac{1}{2}mR^2 + m(4R)^2$. If we had considered five of the disks OO⊕OO with the one at O in the middle, then the total rotational inertia is

$$I = 5(\tfrac{1}{2}mR^2) + 2(m(2R)^2 + m(4R)^2).$$

The pattern is now clear and we can write down the total I for the collection of fifteen disks:

$$I = 15(\tfrac{1}{2}mR^2) + 2(m(2R)^2 + m(4R)^2 + m(6R)^2 + \ldots + m(14R)^2) = \tfrac{2255}{2}mR^2.$$

The generalization to N disks (where N is assumed to be an odd number) is

$$I = \tfrac{1}{6}(2N^2 + 1)NmR^2.$$

In terms of the total mass ($m = M/15$) and the total length ($R = L/30$), we obtain

$$I = 0.083519ML^2 \approx (0.08352)(0.1000\text{ kg})(1.0000\text{ m})^2 = 8.352\times10^{-3}\text{ kg}\cdot\text{m}^2.$$

(b) Comparing to the formula (e) in Table 10-2 (which gives roughly $I = 0.08333\,ML^2$), we find our answer to part (a) is 0.22% lower.

43. (a) Using Table 10-2(c) and Eq. 10-34, the rotational kinetic energy is

$$K = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{1}{2} MR^2\right)\omega^2 = \frac{1}{4}(500\,\text{kg})(200\pi \text{ rad/s})^2(1.0\,\text{m})^2 = 4.9\times 10^7 \text{ J}.$$

(b) We solve $P = K/t$ (where P is the average power) for the operating time t.

$$t = \frac{K}{P} = \frac{4.9\times 10^7\,\text{J}}{8.0\times 10^3 \text{ W}} = 6.2\times 10^3 \text{ s}$$

which we rewrite as $t \approx 1.0 \times 10^2$ min.

44. (a) We apply Eq. 10-33:

$$I_x = \sum_{i=1}^{4} m_i y_i^2 = \left[50(2.0)^2 + (25)(4.0)^2 + 25(-3.0)^2 + 30(4.0)^2\right] \text{g} \cdot \text{cm}^2 = 1.3 \times 10^3 \ \text{g} \cdot \text{cm}^2.$$

(b) For rotation about the y axis we obtain

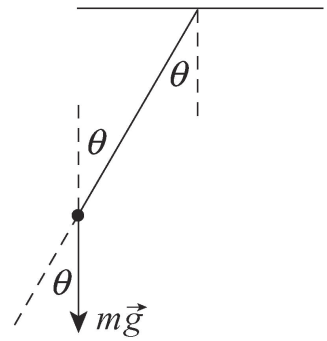
$$I_y = \sum_{i=1}^{4} m_i x_i^2 = 50(2.0)^2 + (25)\ (0)^2 + 25\ (3.0)^2 + 30(2.0)^2 = 5.5 \times 10^2 \ \text{g} \cdot \text{cm}^2.$$

(c) And about the z axis, we find (using the fact that the distance from the z axis is $\sqrt{x^2 + y^2}$)

$$I_z = \sum_{i=1}^{4} m_i \left(x_i^2 + y_i^2\right) = I_x + I_y = 1.3 \times 10^3 + 5.5 \times 10^2 = 1.9 \times 10^2 \ \text{g} \cdot \text{cm}^2.$$

(d) Clearly, the answer to part (c) is $A + B$.

45. Two forces act on the ball, the force of the rod and the force of gravity. No torque about the pivot point is associated with the force of the rod since that force is along the line from the pivot point to the ball. As can be seen from the diagram, the component of the force of gravity that is perpendicular to the rod is mg sin θ. If ℓ is the length of the rod, then the torque associated with this force has magnitude



$$\tau = mg\ell\sin\theta = (0.75)(9.8)(1.25)\sin 30° = 4.6\ \text{N}\cdot\text{m}\,.$$

For the position shown, the torque is counter-clockwise.

46. We compute the torques using $\tau = rF \sin \phi$.

(a) For $\phi = 30°$, $\tau_a = (0.152 \text{ m})(111 \text{ N}) \sin 30° = 8.4 \text{ N} \cdot \text{m}$.

(b) For $\phi = 90°$, $\tau_b = (0.152 \text{ m})(111 \text{ N}) \sin 90° = 17 \text{ N} \cdot \text{m}$.

(c) For $\phi = 180°$, $\tau_c = (0.152 \text{ m})(111 \text{N}) \sin 180° = 0$.

47. We take a torque that tends to cause a counterclockwise rotation from rest to be positive and a torque tending to cause a clockwise rotation to be negative. Thus, a positive torque of magnitude $r_1 F_1 \sin \theta_1$ is associated with $\vec{F}_1$ and a negative torque of magnitude $r_2F_2 \sin \theta_2$ is associated with $\vec{F}_2$. The net torque is consequently

$$\tau = r_1F_1 \sin\theta_1 - r_2F_2 \sin\theta_2 .$$

Substituting the given values, we obtain

$$\tau = (1.30 \text{ m})(4.20 \text{ N}) \sin 75° - (2.15 \text{ m})(4.90 \text{ N}) \sin 60° = -3.85 \text{ N}\cdot\text{m}.$$

48. The net torque is

$$
\begin{aligned}
\tau &= \tau_A + \tau_B + \tau_C = F_A r_A \sin\phi_A - F_B r_B \sin\phi_B + F_C r_C \sin\phi_C \\
&= (10)(8.0)\sin 135° - (16)(4.0)\sin 90° + (19)(3.0)\sin 160° \\
&= 12 \text{ N}\cdot\text{m}.
\end{aligned}
$$

49. (a) We use the kinematic equation $\omega = \omega_0 + \alpha t$, where ω_0 is the initial angular velocity, ω is the final angular velocity, α is the angular acceleration, and t is the time. This gives

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{6.20 \text{ rad / s}}{220 \times 10^{-3} \text{ s}} = 28.2 \text{ rad / s}^2 .$$

(b) If I is the rotational inertia of the diver, then the magnitude of the torque acting on her is

$$\tau = I\alpha = \left(12.0 \text{ kg} \cdot \text{m}^2\right)\left(28.2 \text{ rad / s}^2\right) = 3.38 \times 10^2 \text{ N} \cdot \text{m}.$$

50. The rotational inertia is found from Eq. 10-45.

$$I = \frac{\tau}{\alpha} = \frac{32.0}{25.0} = 1.28\ \mathrm{kg \cdot m^2}$$

51. Combining Eq. 10-45 ($\tau_{net} = I\,\alpha$) with Eq. 10-38 gives $RF_2 - RF_1 = I\alpha$, where $\alpha = \omega / t$ by Eq. 10-12 (with $\omega_0 = 0$). Using item (c) in Table 10-2 and solving for F_2 we find

$$F_2 = \frac{MR\omega}{2t} + F_1 = \frac{(0.02)(0.02)(250)}{2(1.25)} + 0.1 = 0.140 \text{ N}.$$

52. With counterclockwise positive, the angular acceleration α for both masses satisfies $\tau = mgL_1 - mgL_2 = I\alpha = \left(mL_1^2 + mL_2^2\right)\alpha$, by combining Eq. 10-45 with Eq. 10-39 and Eq. 10-33. Therefore, using SI units,

$$\alpha = \frac{g\left(L_1 - L_2\right)}{L_1^2 + L_2^2} = \frac{\left(9.8 \text{ m/s}^2\right)\left(0.20 \text{ m} - 0.80 \text{ m}\right)}{(0.20 \text{ m})^2 + (0.80 \text{ m})^2} = -\,8.65 \text{ rad/s}^2$$

where the negative sign indicates the system starts turning in the clockwise sense. The magnitude of the acceleration vector involves no radial component (yet) since it is evaluated at $t = 0$ when the instantaneous velocity is zero. Thus, for the two masses, we apply Eq. 10-22:

(a) $\left|\vec{a}_1\right| = |\alpha| L_1 = \left(8.65 \text{ rad/s}^2\right)\left(0.20 \text{ m}\right) = 1.7 \text{ m/s}$.

(b) $\left|\vec{a}_2\right| = |\alpha| L_2 = \left(8.65 \text{ rad/s}^2\right)\left(0.80 \text{ m}\right) = 6.9 \text{ m/s}^2$.

53. Combining Eq. 10-34 and Eq. 10-45, we have $RF = I\alpha$, where α is given by ω/t (according to Eq. 10-12, since $\omega_o = 0$ in this case). We also use the fact that

$$I = I_{\text{plate}} + I_{\text{disk}}$$

where $I_{\text{disk}} = \frac{1}{2}MR^2$ (item (c) in Table 10-2). Therefore,

$$I_{\text{plate}} = \frac{RFt}{\omega} - \frac{1}{2}MR^2 = 2.51 \times 10^{-4}\ \text{kg·m}^2.$$

54. According to the sign conventions used in the book, the magnitude of the net torque exerted on the cylinder of mass m and radius R is

$$\tau_{\text{net}} = F_1R - F_2R - F_3r = (6.0\text{ N})(0.12\text{ m}) - (4.0\text{ N})(0.12\text{ m}) - (2.0\text{ N})(0.050\text{ m}) = 71\text{ N}\cdot\text{m}.$$

(a) The resulting angular acceleration of the cylinder (with $I = \frac{1}{2}MR^2$ according to Table 10-2(c)) is

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{71\text{N}\cdot\text{m}}{\frac{1}{2}(2.0\text{kg})(0.12\text{ m})^2} = 9.7\text{rad/s}^2.$$

(b) The direction is counterclockwise (which is the positive sense of rotation).

55. (a) We use constant acceleration kinematics. If down is taken to be positive and a is the acceleration of the heavier block m_2, then its coordinate is given by $y = \frac{1}{2}at^2$, so

$$a = \frac{2y}{t^2} = \frac{2(0.750 \text{ m})}{(5.00 \text{ s})^2} = 6.00 \times 10^{-2} \text{ m/s}^2.$$

Block 1 has an acceleration of 6.00×10^{-2} m/s^2 upward.

(b) Newton's second law for block 2 is $m_2 g - T_2 = m_2 a$, where m_2 is its mass and T_2 is the tension force on the block. Thus,

$$T_2 = m_2(g-a) = (0.500 \text{ kg})\left(9.8 \text{ m/s}^2 - 6.00\times10^{-2} \text{ m/s}^2\right) = 4.87 \text{ N}.$$

(c) Newton's second law for block 1 is $m_1 g - T_1 = -m_1 a$, where T_1 is the tension force on the block. Thus,

$$T_1 = m_1(g+a) = (0.460 \text{ kg})\left(9.8 \text{ m/s}^2 + 6.00\times10^{-2} \text{ m/s}^2\right) = 4.54 \text{ N}.$$

(d) Since the cord does not slip on the pulley, the tangential acceleration of a point on the rim of the pulley must be the same as the acceleration of the blocks, so

$$\alpha = \frac{a}{R} = \frac{6.00\times10^{-2} \text{ m/s}^2}{5.00\times10^{-2} \text{ m}} = 1.20 \text{ rad/s}^2.$$

(e) The net torque acting on the pulley is $\tau = (T_2 - T_1)R$. Equating this to $I\alpha$ we solve for the rotational inertia:

$$I = \frac{(T_2 - T_1)R}{\alpha} = \frac{(4.87 \text{ N} - 4.54 \text{ N})\left(5.00\times10^{-2} \text{ m}\right)}{1.20 \text{ rad/s}^2} = 1.38\times10^{-2} \text{ kg}\cdot\text{m}^2.$$

56. (a) In this case, the force is $mg = (70 \text{ kg})(9.8 \text{ m/s}^2)$, and the "lever arm" (the perpendicular distance from point O to the line of action of the force) is 0.28 m. Thus, the torque (in absolute value) is $(70 \text{ kg})(9.8 \text{ m/s}^2)(0.28 \text{ m})$. Since the moment-of-inertia is $I = 65 \text{ kg}\cdot\text{m}^2$, then Eq. 10-45 gives $|\alpha| = 2.955 \approx 3.0 \text{ rad/s}^2$.

(b) Now we have another contribution ($1.4 \text{ m} \times 300 \text{ N}$) to the net torque, so

$$|\tau_{\text{net}}| = (70 \text{ kg})(9.8 \text{ m/s}^2)(0.28 \text{ m}) + (1.4 \text{ m})(300 \text{ N}) = (65 \text{ kg}\cdot\text{m}^2)\,|\alpha|$$

which leads to $|\alpha| = 9.4 \text{ rad/s}^2$.

57. Since the force acts tangentially at r = 0.10 m, the angular acceleration (presumed positive) is

$$\alpha = \frac{\tau}{I} = \frac{Fr}{I} = \frac{(0.5t + 0.3t^2)(0.10)}{1.0 \times 10^{-3}} = 50t + 30t^2$$

in SI units (rad/s^2).

(a) At t = 3 s, the above expression becomes α = 4.2 × 10^2 rad/s^2.

(b) We integrate the above expression, noting that ω_0 = 0, to obtain the angular speed at t = 3 s:

$$\omega = \int_0^3 \alpha\, dt = \left(25t^2 + 10t^3\right)\Big|_0^3 = 5.0 \times 10^2 \text{ rad/s}.$$

58. (a) We apply Eq. 10-34:

$$K = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{1}{3} mL^2\right)\omega^2 = \frac{1}{6} mL^2\omega^2 = \frac{1}{6}(0.42 \text{ kg})(0.75 \text{ m})^2 (4.0 \text{ rad/s})^2 = 0.63 \text{ J}.$$

(b) Simple conservation of mechanical energy leads to $K = mgh$. Consequently, the center of mass rises by

$$h = \frac{K}{mg} = \frac{mL^2\omega^2}{6mg} = \frac{L^2\omega^2}{6g} = \frac{(0.75 \text{ m})^2 (4.0 \text{ rad/s})^2}{6(9.8 \text{ m/s}^2)} = 0.153 \text{ m} \approx 0.15 \text{ m}.$$

59. The initial angular speed is ω = (280 rev/min)($2\pi/60$) = 29.3 rad/s.

(a) Since the rotational inertia is (Table 10-2(a)) $I = (32\text{ kg})(1.2\text{ m})^2 = 46.1\text{ kg}\cdot\text{m}^2$, the work done is

$$W = \Delta K = 0 - \frac{1}{2}I\omega^2 = -\frac{1}{2}(46.1\text{ kg}\cdot\text{m}^2)(29.3\text{ rad/s})^2$$

which yields $|W| = 1.98 \times 10^4$ J.

(b) The average power (in absolute value) is therefore

$$|P| = \frac{|W|}{\Delta t} = \frac{19.8\times10^3}{15} = 1.32\times10^3\text{ W}.$$

60. (a) The speed of v of the mass m after it has descended d = 50 cm is given by $v^2 = 2ad$ (Eq. 2-16). Thus, using g = 980 cm/s^2, we have

$$v = \sqrt{2ad} = \sqrt{\frac{2(2mg)d}{M+2m}} = \sqrt{\frac{4(50)(980)(50)}{400+2(50)}} = 1.4\times10^2 \text{ cm/s}.$$

(b) The answer is still 1.4×10^2 cm/s = 1.4 m/s, since it is independent of R.

61. With $\omega = (1800)(2\pi/60) = 188.5$ rad/s, we apply Eq. 10-55:

$$P = \tau\omega \quad \Rightarrow \quad \tau = \frac{74600\ \text{W}}{188.5\ \text{rad/s}} = 396\ \text{N}\cdot\text{m}\ .$$

62. (a) We use the parallel-axis theorem to find the rotational inertia:

$$I = I_{\text{com}} + Mh^2 = \frac{1}{2}MR^2 + Mh^2 = \frac{1}{2}(20\text{ kg})(0.10\text{ m})^2 + (20\text{ kg})(0.50\text{ m})^2 = 0.15\text{ kg}\cdot\text{m}^2.$$

(b) Conservation of energy requires that $Mgh = \frac{1}{2}I\omega^2$, where ω is the angular speed of the cylinder as it passes through the lowest position. Therefore,

$$\omega = \sqrt{\frac{2Mgh}{I}} = \sqrt{\frac{2(20\text{ kg})(9.8\text{ m/s}^2)(0.050\text{ m})}{0.15\text{ kg}\cdot\text{m}^2}} = 11\text{ rad/s}.$$

63. We use ℓ to denote the length of the stick. Since its center of mass is $\ell/2$ from either end, its initial potential energy is $\frac{1}{2}mg\ell$, where m is its mass. Its initial kinetic energy is zero. Its final potential energy is zero, and its final kinetic energy is $\frac{1}{2}I\omega^2$, where I is its rotational inertia about an axis passing through one end of the stick and ω is the angular velocity just before it hits the floor. Conservation of energy yields

$$\frac{1}{2}mg\ell = \frac{1}{2}I\omega^2 \Rightarrow \omega = \sqrt{\frac{mg\ell}{I}}.$$

The free end of the stick is a distance ℓ from the rotation axis, so its speed as it hits the floor is (from Eq. 10-18)

$$v = \omega\ell = \sqrt{\frac{mg\ell^3}{I}}.$$

Using Table 10-2 and the parallel-axis theorem, the rotational inertial is $I = \frac{1}{3}m\ell^2$, so

$$v = \sqrt{3g\ell} = \sqrt{3\left(9.8\ \text{m/s}^2\right)\left(1.00\ \text{m}\right)} = 5.42\ \text{m/s}.$$

64. (a) Eq. 10-33 gives

$$I_{\text{total}} = md^2 + m(2d)^2 + m(3d)^2 = 14\,md^2,$$

where d = 0.020 m and m = 0.010 kg. The work done is $W = \Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$, where ω_f = 20 rad/s and ω_i = 0. This gives W = 11.2 mJ.

(b) Now, ω_f = 40 rad/s and ω_i = 20 rad/s, and we get W = 33.6 mJ.

(c) In this case, ω_f = 60 rad/s and ω_i = 40 rad/s. This gives W = 56.0 mJ.

(d) Eq. 10-34 indicates that the slope should be $\frac{1}{2}I$. Therefore, it should be

$$7md^2 = 2.80 \times 10^{-5}\ \text{J} \cdot \text{s}^2/\ \text{rad}^2.$$

65. Using the parallel axis theorem and items (e) and (h) in Table 10-2, the rotational inertia is

$$I = \tfrac{1}{12}mL^2 + m(L/2)^2 + \tfrac{1}{2}mR^2 + m(R + L)^2 = 10.83mR^2 ,$$

where $L = 2R$ has been used. If we take the base of the rod to be at the coordinate origin ($x = 0$, $y = 0$) then the center of mass is at

$$y = \frac{mL/2 + m(L + R)}{m + m} = 2R .$$

Comparing the position shown in the textbook figure to its upside down (inverted) position shows that the change in center of mass position (in absolute value) is $|\Delta y| = 4R$. The corresponding loss in gravitational potential energy is converted into kinetic energy. Thus,

$$K = (2m)g(4R) \quad \Rightarrow \quad \omega = 9.82 \text{ rad/s} .$$

where Eq. 10-34 has been used.

66. From Table 10-2, the rotational inertia of the spherical shell is $2MR^2/3$, so the kinetic energy (after the object has descended distance h) is

$$K = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\omega^2_{\text{sphere}} + \frac{1}{2}I\omega^2_{\text{pulley}} + \frac{1}{2}mv^2.$$

Since it started from rest, then this energy must be equal (in the absence of friction) to the potential energy mgh with which the system started. We substitute v/r for the pulley's angular speed and v/R for that of the sphere and solve for v.

$$v = \sqrt{\frac{mgh}{\frac{1}{2}m + \frac{1}{2}\frac{I}{r^2} + \frac{M}{3}}} = \sqrt{\frac{2gh}{1 + (I/mr^2) + (2M/3m)}}$$

$$= \sqrt{\frac{2(9.8)(0.82)}{1 + 3.0\times10^{-3}/((0.60)(0.050)^2) + 2(4.5)/3(0.60)}} = 1.4 \text{ m/s}$$

67. (a) We use conservation of mechanical energy to find an expression for ω^2 as a function of the angle θ that the chimney makes with the vertical. The potential energy of the chimney is given by $U = Mgh$, where M is its mass and h is the altitude of its center of mass above the ground. When the chimney makes the angle θ with the vertical, $h = (H/2)\cos\theta$. Initially the potential energy is $U_i = Mg(H/2)$ and the kinetic energy is zero. The kinetic energy is $\frac{1}{2}I\omega^2$ when the chimney makes the angle θ with the vertical, where I is its rotational inertia about its bottom edge. Conservation of energy then leads to

$$MgH/2 = Mg(H/2)\cos\theta + \frac{1}{2}I\omega^2 \Rightarrow \omega^2 = (MgH/I)(1-\cos\theta).$$

The rotational inertia of the chimney about its base is $I = MH^2/3$ (found using Table 10-2(e) with the parallel axis theorem). Thus

$$\omega = \sqrt{\frac{3g}{H}(1-\cos\theta)} = \sqrt{\frac{3(9.80\text{ m/s}^2)}{55.0\text{ m}}(1-\cos 35.0°)} = 0.311\text{ rad/s}.$$

(b) The radial component of the acceleration of the chimney top is given by $a_r = H\omega^2$, so

$$a_r = 3g\,(1-\cos\theta) = 3\,(9.80\text{ m/s}^2)(1-\cos 35.0°) = 5.32\text{ m/s}^2.$$

(c) The tangential component of the acceleration of the chimney top is given by $a_t = H\alpha$, where α is the angular acceleration. We are unable to use Table 10-1 since the acceleration is not uniform. Hence, we differentiate

$$\omega^2 = (3g/H)(1-\cos\theta)$$

with respect to time, replacing $d\omega/dt$ with α, and $d\theta/dt$ with ω, and obtain

$$\frac{d\omega^2}{dt} = 2\omega\alpha = (3g/H)\omega\sin\theta \Rightarrow \alpha = (3g/2H)\sin\theta.$$

Consequently,

$$a_t = H\alpha = \frac{3g}{2}\sin\theta = \frac{3(9.80\text{ m/s}^2)}{2}\sin 35.0° = 8.43\text{ m/s}^2.$$

(d) The angle θ at which $a_t = g$ is the solution to $\frac{3g}{2}\sin\theta = g$. Thus, $\sin\theta = 2/3$ and we obtain $\theta = 41.8°$.

68. The rotational inertia of the passengers is (to a good approximation) given by Eq. 10-53: $I = \sum mR^2 = NmR^2$ where N is the number of people and m is the (estimated) mass per person. We apply Eq. 10-52:

$$W = \frac{1}{2} I\omega^2 = \frac{1}{2} NmR^2\omega^2$$

where R = 38 m and $N = 36 \times 60 = 2160$ persons. The rotation rate is constant so that $\omega = \theta/t$ which leads to $\omega = 2\pi/120 = 0.052$ rad/s. The mass (in kg) of the average person is probably in the range $50 \le m \le 100$, so the work should be in the range

$$\frac{1}{2}(2160)(50)(38)^2(0.052)^2 \le W \le \frac{1}{2}(2160)(100)(38)^2(0.052)^2$$
$$2\times10^5 \text{ J} \le W \le 4\times10^5 \text{ J}.$$

69. We choose positive coordinate directions (different choices for each item) so that each is accelerating positively, which will allow us to set $a_2 = a_1 = R\alpha$ (for simplicity, we denote this as a). Thus, we choose rightward positive for $m_2 = M$ (the block on the table), downward positive for $m_1 = M$ (the block at the end of the string) and (somewhat unconventionally) clockwise for positive sense of disk rotation. This means that we interpret θ given in the problem as a positive-valued quantity. Applying Newton's second law to m_1, m_2 and (in the form of Eq. 10-45) to M, respectively, we arrive at the following three equations (where we allow for the possibility of friction f_2 acting on m_2).

$$\begin{aligned} m_1 g - T_1 &= m_1 a_1 \\ T_2 - f_2 &= m_2 a_2 \\ T_1 R - T_2 R &= I\alpha \end{aligned}$$

(a) From Eq. 10-13 (with $\omega_0 = 0$) we find

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \;\Rightarrow\; \alpha = \frac{2\theta}{t^2} = \frac{2(1.30 \text{ rad})}{(0.0910 \text{ s})^2} = 314 \text{ rad/s}^2.$$

(b) From the fact that $a = R\alpha$ (noted above), we obtain

$$a = \frac{2R\theta}{t^2} = \frac{2(0.024 \text{ m})(1.30 \text{ rad})}{(0.0910 \text{ s})^2} = 7.54 \text{ m/s}^2.$$

(c) From the first of the above equations, we find

$$\begin{aligned} T_1 &= m_1(g - a_1) = M\left(g - \frac{2R\theta}{t^2}\right) = (6.20 \text{ kg})\left(9.80 \text{ m/s}^2 - \frac{2(0.024 \text{ m})(1.30 \text{ rad})}{(0.0910 \text{ s})^2}\right) \\ &= 14.0 \text{ N}. \end{aligned}$$

(d) From the last of the above equations, we obtain the second tension:

$$\begin{aligned} T_2 &= T_1 - \frac{I\alpha}{R} = M\left(g - \frac{2R\theta}{t^2}\right) - \frac{2I\theta}{Rt^2} = 14.0 \text{ N} - \frac{(7.40\times10^{-4} \text{ kg}\cdot\text{m}^2)(314 \text{ rad/s}^2)}{0.024 \text{ m}} \\ &= 4.36 \text{ N}. \end{aligned}$$

70. In the calculation below, M_1 and M_2 are the ring masses, R_{1i} and R_{2i} are their inner radii, and R_{1o} and R_{2o} are their outer radii. Referring to item (b) in Table 10-2, we compute

$$I = \tfrac{1}{2} M_1 (R_{1i}{}^2 + R_{1o}{}^2) + \tfrac{1}{2} M_2 (R_{2i}{}^2 + R_{2o}{}^2) \;\; = 0.00346 \text{ kg}\cdot\text{m}^2 .$$

Thus, with Eq. 10-38 ($\tau = rF$ where $r = R_{2o}$) and $\tau = I\alpha$ (Eq. 10-45), we find

$$\alpha = \frac{(0.140)(12.0)}{0.00346} = 485 \text{ rad/s}^2 .$$

Then Eq. 10-12 gives $\omega = \alpha t$ = 146 rad/s.

71. The volume of each disk is $\pi r^2 h$ where we are using h to denote the thickness (which equals 0.00500 m). If we use R (which equals 0.0400 m) for the radius of the larger disk and r (which equals 0.0200 m) for the radius of the smaller one, then the mass of each is $m = \rho\pi r^2 h$ and $M = \rho\pi R^2 h$ where $\rho = 1400$ kg/m^3 is the given density. We now use the parallel axis theorem as well as item (c) in Table 10-2 to obtain the rotation inertia of the two-disk assembly:

$$I = \tfrac{1}{2}MR^2 + \tfrac{1}{2}mr^2 + m(r+R)^2 = \rho\pi h[\tfrac{1}{2}R^4 + \tfrac{1}{2}r^4 + r^2(r+R)^2] = 6.16 \times 10^{-5}\ \text{kg}\cdot\text{m}^2.$$

72. (a) The longitudinal separation between Helsinki and the explosion site is $\Delta\theta = 102° - 25° = 77°$. The spin of the earth is constant at

$$\omega = \frac{1 \text{ rev}}{1 \text{ day}} = \frac{360°}{24 \text{ h}}$$

so that an angular displacement of $\Delta\theta$ corresponds to a time interval of

$$\Delta t = (77°)\left(\frac{24 \text{ h}}{360°}\right) = 5.1 \text{ h}.$$

(b) Now $\Delta\theta = 102° - (-20°) = 122°$ so the required time shift would be

$$\Delta t = (122°)\left(\frac{24 \text{ h}}{360°}\right) = 8.1 \text{ h}.$$

73. We choose positive coordinate directions (different choices for each item) so that each is accelerating positively, which will allow us to set $a_1 = a_2 = R\alpha$ (for simplicity, we denote this as a). Thus, we choose upward positive for m_1, downward positive for m_2 and (somewhat unconventionally) clockwise for positive sense of disk rotation. Applying Newton's second law to m_1m_2 and (in the form of Eq. 10-45) to M, respectively, we arrive at the following three equations.

$$T_1 - m_1 g = m_1 a_1$$
$$m_2 g - T_2 = m_2 a_2$$
$$T_2 R - T_1 R = I\alpha$$

(a) The rotational inertia of the disk is $I = \frac{1}{2} MR^2$ (Table 10-2(c)), so we divide the third equation (above) by R, add them all, and use the earlier equality among accelerations — to obtain:

$$m_2 g - m_1 g = \left(m_1 + m_2 + \frac{1}{2} M \right) a$$

which yields $a = \frac{4}{25} g = 1.57\ \text{m/s}^2$.

(b) Plugging back in to the first equation, we find

$$T_1 = \frac{29}{25} m_1 g = 4.55\ \text{N}$$

where it is important in this step to have the mass in SI units: $m_1 = 0.40$ kg.

(c) Similarly, with $m_2 = 0.60$ kg, we find

$$T_2 = \frac{5}{6} m_2 g = 4.94\ \text{N}.$$

74. (a) Constant angular acceleration kinematics can be used to compute the angular acceleration α. If ω_0 is the initial angular velocity and t is the time to come to rest, then

$$0 = \omega_0 + \alpha t \Rightarrow \alpha = -\frac{\omega_0}{t}$$

which yields –39/32 = –1.2 rev/s or (multiplying by 2π) –7.66 rad/s^2 for the value of α.

(b) We use $\tau = I\alpha$, where τ is the torque and I is the rotational inertia. The contribution of the rod to I is $M\ell^2/12$ (Table 10-2(e)), where M is its mass and ℓ is its length. The contribution of each ball is $m(\ell/2)^2$, where m is the mass of a ball. The total rotational inertia is

$$I = \frac{M\ell^2}{12} + 2\frac{m\ell^2}{4} = \frac{(6.40\text{ kg})(1.20\text{ m})^2}{12} + \frac{(1.06\text{ kg})(1.20\text{ m})^2}{2}$$

which yields $I = 1.53\text{ kg}\cdot\text{m}^2$. The torque, therefore, is

$$\tau = (1.53\text{ kg}\cdot\text{m}^2)(-7.66\text{ rad/s}^2) = -11.7\text{ N}\cdot\text{m}.$$

(c) Since the system comes to rest the mechanical energy that is converted to thermal energy is simply the initial kinetic energy

$$K_i = \frac{1}{2}I\omega_0^2 = \frac{1}{2}(1.53\text{ kg}\cdot\text{m}^2)((2\pi)(39)\text{ rad/s})^2 = 4.59\times10^4\text{ J}.$$

(d) We apply Eq. 10-13:

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = ((2\pi)(39)\text{ rad/s})(32.0\text{ s}) + \frac{1}{2}(-7.66\text{ rad/s}^2)(32.0\text{ s})^2$$

which yields 3920 rad or (dividing by 2π) 624 rev for the value of angular displacement θ.

(e) Only the mechanical energy that is converted to thermal energy can still be computed without additional information. It is 4.59×10^4 J no matter how τ varies with time, as long as the system comes to rest.

75. The *Hint* given in the problem would make the computation in part (a) very straightforward (without doing the integration as we show here), but we present this further level of detail in case that hint is not obvious or — simply — in case one wishes to see how the calculus supports our intuition.

(a) The (centripetal) force exerted on an infinitesimal portion of the blade with mass dm located a distance r from the rotational axis is (Newton's second law) $dF = (dm)\omega^2 r$, where dm can be written as $(M/L)dr$ and the angular speed is

$$\omega = (320)(2\pi/60) = 33.5 \text{ rad/s}.$$

Thus for the entire blade of mass M and length L the total force is given by

$$F = \int dF = \int \omega^2 r\, dm = \frac{M}{L}\int_0^L \omega^2 r\, dr = \frac{M\omega^2 L}{2} = \frac{(110\,\text{kg})\,(33.5 \text{ rad/s})^2\,(7.80\,\text{m})}{2}$$
$$= 4.81\times10^5\,\text{N}.$$

(b) About its center of mass, the blade has $I = ML^2/12$ according to Table 10-2(e), and using the parallel-axis theorem to "move" the axis of rotation to its end-point, we find the rotational inertia becomes $I = ML^2/3$. Using Eq. 10-45, the torque (assumed constant) is

$$\tau = I\alpha = \left(\frac{1}{3}ML^2\right)\left(\frac{\Delta\omega}{\Delta t}\right) = \frac{1}{3}(110\,\text{kg})(7.8\text{ m})^2\left(\frac{33.5\,\text{rad/s}}{6.7\,\text{s}}\right) = 1.12\times10^4\,\text{N}\cdot\text{m}.$$

(c) Using Eq. 10-52, the work done is

$$W = \Delta K = \frac{1}{2}I\omega^2 - 0 = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 = \frac{1}{6}(110\,\text{kg})(7.80\,\text{m})^2(33.5\,\text{rad/s})^2 = 1.25\times10^6\text{ J}.$$

76. The wheel starts turning from rest ($\omega_0 = 0$) at $t = 0$, and accelerates uniformly at $\alpha = 2.00$ rad/s^2 .Between t_1 and t_2 the wheel turns through $\Delta\theta = 90.0$ rad, where $t_2 - t_1 = \Delta t = 3.00$ s. We solve (b) first.

(b) We use Eq. 10-13 (with a slight change in notation) to describe the motion for $t_1 \le t \le t_2$:

$$\Delta\theta = \omega_1 \Delta t + \frac{1}{2}\alpha(\Delta t)^2 \Rightarrow \omega_1 = \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2}$$

which we plug into Eq. 10-12, set up to describe the motion during $0 \le t \le t_1$:

$$\omega_1 = \omega_0 + \alpha t_1 \;\Rightarrow\; \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2} = \alpha t_1 \;\Rightarrow\; \frac{90.0}{3.00} - \frac{(2.00)(3.00)}{2} = (2.00)\,t_1$$

yielding $t_1 = 13.5$ s.

(a) Plugging into our expression for ω_1 (in previous part) we obtain

$$\omega_1 = \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2} = \frac{90.0}{3.00} - \frac{(2.00)(3.00)}{2} = 27.0 \text{ rad / s.}$$

77. To get the time to reach the maximum height, we use Eq. 4-23, setting the left-hand side to zero. Thus, we find

$$t = \frac{(60\text{ m/s})\sin(20^{\circ})}{9.8\text{ m/s}^2} = 2.094\text{ s}.$$

Then (assuming $\alpha = 0$) Eq. 10-13 gives

$$\theta - \theta_0 = \omega_0 t = (90\text{ rad/s})(2.094\text{ s}) = 188\text{ rad},$$

which is equivalent to roughly 30 rev.

78. We choose ± directions such that the initial angular velocity is $\omega_0 = -317$ rad/s and the values for α, τ and F are positive.

(a) Combining Eq. 10-12 with Eq. 10-45 and Table 10-2(f) (and using the fact that $\omega = 0$) we arrive at the expression

$$\tau = \left(\frac{2}{5}MR^2\right)\left(-\frac{\omega_0}{t}\right) = -\frac{2}{5}\frac{MR^2\omega_0}{t}.$$

With $t = 15.5$ s, $R = 0.226$ m and $M = 1.65$ kg, we obtain $\tau = 0.689$ N · m.

(b) From Eq. 10-40, we find $F = \tau / R = 3.05$ N.

(c) Using again the expression found in part (a), but this time with $R = 0.854$ m, we get $\tau = 9.84\ \text{N}\cdot\text{m}$.

(d) Now, $F = \tau / R = 11.5$ N.

79. The center of mass is initially at height $h = \frac{L}{2}\sin 40°$ when the system is released (where $L = 2.0$ m). The corresponding potential energy Mgh (where $M = 1.5$ kg) becomes rotational kinetic energy $\frac{1}{2}I\omega^2$ as it passes the horizontal position (where I is the rotational inertia about the pin). Using Table 10-2 (e) and the parallel axis theorem, we find

$$I = \tfrac{1}{12}ML^2 + M(L/2)^2 = \tfrac{1}{3}ML^2.$$

Therefore,

$$Mg\frac{L}{2}\sin 40° = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 \quad \Rightarrow \quad \omega = \sqrt{\frac{3g\sin 40°}{L}} = 3.1 \text{ rad/s}.$$

80. (a) Eq. 10-12 leads to $\alpha = -\omega_o / t = -(25.0 \text{ rad/s})/(20.0 \text{ s}) = -1.25 \text{ rad/s}^2$.

(b) Eq. 10-15 leads to $\theta = \frac{1}{2}\omega_o t = \frac{1}{2}(25.0 \text{ rad/s})(20.0 \text{ s}) = 250 \text{ rad}$.

(c) Dividing the previous result by 2π we obtain θ= 39.8 rev.

81. (a) With $r = 0.780$ m, the rotational inertia is

$$I = Mr^2 = (1.30 \text{ kg})(0.780 \text{ m})^2 = 0.791 \text{ kg}\cdot\text{m}^2.$$

(b) The torque that must be applied to counteract the effect of the drag is

$$\tau = rf = (0.780 \text{ m})(2.30\times10^{-2} \text{ N}) = 1.79\times10^{-2} \text{ N}\cdot\text{m}.$$

82. The motion consists of two stages. The first, the interval $0 \le t \le 20$ s, consists of constant angular acceleration given by

$$\alpha = \frac{5.0 \text{ rad/s}}{2.0 \text{ s}} = 2.5 \text{ rad/s}^2 .$$

The second stage, $20 < t \le 40$ s, consists of constant angular velocity $\omega = \Delta\theta / \Delta t$. Analyzing the first stage, we find

$$\theta_1 = \frac{1}{2}\alpha t^2 \bigg|_{t=20} = 500 \text{ rad}, \quad \omega = \alpha t\big|_{t=20} = 50 \text{ rad/s}.$$

Analyzing the second stage, we obtain

$$\theta_2 = \theta_1 + \omega\Delta t = 500 \text{ rad} + (50 \text{ rad/s})(20 \text{ s}) = 1.5\times 10^3 \text{ rad}.$$

83. The magnitude of torque is the product of the force magnitude and the distance from the pivot to the line of action of the force. In our case, it is the gravitational force that passes through the walker's center of mass. Thus,

$$\tau = I\alpha = rF = rmg.$$

(a) Without the pole, with $I = 15\ \mathrm{kg \cdot m^2}$, the angular acceleration is

$$\alpha = \frac{rF}{I} = \frac{rmg}{I} = \frac{(0.050\ \mathrm{m})(70\ \mathrm{kg})(9.8\ \mathrm{m/s^2})}{15\ \mathrm{kg \cdot m^2}} = 2.3\ \mathrm{rad/s^2}.$$

(b) When the walker carries a pole, the torque due to the gravitational force through the pole's center of mass opposes the torque due to the gravitational force that passes through the walker's center of mass. Therefore,

$$\tau_{\mathrm{net}} = \sum_i r_i F_i = (0.050\ \mathrm{m})(70\ \mathrm{kg})(9.8\ \mathrm{m/s^2}) - (0.10\ \mathrm{m})(14\ \mathrm{kg})(9.8\ \mathrm{m/s^2}) = 20.58\ \mathrm{N \cdot m},$$

and the resulting angular acceleration is

$$\alpha = \frac{\tau_{\mathrm{net}}}{I} = \frac{20.58\ \mathrm{N \cdot m}}{15\ \mathrm{kg \cdot m^2}} \approx 1.4\ \mathrm{rad/s^2}.$$

84. The angular displacements of disks A and B can be written as:

$$\theta_A = \omega_A t, \quad \theta_B = \frac{1}{2}\alpha_B t^2.$$

(a) The time when $\theta_A = \theta_B$ is given by

$$\omega_A t = \frac{1}{2}\alpha_B t^2 \quad \Rightarrow \quad t = \frac{2\omega_A}{\alpha_B} = \frac{2(9.5 \text{ rad/s})}{(2.2 \text{ rad/s}^2)} = 8.6 \text{ s}.$$

(b) The difference in the angular displacement is

$$\Delta\theta = \theta_A - \theta_B = \omega_A t - \frac{1}{2}\alpha_B t^2 = 9.5t - 1.1t^2.$$

For their reference lines to align momentarily, we only require $\Delta\theta = 2\pi N$, where N is an integer. The quadratic equation can be readily solve to yield

$$t_N = \frac{9.5 \pm \sqrt{(9.5)^2 - 4(1.1)(2\pi N)}}{2(1.1)} = \frac{9.5 \pm \sqrt{90.25 - 27.6N}}{2.2}.$$

The solution $t_0 = 8.63$ s (taking the positive root) coincides with the result obtained in (a), while $t_0 = 0$ (taking the negative root) is the moment when both disks begin to rotate. In fact, two solutions exist for N = 0, 1, 2, and 3.

85. Eq. 10-40 leads to $\tau = mgr = (70 \text{ kg})(9.8 \text{ m/s}^2)(0.20 \text{ m}) = 1.4 \times 10^2 \text{ N}\cdot\text{m}$.

86. (a) Using Eq. 10-15, we have 60.0 rad $= \frac{1}{2}(\omega_1 + \omega_2)(6.00\text{ s})$. With ω_2 = 15.0 rad/s, then ω_1 = 5.00 rad/s.

(b) Eq. 10-12 gives α = (15.0 rad/s – 5.0 rad/s)/(6.00 s) = 1.67 rad/s^2.

(c) Interpreting ω now as ω_1 and θ as θ_1 = 10.0 rad (and $\omega_0 = 0$) Eq. 10-14 leads to

$$\theta_o = -\frac{\omega_1^2}{2\alpha} + \theta_1 = 2.50\text{ rad}.$$

87. With rightward positive for the block and clockwise negative for the wheel (as is conventional), then we note that the tangential acceleration of the wheel is of opposite sign from the block's acceleration (which we simply denote as a); that is, $a_t = -a$. Applying Newton's second law to the block leads to $P - T = ma$, where $m = 2.0$ kg. Applying Newton's second law (for rotation) to the wheel leads to $-TR = I\alpha$, where $I = 0.050\ \text{kg}\cdot\text{m}^2$.

Noting that $R\alpha = a_t = -a$, we multiply this equation by R and obtain

$$-TR^2 = -Ia \Rightarrow T = a\frac{I}{R^2}.$$

Adding this to the above equation (for the block) leads to $P = (m + I/R^2)a$.
Thus, $a = 0.92\ \text{m/s}^2$ and therefore $\alpha = -4.6\ \text{rad/s}^2$ (or $|\alpha| = 4.6\ \text{rad/s}^2$), where the negative sign in α should not be mistaken for a deceleration (it simply indicates the clockwise sense to the motion).

88. (a) The time for one revolution is the circumference of the orbit divided by the speed v of the Sun: $T = 2\pi R/v$, where R is the radius of the orbit. We convert the radius:

$$R = \left(2.3\times10^{4}\ \text{ly}\right)\left(9.46\times10^{12}\ \text{km}/\text{ly}\right) = 2.18\times10^{17}\ \text{km}$$

where the ly $\leftrightarrow$ km conversion can be found in Appendix D or figured "from basics" (knowing the speed of light). Therefore, we obtain

$$T = \frac{2\pi\left(2.18\times10^{17}\ \text{km}\right)}{250\ \text{km}/\text{s}} = 5.5\times10^{15}\ \text{s}.$$

(b) The number of revolutions N is the total time t divided by the time T for one revolution; that is, $N = t/T$. We convert the total time from years to seconds and obtain

$$N = \frac{\left(4.5\times10^{9}\ \text{y}\right)\left(3.16\times10^{7}\ \text{s}/\text{y}\right)}{5.5\times10^{15}\ \text{s}} = 26.$$

89. We assume the sense of initial rotation is positive. Then, with $\omega_0 > 0$ and $\omega = 0$ (since it stops at time t), our angular acceleration is negative-valued.

(a) The angular acceleration is constant, so we can apply Eq. 10-12 ($\omega = \omega_0 + \alpha t$). To obtain the requested units, we have $t = 30/60 = 0.50$ min. Thus,

$$\alpha = -\frac{33.33\ \text{rev/min}}{0.50\,\text{min}} = -66.7\ \text{rev/min}^2 \approx -67\ \text{rev/min}^2.$$

(b) We use Eq. 10-13:

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (33.33\ \text{rev/min})(0.50\ \text{min}) + \frac{1}{2}(-66.7\text{rev/min}^2)(0.50\ \text{min})^2 = 8.3\ \text{rev}.$$

90. We use conservation of mechanical energy. The center of mass is at the midpoint of the cross bar of the **H** and it drops by $L/2$, where L is the length of any one of the rods. The gravitational potential energy decreases by $MgL/2$, where M is the mass of the body. The initial kinetic energy is zero and the final kinetic energy may be written $\frac{1}{2}I\omega^2$, where I is the rotational inertia of the body and ω is its angular velocity when it is vertical. Thus,

$$0 = -MgL/2 + \frac{1}{2}I\omega^2 \Rightarrow \omega = \sqrt{MgL/I}.$$

Since the rods are thin the one along the axis of rotation does not contribute to the rotational inertia. All points on the other leg are the same distance from the axis of rotation, so that leg contributes $(M/3)L^2$, where $M/3$ is its mass. The cross bar is a rod that rotates around one end, so its contribution is $(M/3)L^2/3 = ML^2/9$. The total rotational inertia is

$$I = (ML^2/3) + (ML^2/9) = 4ML^2/9.$$

Consequently, the angular velocity is

$$\omega = \sqrt{\frac{MgL}{I}} = \sqrt{\frac{MgL}{4ML^2/9}} = \sqrt{\frac{9g}{4L}} = \sqrt{\frac{9(9.800 \text{ m/s}^2)}{4(0.600 \text{ m})}} = 6.06 \text{ rad/s}.$$

91. (a) According to Table 10-2, the rotational inertia formulas for the cylinder (radius *R*) and the hoop (radius *r*) are given by

$$I_C = \frac{1}{2}MR^2 \;\text{ and }\; I_H = Mr^2.$$

Since the two bodies have the same mass, then they will have the same rotational inertia if

$$R^2/2 = R_H^2 \;\rightarrow\; R_H = R/\sqrt{2}\,.$$

(b) We require the rotational inertia to be written as $I = Mk^2$, where *M* is the mass of the given body and *k* is the radius of the "equivalent hoop." It follows directly that $k = \sqrt{I/M}$.

92. (a) We use $\tau = I\alpha$, where τ is the net torque acting on the shell, I is the rotational inertia of the shell, and α is its angular acceleration. Therefore,

$$I = \frac{\tau}{\alpha} = \frac{960 \text{ N}\cdot\text{m}}{6.20 \text{ rad/s}^2} = 155 \text{ kg}\cdot\text{m}^2.$$

(b) The rotational inertia of the shell is given by $I = (2/3)\, MR^2$ (see Table 10-2 of the text). This implies

$$M = \frac{3I}{2R^2} = \frac{3\left(155 \text{ kg}\cdot\text{m}^2\right)}{2\left(1.90 \text{ m}\right)^2} = 64.4 \text{ kg}.$$

93. We choose positive coordinate directions so that each is accelerating positively, which will allow us to set $a_{box} = R\alpha$ (for simplicity, we denote this as a). Thus, we choose downhill positive for the m = 2.0 kg box and (as is conventional) counterclockwise for positive sense of wheel rotation. Applying Newton's second law to the box and (in the form of Eq. 10-45) to the wheel, respectively, we arrive at the following two equations (using θ as the incline angle 20°, not as the angular displacement of the wheel).

$$mg\sin\theta - T = ma$$
$$TR = I\alpha$$

Since the problem gives a = 2.0 m/s^2, the first equation gives the tension $T = m\,(g\sin\theta - a)$ = 2.7 N. Plugging this and R = 0.20 m into the second equation (along with the fact that $\alpha = a/R$) we find the rotational inertia

$$I = TR^2/a = 0.054\ \text{kg}\cdot\text{m}^2.$$

94. Analyzing the forces tending to drag the $M = 5124$ kg stone down the oak beam, we find

$$F = Mg\left(\sin\theta + \mu_s \cos\theta\right)$$

where $\mu_s = 0.22$ (static friction is assumed to be at its maximum value) and the incline angle θ for the oak beam is $\sin^{-1}(3.9/10) = 23°$ (but the incline angle for the spruce log is the complement of that). We note that the component of the weight of the workers (N of them) which is perpendicular to the spruce log is $Nmg\cos(90° - \theta) = Nmg\sin\theta$, where $m = 85$ kg. The corresponding torque is therefore $Nmg\ell\sin\theta$ where $\ell = 4.5 - 0.7 = 3.8$ m. This must (at least) equal the magnitude of torque due to F, so with $r = 0.7$ m, we have

$$Mgr\left(\sin\theta + \mu_s \cos\theta\right) = Ngm\ell\sin\theta.$$

This expression yields $N \approx 17$ for the number of workers.

95. The centripetal acceleration at a point P which is r away from the axis of rotation is given by Eq. 10-23: $a = v^2/r = \omega^2 r$, where $v = \omega r$, with $\omega = 2000$ rev/min $\approx$ 209.4 rad/s.

(a) If points A and P are at a radial distance r_A=1.50 m and r = 0.150 m from the axis, the difference in their acceleration is

$$\Delta a = a_A - a = \omega^2 (r_A - r) = (209.4 \text{ rad/s})^2 (1.50 \text{ m} - 0.150 \text{ m}) \approx 5.92\times10^4 \text{ m/s}^2$$

(b) The slope is given by $a/r = \omega^2 = 4.39\times10^4 \, /\text{s}^2$.

96. Let T be the tension on the rope. From Newton's second law, we have

$$T - mg = ma \quad \Rightarrow \quad T = m(g + a).$$

Since the box has an upward acceleration a = 0.80 m/s^2, the tension is given by

$$T = (30 \text{ kg})(9.8 \text{ m/s}^2 + 0.8 \text{ m/s}^2) = 318 \text{ N}.$$

The rotation of the device is described by $F_{\text{app}}R - Tr = I\alpha = Ia/r$. The moment of inertia can then be obtained as

$$I = \frac{r(F_{\text{app}}R - Tr)}{a} = \frac{(0.20 \text{ m})[(140 \text{ N})(0.50 \text{ m}) - (318 \text{ N})(0.20 \text{ m})]}{0.80 \text{ m/s}^2} = 1.6 \text{ kg} \cdot \text{m}^2$$

97. The distances from P to the particles are as follows:

$$
\begin{aligned}
r_1 &= a \text{ for } m_1 = 2M(\text{lower left}) \\
r_2 &= \sqrt{b^2 - a^2} \text{ for } m_2 = M(\text{top}) \\
r_3 &= a \text{ for } m_1 = 2M(\text{lower right})
\end{aligned}
$$

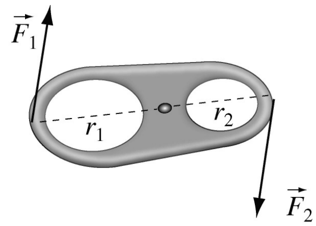
The rotational inertia of the system about P is

$$I = \sum_{i=1}^{3} m_i r_i^2 = \left(3a^2 + b^2\right)M$$

which yields $I = 0.208\ \text{kg}\cdot\text{m}^2$ for M = 0.40 kg, a = 0.30 m and b = 0.50 m. Applying Eq. 10-52, we find

$$W = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(0.208\ \text{kg}\cdot\text{m}^2\right)\left(5.0\ \text{rad/s}\right)^2 = 2.6\ \text{J}.$$

98. In the figure below, we show a pull tab of a beverage can. Since the tab is pivoted, when pulling on one end upward with a force $\vec{F}_1$, a force $\vec{F}_2$ will be exerted on the other end. The torque produced by $\vec{F}_1$ must be balanced by the torque produced by $\vec{F}_2$ so that the tab does not rotate.



The two forces are related by

$$r_1 F_1 = r_2 F_2$$

where $r_1 \approx 1.8 \text{ cm}$ and $r_2 \approx 0.73 \text{ cm}$. Thus, if $F_1 = 10$ N,

$$F_2 = \left(\frac{r_1}{r_2}\right) F_1 \approx \left(\frac{1.8 \text{ cm}}{0.73 \text{ cm}}\right)(10 \text{ N}) \approx 25 \text{ N}.$$

99. (a) We apply Eq. 10-18, using the subscript J for the Jeep.

$$\omega = \frac{v_J}{r_J} = \frac{114 \text{ km/h}}{0.100 \text{ km}}$$

which yields 1140 rad/h or (dividing by 3600) 0.32 rad/s for the value of the angular speed ω.

(b) Since the cheetah has the same angular speed, we again apply Eq. 10-18, using the subscript c for the cheetah.

$$v_c = r_c\omega = (92\,\text{m})\,(1140 \text{ rad/h}) = 1.048 \times 10^5 \text{ m/h} \approx 1.0 \times 10^2 \text{ km/h}$$

for the cheetah's speed.

100. Using Eq. 10-7 and Eq. 10-18, the average angular acceleration is

$$\alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t} = \frac{\Delta v}{r\Delta t} = \frac{25-12}{(0.75/2)(6.2)} = 5.6\,\text{rad/s}^2 \ .$$

101. We make use of Table 10-2(e) and the parallel-axis theorem in Eq. 10-36.

(a) The moment of inertia is

$$I = \frac{1}{12}ML^2 + Mh^2 = \frac{1}{12}(3.0\text{ kg})(4.0\text{ m})^2 + (3.0\text{ kg})(1.0\text{ m})^2 = 7.0\text{ kg}\cdot\text{m}^2.$$

(b) The rotational kinetic energy is

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 \quad\Rightarrow\quad \omega = \sqrt{\frac{2K_{\text{rot}}}{I}} = \sqrt{\frac{2(20\text{ J})}{7\text{ kg}\cdot\text{m}^2}} = 2.4\text{ rad/s}$$

The linear speed of the end *B* is given by $v_B = \omega r_{AB} = (2.4\text{ rad/s})(3.00\text{ m}) = 7.2\text{ m/s}$, where r_{AB} is the distance between *A* and *B*.

(c) The maximum angle θ is attained when all the rotational kinetic energy is transformed into potential energy. Moving from the vertical position ($\theta = 0$) to the maximum angle θ, the center of mass is elevated by $\Delta y = d_{AC}(1-\cos\theta)$, where d_{AC} = 1.00 m is the distance between *A* and the center of mass of the rod. Thus, the change in potential energy is

$$\Delta U = mg\Delta y = mgd_{AC}(1-\cos\theta) \quad\Rightarrow\quad 20\text{ J} = (3.0\text{ kg})(9.8\text{ m/s}^2)(1.0\text{ m})(1-\cos\theta)$$

which yields $\cos\theta = 0.32$, or $\theta \approx 71°$.

102. (a) The linear speed at $t = 15.0$ s is

$$v = a_t t = \left(0.500\,\text{m/s}^2\right)(15.0\,\text{s}) = 7.50\,\text{m/s}\ .$$

The radial (centripetal) acceleration at that moment is

$$a_r = \frac{v^2}{r} = \frac{(7.50\,\text{m/s})^2}{30.0\,\text{m}} = 1.875\,\text{m/s}^2\ .$$

Thus, the net acceleration has magnitude:

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{\left(0.500\,\text{m/s}^2\right)^2 + \left(1.875\,\text{m/s}^2\right)^2} = 1.94\,\text{m/s}^2\ .$$

(b) We note that $\vec{a}_t \,||\, \vec{v}$. Therefore, the angle between $\vec{v}$ and $\vec{a}$ is

$$\tan^{-1}\left(\frac{a_r}{a_t}\right) = \tan^{-1}\left(\frac{1.875}{0.5}\right) = 75.1°$$

so that the vector is pointing more toward the center of the track than in the direction of motion.

103. (a) Using Eq. 10-1, the angular displacement is

$$\theta = \frac{5.6\,\text{m}}{8.0\times 10^{-2}\ \text{m}} = 1.4\times 10^{2}\ \text{rad}\ .$$

(b) We use $\theta = \frac{1}{2}\alpha t^2$ (Eq. 10-13) to obtain *t*:

$$t = \sqrt{\frac{2\theta}{\alpha}} = \sqrt{\frac{2\left(1.4\times 10^{2}\ \text{rad}\right)}{1.5\,\text{rad/s}^2}} = 14\,\text{s}\ .$$

104. We apply Eq. 10-12 twice, assuming the sense of rotation is positive. We have $\omega > 0$ and $\alpha < 0$. Since the angular velocity at t = 1 min is ω_1 = (0.90)(250) = 225 rev/min, we have

$$\omega_1 = \omega_0 + \alpha t \Rightarrow a = \frac{225 - 250}{1} = -25 \text{ rev / min}^2 .$$

Next, between t = 1 min and t = 2 min we have the interval Δt = 1 min. Consequently, the angular velocity at t = 2 min is

$$\omega_2 = \omega_1 + \alpha \Delta t = 225 + (-25)(1) = 200 \text{ rev / min} .$$

105. (a) Using Table 10-2(c), the rotational inertia is

$$I = \frac{1}{2}mR^2 = \frac{1}{2}(1210\ \text{kg})\left(\frac{1.21\ \text{m}}{2}\right)^2 = 221\ \text{kg}\cdot\text{m}^2.$$

(b) The rotational kinetic energy is, by Eq. 10-34,

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(2.21\times10^2\ \text{kg}\cdot\text{m}^2)[(1.52\ \text{rev/s})(2\pi\,\text{rad/rev})]^2 = 1.10\times10^4\ \text{J}.$$

106. (a) We obtain

$$\omega = \frac{(33.33 \text{ rev/min})(2\pi \text{ rad/rev})}{60 \text{ s/min}} = 3.5 \text{ rad/s}.$$

(b) Using Eq. 10-18, we have $v = r\omega = (15)(3.49) = 52$ cm/s.

(c) Similarly, when r = 7.4 cm we find $v = r\omega$ = 26 cm/s. The goal of this exercise is to observe what is and is not the same at different locations on a body in rotational motion (ω is the same, v is not), as well as to emphasize the importance of radians when working with equations such as Eq. 10-18.

107. With v = 50(1000/3600) = 13.9 m/s, Eq. 10-18 leads to

$$\omega = \frac{v}{r} = \frac{13.9}{110} = 0.13 \text{ rad / s.}$$

108. (a) The angular speed ω associated with Earth's spin is $\omega = 2\pi/T$, where $T = 86400$s (one day). Thus,

$$\omega = \frac{2\pi}{86400 \text{ s}} = 7.3\times10^{-5} \text{ rad/s}$$

and the angular acceleration α required to accelerate the Earth from rest to ω in one day is $\alpha = \omega/T$. The torque needed is then

$$\tau = I\alpha = \frac{I\omega}{T} = \frac{(9.7\times10^{37} \text{ kg}\cdot\text{m}^2)(7.3\times10^{-5} \text{ rad/s})}{86400\,\text{s}} = 8.2\times10^{28} \text{ N}\cdot\text{m}$$

where we used

$$I = \frac{2}{5}M\,R^2 = \frac{2}{5}\left(5.98\times10^{24} \text{ kg}\right)\left(6.37\times10^{6} \text{ m}\right)^2 = 9.7\times10^{37} \text{ kg}\cdot\text{m}^2$$

for Earth's rotational inertia.

(b) Using the values from part (a), the kinetic energy of the Earth associated with its rotation about its own axis is $K = \frac{1}{2}I\omega^2 = 2.6\times10^{29}\text{J}$. This is how much energy would need to be supplied to bring it (starting from rest) to the current angular speed.

(c) The associated power is

$$P = \frac{K}{T} = \frac{2.57\times10^{29}\text{J}}{86400 \text{ s}} = 3.0\times10^{24} \text{ W}.$$

109. The translational kinetic energy of the molecule is

$$K_t = \frac{1}{2}mv^2 = \frac{1}{2}(5.30\times10^{-26}\ \text{kg})(500\ \text{m/s})^2 = 6.63\times10^{-21}\ \text{J}.$$

With $I = 1.94\times10^{-46}\ \text{kg}\cdot\text{m}^2$, we employ Eq. 10-34:

$$K_r = \frac{2}{3}K_t \quad \Rightarrow \quad \frac{1}{2}I\omega^2 = \frac{2}{3}(6.63\times10^{-21}\ \text{J})$$

which leads to $\omega = 6.75\times10^{12}$ rad/s.

110. (a) The rotational inertia relative to the specified axis is

$$I = \sum m_i r_i^2 = (2M)L^2 + (2M)L^2 + M(2L)^2$$

which is found to be $I = 4.6 \text{ kg} \cdot \text{m}^2$. Then, with $\omega = 1.2$ rad/s, we obtain the kinetic energy from Eq. 10-34:

$$K = \frac{1}{2} I\omega^2 = 3.3 \text{ J}.$$

(b) In this case the axis of rotation would appear as a standard y axis with origin at P. Each of the $2M$ balls are a distance of $r = L \cos 30°$ from that axis. Thus, the rotational inertia in this case is

$$I = \sum m_i r_i^2 = (2M)r^2 + (2M)r^2 + M(2L)^2$$

which is found to be $I = 4.0 \text{ kg} \cdot \text{m}^2$. Again, from Eq. 10-34 we obtain the kinetic energy

$$K = \frac{1}{2} I\omega^2 = 2.9 \text{ J}.$$

111. (a) The linear speed of a point on belt 1 is

$$v_1 = r_A \omega_A = (15 \text{ cm})(10 \text{ rad/s}) = 1.5\times10^2 \text{ cm/s}.$$

(b) The angular speed of pulley B is

$$r_B \omega_B = r_A \omega_A \quad \Rightarrow \quad \omega_B = \frac{r_A \omega_A}{r_B} = \left(\frac{15 \text{ cm}}{10 \text{ cm}}\right)(10 \text{ rad/s}) = 15 \text{ rad/s}.$$

(c) Since the two pulleys are rigidly attached to each other, the angular speed of pulley B' is the same as that of pulley B, i.e., $\omega'_B = 15 \text{ rad/s}$.

(d) The linear speed of a point on belt 2 is

$$v_2 = r_{B'} \omega'_B = (5 \text{ cm})(15 \text{ rad/s}) = 75 \text{ cm/s}.$$

(e) The angular speed of pulley C is

$$r_C \omega_C = r_{B'} \omega'_B \quad \Rightarrow \quad \omega_C = \frac{r_{B'} \omega'_B}{r_C} = \left(\frac{5 \text{ cm}}{25 \text{ cm}}\right)(15 \text{ rad/s}) = 3.0 \text{ rad/s}$$

112. (a) The particle at A has $r = 0$ with respect to the axis of rotation. The particle at B is $r = L = 0.50$ m from the axis; similarly for the particle directly above A in the figure. The particle diagonally opposite A is a distance $r = \sqrt{2}L = 0.71\text{ m}$ from the axis. Therefore,

$$I = \sum m_i r_i^2 = 2mL^2 + m\left(\sqrt{2}L\right)^2 = 0.20\text{ kg}\cdot\text{m}^2.$$

(b) One imagines rotating the figure (about point A) clockwise by 90° and noting that the center of mass has fallen a distance equal to L as a result. If we let our reference position for gravitational potential be the height of the center of mass at the instant AB swings through vertical orientation, then

$$K_0 + U_0 = K + U \quad \Rightarrow \quad 0 + (4m)gh_0 = K + 0.$$

Since $h_0 = L = 0.50$ m, we find $K = 3.9$ J. Then, using Eq. 10-34, we obtain

$$K = \frac{1}{2} I_A \omega^2 \Rightarrow \omega = 6.3\text{ rad/s}.$$

113. Using Eq. 10-12, we have

$$\omega = \omega_0 + \alpha t \Rightarrow \alpha = \frac{2.6 \text{ rad/s} - 8.0 \text{ rad/s}}{3.0 \text{ s}} = -1.8 \text{ rad/s}^2.$$

Using this value in Eq. 10-14 leads to

$$\omega^2 = \omega_0^2 + 2\alpha\theta \;\Rightarrow\; \theta = \frac{0 - (8.0 \text{ rad/s})^2}{2\left(-1.8 \text{ rad/s}^2\right)} = 18 \text{ rad.}$$

114. We make use of Table 10-2(e) as well as the parallel-axis theorem, Eq. 10-34, where needed. We use ℓ (as a subscript) to refer to the long rod and *s* to refer to the short rod.

(a) The rotational inertia is

$$I = I_s + I_\ell = \frac{1}{12} m_s L_s^2 + \frac{1}{3} m_\ell L_\ell^2 = 0.019 \text{ kg} \cdot \text{m}^2 .$$

(b) We note that the center of the short rod is a distance of $h = 0.25$ m from the axis. The rotational inertia is

$$I = I_s + I_\ell = \frac{1}{12} m_s L_s^2 + m_s h^2 + \frac{1}{12} m_\ell L_\ell^2$$

which again yields $I = 0.019 \text{ kg} \cdot \text{m}^2$.

115. We employ energy methods in this solution; thus, considerations of positive versus negative sense (regarding the rotation of the wheel) are not relevant.

(a) The speed of the box is related to the angular speed of the wheel by $v = R\omega$, so that

$$K_{\text{box}} = \frac{1}{2} m_{\text{box}} v^2 \Rightarrow v = \sqrt{\frac{2K_{\text{box}}}{m_{\text{box}}}} = 1.41 \text{ m/s}$$

implies that the angular speed is ω = 1.41/0.20 = 0.71 rad/s. Thus, the kinetic energy of rotation is $\frac{1}{2} I\omega^2 = 10.0$ J.

(b) Since it was released from rest at what we will consider to be the reference position for gravitational potential, then (with SI units understood) energy conservation requires

$$K_0 + U_0 = K + U \quad \Rightarrow \quad 0 + 0 = (6.0 + 10.0) + m_{\text{box}} g(-h).$$

Therefore, h = 16.0/58.8 = 0.27 m.

116. (a) One particle is on the axis, so $r = 0$ for it. For each of the others, the distance from the axis is

$$r = (0.60 \text{ m}) \sin 60° = 0.52 \text{ m}.$$

Therefore, the rotational inertia is $I = \sum m_i r_i^2 = 0.27 \text{ kg} \cdot \text{m}^2$.

(b) The two particles that are nearest the axis are each a distance of $r = 0.30$ m from it. The particle "opposite" from that side is a distance $r = (0.60 \text{ m}) \sin 60° = 0.52$ m from the axis. Thus, the rotational inertia is

$$I = \sum m_i r_i^2 = 0.22 \text{ kg} \cdot \text{m}^2.$$

(c) The distance from the axis for each of the particles is $r = \frac{1}{2}(0.60 \text{ m}) \sin 60°$. The rotational inertia is

$$I = 3(0.50 \text{ kg})(0.26 \text{ m})^2 = 0.10 \text{ kg} \cdot \text{m}^2.$$

Chapter 11

1. The velocity of the car is a constant

$$\vec{v} = +(80 \text{ km/h})(1000 \text{ m/km})(1 \text{ h}/3600 \text{ s})\,\hat{\text{i}} = (+22\,\text{m/s})\hat{\text{i}},$$

and the radius of the wheel is $r = 0.66/2 = 0.33$ m.

(a) In the car's reference frame (where the lady perceives herself to be at rest) the road is moving towards the rear at $\vec{v}_{\text{road}} = -v = -22\,\text{m/s}$, and the motion of the tire is purely rotational. In this frame, the center of the tire is "fixed" so $v_{\text{center}} = 0$.

(b) Since the tire's motion is only rotational (not translational) in this frame, Eq. 10-18 gives $\vec{v}_{\text{top}} = (+22\,\text{m/s})\hat{\text{i}}$.

(c) The bottom-most point of the tire is (momentarily) in firm contact with the road (not skidding) and has the same velocity as the road: $\vec{v}_{\text{bottom}} = (-22\,\text{m/s})\hat{\text{i}}$. This also follows from Eq. 10-18.

(d) This frame of reference is not accelerating, so "fixed" points within it have zero acceleration; thus, $a_{\text{center}} = 0$.

(e) Not only is the motion purely rotational in this frame, but we also have ω = constant, which means the only acceleration for points on the rim is radial (centripetal). Therefore, the magnitude of the acceleration is

$$a_{\text{top}} = \frac{v^2}{r} = \frac{(22 \text{ m/s})^2}{0.33 \text{ m}} = 1.5\times10^3 \text{ m/s}^2 .$$

(f) The magnitude of the acceleration is the same as in part (d): $a_{\text{bottom}} = 1.5 \times 10^3$ m/s^2.

(g) Now we examine the situation in the road's frame of reference (where the road is "fixed" and it is the car that appears to be moving). The center of the tire undergoes purely translational motion while points at the rim undergo a combination of translational and rotational motions. The velocity of the center of the tire is $\vec{v} = (+22\,\text{m/s})\hat{\text{i}}$.

(h) In part (b), we found $\vec{v}_{\text{top,car}} = +v$ and we use Eq. 4-39:

$$\vec{v}_{\text{top, ground}} = \vec{v}_{\text{top, car}} + \vec{v}_{\text{car, ground}} = v\,\hat{\text{i}} + v\,\hat{\text{i}} = 2v\,\hat{\text{i}}$$

which yields $2v = +44$ m/s. This is consistent with Fig. 11-3(c).

(i) We can proceed as in part (h) or simply recall that the bottom-most point is in firm contact with the (zero-velocity) road. Either way – the answer is zero.

(j) The translational motion of the center is constant; it does not accelerate.

(k) Since we are transforming between constant-velocity frames of reference, the accelerations are unaffected. The answer is as it was in part (e): 1.5×10^3 m/s^2.

(1) As explained in part (k), $a = 1.5 \times 10^3$ m/s^2.

2. The initial speed of the car is

$$v = (80 \text{ km/h})(1000 \text{ m/km})(1 \text{ h}/3600 \text{ s}) = 22.2 \text{ m/s}.$$

The tire radius is $R = 0.750/2 = 0.375$ m.

(a) The initial speed of the car is the initial speed of the center of mass of the tire, so Eq. 11-2 leads to

$$\omega_0 = \frac{v_{\text{com}0}}{R} = \frac{22.2 \text{ m/s}}{0.375 \text{ m}} = 59.3 \text{ rad/s}.$$

(b) With $\theta = (30.0)(2\pi) = 188$ rad and $\omega = 0$, Eq. 10-14 leads to

$$\omega^2 = \omega_0^2 + 2\alpha\theta \;\Rightarrow\; |\alpha| = \frac{(59.3 \text{ rad/s})^2}{2(188 \text{ rad})} = 9.31 \text{ rad/s}^2.$$

(c) Eq. 11-1 gives $R\theta = 70.7$ m for the distance traveled.

3. Let M be the mass of the car (presumably including the mass of the wheels) and v be its speed. Let I be the rotational inertia of one wheel and ω be the angular speed of each wheel. The kinetic energy of rotation is

$$K_{\text{rot}} = 4\left(\frac{1}{2} I\omega^2\right),$$

where the factor 4 appears because there are four wheels. The total kinetic energy is given by $K = \frac{1}{2} Mv^2 + 4(\frac{1}{2} I\omega^2)$. The fraction of the total energy that is due to rotation is

$$\text{fraction} = \frac{K_{\text{rot}}}{K} = \frac{4I\omega^2}{Mv^2 + 4I\omega^2}.$$

For a uniform disk (relative to its center of mass) $I = \frac{1}{2} mR^2$ (Table 10-2(c)). Since the wheels roll without sliding $\omega = v/R$ (Eq. 11-2). Thus the numerator of our fraction is

$$4I\omega^2 = 4\left(\frac{1}{2} mR^2\right)\left(\frac{v}{R}\right)^2 = 2mv^2$$

and the fraction itself becomes

$$\text{fraction} = \frac{2mv^2}{Mv^2 + 2mv^2} = \frac{2m}{M + 2m} = \frac{2(10)}{1000} = \frac{1}{50} = 0.020.$$

The wheel radius cancels from the equations and is not needed in the computation.

4. We use the results from section 11.3.

(a) We substitute $I = \frac{2}{5} M R^2$ (Table 10-2(f)) and $a = -\,0.10g$ into Eq. 11-10:

$$-0.10g = -\frac{g\sin\theta}{1+\left(\frac{2}{5}MR^2\right)/MR^2} = -\frac{g\sin\theta}{7/5}$$

which yields $\theta = \sin^{-1}(0.14) = 8.0°$.

(b) The acceleration would be more. We can look at this in terms of forces or in terms of energy. In terms of forces, the uphill static friction would then be absent so the downhill acceleration would be due only to the downhill gravitational pull. In terms of energy, the rotational term in Eq. 11-5 would be absent so that the potential energy it started with would simply become $\frac{1}{2}mv^2$ (without it being "shared" with another term) resulting in a greater speed (and, because of Eq. 2-16, greater acceleration).

5. By Eq. 10-52, the work required to stop the hoop is the negative of the initial kinetic energy of the hoop. The initial kinetic energy is $K = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$ (Eq. 11-5), where $I = mR^2$ is its rotational inertia about the center of mass, m = 140 kg, and v = 0.150 m/s is the speed of its center of mass. Eq. 11-2 relates the angular speed to the speed of the center of mass: $\omega = v/R$. Thus,

$$K = \frac{1}{2} mR^2 \left(\frac{v^2}{R^2} \right) + \frac{1}{2} mv^2 = mv^2 = (140\ \text{kg})(0.150\ \text{m/s})^2$$

which implies that the work required is – 3.15 J.

6. From $I = \frac{2}{3} MR^2$ (Table 10-2(g)) we find

$$M = \frac{3I}{2R^2} = \frac{3\left(0.040\ \text{kg}\cdot\text{m}^2\right)}{2\left(0.15\ \text{m}\right)^2} = 2.7\ \text{kg}.$$

It also follows from the rotational inertia expression that $\frac{1}{2} I\omega^2 = \frac{1}{3} MR^2\omega^2$. Furthermore, it rolls without slipping, $v_{\text{com}} = R\omega$, and we find

$$\frac{K_{\text{rot}}}{K_{\text{com}} + K_{\text{rot}}} = \frac{\frac{1}{3} MR^2\omega^2}{\frac{1}{2} mR^2\omega^2 + \frac{1}{3} MR^2\omega^2}.$$

(a) Simplifying the above ratio, we find $K_{\text{rot}}/K = 0.4$. Thus, 40% of the kinetic energy is rotational, or

$$K_{\text{rot}} = (0.4)(20\ \text{J}) = 8.0\ \text{J}.$$

(b) From $K_{\text{rot}} = \frac{1}{3} M R^2\omega^2 = 8.0\,\text{J}$ (and using the above result for M) we find

$$\omega = \frac{1}{0.15\ \text{m}}\sqrt{\frac{3(8.0\ \text{J})}{2.7\ \text{kg}}} = 20\ \text{rad/s}$$

which leads to $v_{\text{com}} = (0.15\ \text{m})(20\ \text{rad/s}) = 3.0\ \text{m/s}$.

(c) We note that the inclined distance of 1.0 m corresponds to a height $h = 1.0 \sin 30° = 0.50$ m. Mechanical energy conservation leads to

$$K_i = K_f + U_f \quad \Rightarrow \quad 20\,\text{J} = K_f + Mgh$$

which yields (using the values of M and h found above) $K_f = 6.9$ J.

(d) We found in part (a) that 40% of this must be rotational, so

$$\frac{1}{3} MR^2\omega_f^2 = (0.40)K_f \Rightarrow \omega_f = \frac{1}{0.15\ \text{m}}\sqrt{\frac{3(0.40)(6.9\ \text{J})}{2.7\ \text{kg}}}$$

which yields $\omega_f = 12$ rad/s and leads to

$$v_{\text{com}\,f} = R\omega_f = (0.15\ \text{m})(12\ \text{rad/s}) = 1.8\ \text{m/s}.$$

7. With $\vec{F}_{\text{app}} = (10\ \text{N})\hat{\text{i}}$, we solve the problem by applying Eq. 9-14 and Eq. 11-37.

(a) Newton's second law in the x direction leads to

$$F_{\text{app}} - f_s = ma \quad \Rightarrow f_s = 10\,\text{N} - (10\,\text{kg})\left(0.60\ \text{m/s}^2\right) = 4.0\ \text{N}.$$

In unit vector notation, we have $\vec{f}_s = (-4.0\ \text{N})\hat{\text{i}}$ which points leftward.

(b) With R = 0.30 m, we find the magnitude of the angular acceleration to be

$$|\alpha| = |a_{\text{com}}| / R = 2.0\ \text{rad/s}^2,$$

from Eq. 11-6. The only force not directed towards (or away from) the center of mass is $\vec{f}_s$, and the torque it produces is clockwise:

$$|\tau| = I\,|\alpha| \quad \Rightarrow \quad (0.30\,\text{m})(4.0\,\text{N}) = I\left(2.0\,\text{rad/s}^2\right)$$

which yields the wheel's rotational inertia about its center of mass: $I = 0.60\,\text{kg}\cdot\text{m}^2$.

8. Using the floor as the reference position for computing potential energy, mechanical energy conservation leads to

$$U_{\text{release}} = K_{\text{top}} + U_{\text{top}} \quad \Rightarrow \quad mgh = \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}I\omega^2 + mg\left(2R\right).$$

Substituting $I = \frac{2}{5}mr^2$ (Table 10-2(f)) and $\omega = v_{\text{com}}/r$ (Eq. 11-2), we obtain

$$mgh = \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_{\text{com}}}{r}\right)^2 + 2mgR \quad \Rightarrow \quad gh = \frac{7}{10}v_{\text{com}}^2 + 2gR$$

where we have canceled out mass *m* in that last step.

(a) To be on the verge of losing contact with the loop (at the top) means the normal force is vanishingly small. In this case, Newton's second law along the vertical direction (+*y* downward) leads to

$$mg = ma_r \Rightarrow g = \frac{v_{\text{com}}^2}{R - r}$$

where we have used Eq. 10-23 for the radial (centripetal) acceleration (of the center of mass, which at this moment is a distance $R - r$ from the center of the loop). Plugging the result $v_{\text{com}}^2 = g\left(R - r\right)$ into the previous expression stemming from energy considerations gives

$$gh = \frac{7}{10}\left(g\right)\left(R - r\right) + 2gR$$

which leads to $h = 2.7R - 0.7r \approx 2.7R$. With R = 14.0 cm , we have h = (2.7)(14.0 cm) = 37.8 cm.

(b) The energy considerations shown above (now with $h = 6R$) can be applied to point Q (which, however, is only at a height of R) yielding the condition

$$g\left(6R\right) = \frac{7}{10}v_{\text{com}}^2 + gR$$

which gives us $v_{\text{com}}^2 = 50gR/7$. Recalling previous remarks about the radial acceleration, Newton's second law applied to the horizontal axis at Q leads to

$$N = m\frac{v_{\text{com}}^2}{R - r} = m\frac{50gR}{7\left(R - r\right)}$$

which (for $R \gg r$) gives

$$N \approx \frac{50mg}{7} = \frac{50(2.80\times10^{-4}\ \text{kg})(9.80\ \text{m/s}^2)}{7} = 1.96\times10^{-2}\ \text{N}.$$

(b) The direction is toward the center of the loop.

9. (a) We find its angular speed as it leaves the roof using conservation of energy. Its initial kinetic energy is $K_i = 0$ and its initial potential energy is $U_i = Mgh$ where $h = 6.0\sin 30° = 3.0$ m (we are using the edge of the roof as our reference level for computing U). Its final kinetic energy (as it leaves the roof) is (Eq. 11-5)

$$K_f = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2 .$$

Here we use v to denote the speed of its center of mass and ω is its angular speed — at the moment it leaves the roof. Since (up to that moment) the ball rolls without sliding we can set $v = R\omega = v$ where $R = 0.10$ m. Using $I = \tfrac{1}{2}MR^2$ (Table 10-2(c)), conservation of energy leads to

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}MR^2\omega^2 + \frac{1}{4}MR^2\omega^2 = \frac{3}{4}MR^2\omega^2 .$$

The mass M cancels from the equation, and we obtain

$$\omega = \frac{1}{R}\sqrt{\frac{4}{3}gh} = \frac{1}{0.10\text{ m}}\sqrt{\frac{4}{3}\left(9.8\text{ m/s}^2\right)\left(3.0\text{ m}\right)} = 63\text{ rad/s}.$$

(b) Now this becomes a projectile motion of the type examined in Chapter 4. We put the origin at the position of the center of mass when the ball leaves the track (the "initial" position for this part of the problem) and take $+x$ leftward and $+y$ downward. The result of part (a) implies $v_0 = R\omega = 6.3$ m/s, and we see from the figure that (with these positive direction choices) its components are

$$v_{0x} = v_0\cos 30° = 5.4\text{ m/s}$$
$$v_{0y} = v_0\sin 30° = 3.1\text{ m/s}.$$

The projectile motion equations become

$$x = v_{0x}t \quad\text{and}\quad y = v_{0y}t + \frac{1}{2}gt^2 .$$

We first find the time when $y = H = 5.0$ m from the second equation (using the quadratic formula, choosing the positive root):

$$t = \frac{-v_{0y} + \sqrt{v_{0y}^2 + 2gH}}{g} = 0.74\text{ s}.$$

Then we substitute this into the x equation and obtain $x = \left(5.4\text{ m/s}\right)\left(0.74\text{ s}\right) = 4.0\text{ m}.$

10. We plug $a = -3.5$ m/s^2 (where the magnitude of this number was estimated from the "rise over run" in the graph), $\theta = 30°$, $M = 0.50$ kg and $R = 0.060$ m into Eq. 11-10 and solve for the rotational inertia. We find $I = 7.2 \times 10^{-4}$ kg·m^2.

11. To find where the ball lands, we need to know its speed as it leaves the track (using conservation of energy). Its initial kinetic energy is $K_i = 0$ and its initial potential energy is $U_i = M\,gH$. Its final kinetic energy (as it leaves the track) is $K_f = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$ (Eq. 11-5) and its final potential energy is $M\,gh$. Here we use v to denote the speed of its center of mass and ω is its angular speed — at the moment it leaves the track. Since (up to that moment) the ball rolls without sliding we can set $\omega = v/R$. Using $I = \frac{2}{5}MR^2$ (Table 10-2(f)), conservation of energy leads to

$$MgH = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + Mgh = \frac{1}{2}Mv^2 + \frac{2}{10}Mv^2 + Mgh = \frac{7}{10}Mv^2 + Mgh.$$

The mass M cancels from the equation, and we obtain

$$v = \sqrt{\frac{10}{7}g(H-h)} = \sqrt{\frac{10}{7}\left(9.8\text{ m/s}^2\right)\left(6.0\text{ m} - 2.0\text{ m}\right)} = 7.48\text{ m/s}.$$

Now this becomes a projectile motion of the type examined in Chapter 4. We put the origin at the position of the center of mass when the ball leaves the track (the "initial" position for this part of the problem) and take $+x$ rightward and $+y$ downward. Then (since the initial velocity is purely horizontal) the projectile motion equations become

$$x = vt \text{ and } y = -\frac{1}{2}gt^2.$$

Solving for x at the time when $y = h$, the second equation gives $t = \sqrt{2h/g}$. Then, substituting this into the first equation, we find

$$x = v\sqrt{\frac{2h}{g}} = (7.48\text{ m/s})\sqrt{\frac{2(2.0\text{ m})}{9.8\text{ m/s}^2}} = 4.8\text{ m}.$$

12. (a) Let the turning point be designated P. By energy conservation, the mechanical energy at $x = 7.0$ m is equal to the mechanical energy at P. Thus, with Eq. 11-5, we have

$$75 \text{ J} = \tfrac{1}{2} m v_p^2 + \tfrac{1}{2} I_{com}\, \omega_p^2 + U_p$$

Using item (*f*) of Table 10-2 and Eq. 11-2 (which means, if this is to be a turning point, that $\omega_p = v_p = 0$), we find $U_p = 75$ J. On the graph, this seems to correspond to $x = 2.0$ m, and we conclude that there is a turning point (and this is it). The ball, therefore, does not reach the origin.

(b) We note that there is no point (on the graph, to the right of $x = 7.0$ m) which is shown "higher" than 75 J, so we suspect that there is no turning point in this direction, and we seek the velocity v_p at $x = 13$ m. If we obtain a real, nonzero answer, then our suspicion is correct (that it does reach this point P at $x = 13$ m). By energy conservation, the mechanical energy at $x = 7.0$ m is equal to the mechanical energy at P. Therefore,

$$75 \text{ J} = \tfrac{1}{2} m v_p^2 + \tfrac{1}{2} I_{com}\, \omega_p^2 + U_p$$

Again, using item (*f*) of Table 11-2, Eq. 11-2 (less trivially this time) and $U_p = 60$ J (from the graph), as well as the numerical data given in the problem, we find $v_p = 7.3$ m/s.

13. (a) We choose clockwise as the negative rotational sense and rightwards as the positive translational direction. Thus, since this is the moment when it begins to roll smoothly, Eq. 11-2 becomes $v_{\text{com}} = -R\omega = (-0.11 \text{ m})\omega.$

This velocity is positive-valued (rightward) since ω is negative-valued (clockwise) as shown in Fig. 11-57.

(b) The force of friction exerted on the ball of mass m is $-\mu_k mg$ (negative since it points left), and setting this equal to ma_{com} leads to

$$a_{\text{com}} = -\mu g = -(0.21)\left(9.8 \text{ m/s}^2\right) = -2.1 \text{ m/s}^2$$

where the minus sign indicates that the center of mass acceleration points left, opposite to its velocity, so that the ball is decelerating.

(c) Measured about the center of mass, the torque exerted on the ball due to the frictional force is given by $\tau = -\mu mgR$. Using Table 10-2(f) for the rotational inertia, the angular acceleration becomes (using Eq. 10-45)

$$\alpha = \frac{\tau}{I} = \frac{-\mu mgR}{2mR^2/5} = \frac{-5\mu g}{2R} = \frac{-5(0.21)\left(9.8 \text{ m/s}^2\right)}{2(0.11 \text{ m})} = -47 \text{ rad/s}^2$$

where the minus sign indicates that the angular acceleration is clockwise, the same direction as ω (so its angular motion is "speeding up'').

(d) The center-of-mass of the sliding ball decelerates from $v_{\text{com},0}$ to v_{com} during time t according to Eq. 2-11: $v_{\text{com}} = v_{\text{com},0} - \mu gt$. During this time, the angular speed of the ball increases (in magnitude) from zero to $|\omega|$ according to Eq. 10-12:

$$|\omega| = |\alpha|\, t = \frac{5\mu gt}{2R} = \frac{v_{\text{com}}}{R}$$

where we have made use of our part (a) result in the last equality. We have two equations involving v_{com}, so we eliminate that variable and find

$$t = \frac{2v_{\text{com},0}}{7\mu g} = \frac{2(8.5 \text{ m/s})}{7(0.21)\left(9.8 \text{ m/s}^2\right)} = 1.2 \text{ s}.$$

(e) The skid length of the ball is (using Eq. 2-15)

$$\Delta x = v_{\text{com},0}t - \frac{1}{2}(\mu g)t^2 = (8.5 \text{ m/s})(1.2 \text{ s}) - \frac{1}{2}(0.21)\left(9.8 \text{ m/s}^2\right)(1.2 \text{ s})^2 = 8.6 \text{ m}.$$

(f) The center of mass velocity at the time found in part (d) is

$$v_{\text{com}} = v_{\text{com},0} - \mu gt = 8.5 \text{ m/s} - (0.21)\left(9.8 \text{ m/s}^2\right)(1.2 \text{ s}) = 6.1 \text{ m/s}.$$

14. To find the center of mass speed v on the plateau, we use the projectile motion equations of Chapter 4. With $v_{oy} = 0$ (and using "h" for h_2) Eq. 4-22 gives the time-of-flight as $t = \sqrt{2h/g}$. Then Eq. 4-21 (squared, and using d for the horizontal displacement) gives $v^2 = gd^2/2h$. Now, to find the speed v_p at point P, we apply energy conservation, i.e., mechanical energy on the plateau is equal to the mechanical energy at P. With Eq. 11-5, we obtain

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}I_{com}\,\omega^2 + mgh_1 = \tfrac{1}{2}mv_p^{\,2} + \tfrac{1}{2}I_{com}\,\omega_p^{\,2}$$

Using item (f) of Table 10-2, Eq. 11-2, and our expression (above) $v^2 = gd^2/2h$, we obtain

$$gd^2/2h + 10gh_1/7 = v_p^{\,2}$$

which yields (using the values stated in the problem) $v_p = 1.34$ m/s.

15. The physics of a rolling object usually requires a separate and very careful discussion (above and beyond the basics of rotation discussed in chapter 10); this is done in the first three sections of chapter 11. Also, the normal force on something (which is here the center of mass of the ball) following a circular trajectory is discussed in section 6-6 (see particularly sample problem 6-7). Adapting Eq. 6-19 to the consideration of forces at the *bottom* of an arc, we have

$$F_N - Mg = Mv^2/r$$

which tells us (since we are given $F_N = 2Mg$) that the center of mass speed (squared) is $v^2 = gr$, where r is the arc radius (0.48 m) Thus, the ball's angular speed (squared) is

$$\omega^2 = v^2/R^2 = gr/R^2,$$

where R is the ball's radius. Plugging this into Eq. 10-5 and solving for the rotational inertia (about the center of mass), we find

$$I_{\text{com}} = 2MhR^2/r - MR^2 = MR^2[2(0.36/0.48) - 1]\ .$$

Thus, using the β notation suggested in the problem, we find

$$\beta = 2(0.36/0.48) - 1 = 0.50.$$

16. The physics of a rolling object usually requires a separate and very careful discussion (above and beyond the basics of rotation discussed in chapter 11); this is done in the first three sections of Chapter 11. Using energy conservation with Eq. 11-5 and solving for the rotational inertia (about the center of mass), we find

$$I_{\text{com}} = 2MhR^2/r - MR^2 = MR^2[2g(H-h)/v^2 - 1] \,.$$

Thus, using the β notation suggested in the problem, we find

$$\beta = 2g(H-h)/v^2 - 1.$$

To proceed further, we need to find the center of mass speed v, which we do using the projectile motion equations of Chapter 4. With $v_{oy} = 0$, Eq. 4-22 gives the time-of-flight as $t = \sqrt{2h/g}$. Then Eq. 4-21 (squared, and using d for the horizontal displacement) gives $v^2 = gd^2/2h$. Plugging this into our expression for β gives

$$2g(H-h)/v^2 - 1 = 4h(H-h)/d^2 - 1$$

Therefore, with the values given in the problem, we find β = 0.25.

17. (a) The derivation of the acceleration is found in §11-4; Eq. 11-13 gives

$$a_{\text{com}} = -\frac{g}{1 + I_{\text{com}} / MR_0^2}$$

where the positive direction is upward. We use $I_{\text{com}} = 950\ \text{g}\cdot\text{cm}^2$, M =120g, R_0 = 0.320 cm and g = 980 cm/s^2 and obtain

$$|a_{\text{com}}| = \frac{980\ \text{cm/s}^2}{1 + \left(950\ \text{g}\cdot\text{cm}^2\right) / (120\ \text{g})(0.32\ \text{cm})^2} = 12.5\ \text{cm/s}^2 \approx 13\ \text{cm/s}^2.$$

(b) Taking the coordinate origin at the initial position, Eq. 2-15 leads to $y_{\text{com}} = \frac{1}{2} a_{\text{com}} t^2$. Thus, we set y_{com} = – 120 cm, and find

$$t = \sqrt{\frac{2y_{\text{com}}}{a_{\text{com}}}} = \sqrt{\frac{2(-120\,\text{cm})}{-12.5\ \text{cm/s}^2}} = 4.38\ \text{s} \approx 4.4\ \text{s}.$$

(c) As it reaches the end of the string, its center of mass velocity is given by Eq. 2-11:

$$v_{\text{com}} = a_{\text{com}} t = \left(-12.5\ \text{cm/s}^2\right)(4.38\,\text{s}) = -54.8\ \text{cm/s},$$

so its linear speed then is approximately $|v_{\text{com}}| = 55$ cm/s.

(d) The translational kinetic energy is

$$\tfrac{1}{2} m v_{\text{com}}^2 = \tfrac{1}{2}(0.120\ \text{kg})(0.548\ \text{m/s})^2 = 1.8\times 10^{-2}\ \text{J}.$$

(e) The angular velocity is given by $\omega = -\, v_{\text{com}}/R_0$ and the rotational kinetic energy is

$$\frac{1}{2} I_{\text{com}} \omega^2 = \frac{1}{2} I_{\text{com}} \frac{v_{\text{com}}^2}{R_0^2} = \frac{1}{2} \frac{\left(9.50\times 10^{-5}\ \text{kg}\cdot\text{m}^2\right)(0.548\ \text{m/s})^2}{\left(3.2\times 10^{-3}\ \text{m}\right)^2}$$

which yields K_{rot} = 1.4 J.

(f) The angular speed is

$$\omega = \frac{|v_{\text{com}}|}{R_0} = \frac{0.548\ \text{m/s}}{3.2\times 10^{-3}\ \text{m}} = 1.7\times 10^2\ \text{rad/s} = 27\,\text{rev/s}.$$

18. (a) The derivation of the acceleration is found in § 11-4; Eq. 11-13 gives

$$a_{\text{com}} = -\frac{g}{1 + I_{\text{com}} / MR_0^2}$$

where the positive direction is upward. We use $I_{\text{com}} = MR^2/2$ where the radius is R = 0.32 m and M = 116 kg is the *total* mass (thus including the fact that there are two disks) and obtain

$$a = -\frac{g}{1 + (MR^2/2)/MR_0^2} = \frac{g}{1 + (R/R_0)^2/2}$$

which yields $a = -g/51$ upon plugging in $R_0 = R/10 = 0.032$ m. Thus, the magnitude of the center of mass acceleration is 0.19 m/s^2.

(b) As observed in §11-4, our result in part (a) applies to both the descending and the rising yoyo motions.

(c) The external forces on the center of mass consist of the cord tension (upward) and the pull of gravity (downward). Newton's second law leads to

$$T - Mg = ma \Rightarrow T = M\left(g - \frac{g}{51}\right) = 1.1 \times 10^3 \text{ N}.$$

(d) Our result in part (c) indicates that the tension is well below the ultimate limit for the cord.

(e) As we saw in our acceleration computation, all that mattered was the ratio R/R_0 (and, of course, g). So if it's a scaled-up version, then such ratios are unchanged and we obtain the same result.

(f) Since the tension also depends on mass, then the larger yoyo will involve a larger cord tension.

19. If we write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$\left(yF_z - zF_y\right)\hat{\text{i}} + \left(zF_x - xF_z\right)\hat{\text{j}} + \left(xF_y - yF_x\right)\hat{\text{k}}.$$

(a) In the above expression, we set (with SI units understood) $x = 0$, $y = -4.0$, $z = 3.0$, $F_x = 2.0$, $F_y = 0$ and $F_z = 0$. Then we obtain

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(6.0\hat{\text{j}} + 8.0\hat{\text{k}}\right)\text{N}\cdot\text{m}.$$

This has magnitude $\sqrt{(6.0\ \text{N}\cdot\text{m})^2 + (8.0\ \text{N}\cdot\text{m})^2} = 10\ \text{N}\cdot\text{m}$ and is seen to be parallel to the yz plane. Its angle (measured counterclockwise from the $+y$ direction) is $\tan^{-1}\left(8/6\right) = 53°$.

(b) In the above expression, we set $x = 0$, $y = -4.0$, $z = 3.0$, $F_x = 0$, $F_y = 2.0$ and $F_z = 4.0$. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F} = (-22\,\text{N}\cdot\text{m})\hat{\text{i}}$. This has magnitude $22\ \text{N}\cdot\text{m}$ and points in the $-x$ direction.

20. If we write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$\left(yF_z - zF_y\right)\hat{\text{i}} + \left(zF_x - xF_z\right)\hat{\text{j}} + \left(xF_y - yF_x\right)\hat{\text{k}}.$$

(a) In the above expression, we set (with SI units understood) $x = -2.0$, $y = 0$, $z = 4.0$, $F_x = 6.0$, $F_y = 0$ and $F_z = 0$. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F} = (24\,\text{N}\cdot\text{m})\hat{\text{j}}$.

(b) The values are just as in part (a) with the exception that now $F_x = -6.0$. We find $\vec{\tau} = \vec{r} \times \vec{F} = (-24\,\text{N}\cdot\text{m})\hat{\text{j}}$.

(c) In the above expression, we set $x = -2.0$, $y = 0$, $z = 4.0$, $F_x = 0$, $F_y = 0$ and $F_z = 6.0$. We get $\vec{\tau} = \vec{r} \times \vec{F} = (12\,\text{N}\cdot\text{m})\hat{\text{j}}$.

(d) The values are just as in part (c) with the exception that now $F_z = -6.0$. We find $\vec{\tau} = \vec{r} \times \vec{F} = (-12\,\text{N}\cdot\text{m})\hat{\text{j}}$.

21. If we write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$\left(yF_z - zF_y\right)\hat{\text{i}} + \left(zF_x - xF_z\right)\hat{\text{j}} + \left(xF_y - yF_x\right)\hat{\text{k}}.$$

With (using SI units) $x = 0$, $y = -4.0$, $z = 5.0$, $F_x = 0$, $F_y = -2.0$ and $F_z = 3.0$ (these latter terms being the individual forces that contribute to the net force), the expression above yields

$$\vec{\tau} = \vec{r} \times \vec{F} = (-2.0\,\text{N}\cdot\text{m})\hat{\text{i}}.$$

22. If we write $\vec{r}\,' = x'\,\hat{\mathrm{i}} + y'\,\hat{\mathrm{j}} + z'\,\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r}\,' \times \vec{F}$ is equal to

$$\left(y'F_z - z'F_y\right)\hat{\mathrm{i}} + \left(z'F_x - x'F_z\right)\hat{\mathrm{j}} + \left(x'F_y - y'F_x\right)\hat{\mathrm{k}}.$$

(a) Here, $\vec{r}\,' = \vec{r}$ where $\vec{r} = 3.0\hat{\mathrm{i}} - 2.0\hat{\mathrm{j}} + 4.0\hat{\mathrm{k}}$, and $\vec{F} = \vec{F}_1$. Thus, dropping the prime in the above expression, we set (with SI units understood) $x = 3.0$, $y = -2.0$, $z = 4.0$, $F_x = 3.0$, $F_y = -4.0$ and $F_z = 5.0$. Then we obtain

$$\vec{\tau} = \vec{r} \times \vec{F}_1 = \left(6.0\,\hat{\mathrm{i}} - 3.0\,\hat{\mathrm{j}} - 6.0\,\hat{\mathrm{k}}\right)\ \mathrm{N \cdot m}.$$

(b) This is like part (a) but with $\vec{F} = \vec{F}_2$. We plug in $F_x = -3.0$, $F_y = -4.0$ and $F_z = -5.0$ and obtain

$$\vec{\tau} = \vec{r} \times \vec{F}_2 = \left(26\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} - 18\,\hat{\mathrm{k}}\right)\ \mathrm{N \cdot m}.$$

(c) We can proceed in either of two ways. We can add (vectorially) the answers from parts (a) and (b), or we can first add the two force vectors and then compute $\vec{\tau} = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2\right)$ (these total force components are computed in the next part). The result is

$$\vec{\tau} = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2\right) = \left(32\,\hat{\mathrm{i}} - 24\,\hat{\mathrm{k}}\right)\ \mathrm{N \cdot m}.$$

(d) Now $\vec{r}\,' = \vec{r} - \vec{r}_{\mathrm{o}}$ where $\vec{r}_{\mathrm{o}} = 3.0\hat{\mathrm{i}} + 2.0\hat{\mathrm{j}} + 4.0\hat{\mathrm{k}}$. Therefore, in the above expression, we set $x' = 0, y' = -4.0, z' = 0,$ and

$$\begin{aligned} F_x &= 3.0 - 3.0 = 0 \\ F_y &= -4.0 - 4.0 = -8.0 \\ F_z &= 5.0 - 5.0 = 0. \end{aligned}$$

We get $\vec{\tau} = \vec{r}\,' \times \left(\vec{F}_1 + \vec{F}_2\right) = 0.$

23. If we write $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$\left(yF_z - zF_y\right)\hat{\mathrm{i}} + \left(zF_x - xF_z\right)\hat{\mathrm{j}} + \left(xF_y - yF_x\right)\hat{\mathrm{k}}.$$

(a) Plugging in, we find $\vec{\tau} = \left[(3.0\,\mathrm{m})(6.0\,\mathrm{N}) - (4.0\,\mathrm{m})(-8.0\,\mathrm{N})\right]\hat{\mathrm{k}} = (50\,\mathrm{N}\cdot\mathrm{m})\,\hat{\mathrm{k}}.$

(b) We use Eq. 3-27, $|\vec{r} \times \vec{F}| = rF\sin\phi$, where ϕ is the angle between $\vec{r}$ and $\vec{F}$. Now $r = \sqrt{x^2 + y^2} = 5.0$ m and $F = \sqrt{F_x^2 + F_y^2} = 10$ N. Thus,

$$rF = (5.0\ \mathrm{m})(10\ \mathrm{N}) = 50\ \mathrm{N}\cdot\mathrm{m},$$

the same as the magnitude of the vector product calculated in part (a). This implies $\sin\phi = 1$ and $\phi = 90°$.

24. Eq. 11-14 (along with Eq. 3-30) gives

$$\vec{\tau} = \vec{r} \times \vec{F} = 4.00\hat{\text{i}} + (12.0 + 2.00F_x)\hat{\text{j}} + (14.0 + 3.00F_x)\hat{\text{k}}$$

with SI units understood. Comparing this with the known expression for the torque (given in the problem statement), we see that F_x must satisfy two conditions:

$$12.0 + 2.00F_x = 2.00 \quad \text{and} \quad 14.0 + 3.00F_x = -1.00.$$

The answer ($F_x = -5.00$ N) satisfies both conditions.

25. We use the notation $\vec{r}\,'$ to indicate the vector pointing from the axis of rotation directly to the position of the particle. If we write $\vec{r}\,' = x'\,\hat{\mathrm{i}} + y'\,\hat{\mathrm{j}} + z'\,\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r}\,' \times \vec{F}$ is equal to

$$\left(y'F_z - z'F_y\right)\hat{\mathrm{i}} + \left(z'F_x - x'F_z\right)\hat{\mathrm{j}} + \left(x'F_y - y'F_x\right)\hat{\mathrm{k}}.$$

(a) Here, $\vec{r}\,' = \vec{r}$. Dropping the primes in the above expression, we set (with SI units understood) $x = 0$, $y = 0.5$, $z = -2.0$, $F_x = 2.0$, $F_y = 0$ and $F_z = -3.0$. Then we obtain

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(-1.5\hat{\mathrm{i}} - 4.0\hat{\mathrm{j}} - 1.0\hat{\mathrm{k}}\right)\,\mathrm{N\cdot m}.$$

(b) Now $\vec{r}\,' = \vec{r} - \vec{r}_{\mathrm{o}}$ where $\vec{r}_{\mathrm{o}} = 2.0\hat{\mathrm{i}} - 3.0\hat{\mathrm{k}}$. Therefore, in the above expression, we set $x' = -2.0, y' = 0.5, z' = 1.0, F_x = 2.0, F_y = 0$ and $F_z = -3.0$. Thus, we obtain

$$\vec{\tau} = \vec{r}\,' \times \vec{F} = \left(-1.5\,\hat{\mathrm{i}} - 4.0\,\hat{\mathrm{j}} - 1.0\hat{\mathrm{k}}\right)\,\mathrm{N\cdot m}.$$

26. If we write $\vec{r}' = x'\,\hat{\mathrm{i}} + y'\,\hat{\mathrm{j}} + z'\,\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r}' = \vec{v}$ is equal to

$$\left(y'v_z - z'v_y\right)\hat{\mathrm{i}} + \left(z'v_x - x'v_z\right)\hat{\mathrm{j}} + \left(x'v_y - y'v_x\right)\hat{\mathrm{k}}.$$

(a) Here, $\vec{r}' = \vec{r}$ where $\vec{r} = 3.0\,\hat{\mathrm{i}} - 4.0\,\hat{\mathrm{j}}$. Thus, dropping the primes in the above expression, we set (with SI units understood) $x = 3.0, y = -4.0, z = 0, v_x = 30, v_y = 60$ and $v_z = 0$. Then (with $m = 2.0$ kg) we obtain

$$\vec{\ell} = m\left(\vec{r} \times \vec{v}\right) = (6.0 \times 10^2\ \mathrm{kg \cdot m^2/s})\hat{\mathrm{k}}.$$

(b) Now $\vec{r}' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = -2.0\hat{\mathrm{i}} - 2.0\hat{\mathrm{j}}$. Therefore, in the above expression, we set $x' = 5.0, y' = -2.0, z' = 0, v_x = 30, v_y = 60$ and $v_z = 0$. We get

$$\vec{\ell} = m\left(\vec{r}' \times \vec{v}\right) = (7.2 \times 10^2\ \ \mathrm{kg \cdot m^2/s})\hat{\mathrm{k}}.$$

27. For the 3.1 kg particle, Eq. 11-21 yields

$$\ell_1 = r_{\perp 1} m v_1 = (2.8\ \text{m})(3.1\ \text{kg})(3.6\ \text{m/s}) = 31.2\ \text{kg}\cdot\text{m}^2/\text{s}.$$

Using the right-hand rule for vector products, we find this $(\vec{r}_1 \times \vec{p}_1)$ is out of the page, or along the +*z* axis, perpendicular to the plane of Fig. 11-40. And for the 6.5 kg particle, we find

$$\ell_2 = r_{\perp 2} m v_2 = (1.5\ \text{m})(6.5\ \text{kg})(2.2\ \text{m/s}) = 21.4\ \text{kg}\cdot\text{m}^2/\text{s}.$$

And we use the right-hand rule again, finding that this $(\vec{r}_2 \times \vec{p}_2)$ is into the page, or in the –*z* direction.

(a) The two angular momentum vectors are in opposite directions, so their vector sum is the *difference* of their magnitudes: $L = \ell_1 - \ell_2 = 9.8\ \text{kg}\cdot\text{m}^2/\text{s}$.

(b) The direction of the net angular momentum is along the +*z* axis.

28. We note that the component of $\vec{v}$ perpendicular to $\vec{r}$ has magnitude $v \sin \theta_2$ where $\theta_2 = 30°$. A similar observation applies to $\vec{F}$.

(a) Eq. 11-20 leads to $\ell = rmv_\perp = (3.0\text{ m})(2.0\text{ kg})(4.0\text{ m/s})\sin 30° = 12\ \text{kg}\cdot\text{m}^2/\text{s}$.

(b) Using the right-hand rule for vector products, we find $\vec{r} \times \vec{p}$ points out of the page, or along the +z axis, perpendicular to the plane of the figure.

(c) Eq. 10-38 leads to $\tau = rF \sin \theta_2 = (3.0\text{ m})(2.0\text{ N})\sin 30° = 3.0\,\text{N}\cdot\text{m}$.

(d) Using the right-hand rule for vector products, we find $\vec{r} \times \vec{F}$ is also out of the page, or along the +z axis, perpendicular to the plane of the figure.

29. (a) We use $\vec{\ell} = m\vec{r} \times \vec{v}$, where $\vec{r}$ is the position vector of the object, $\vec{v}$ is its velocity vector, and m is its mass. Only the x and z components of the position and velocity vectors are nonzero, so Eq. 3-30 leads to $\vec{r} \times \vec{v} = \left(-xv_z + zv_z\right)\hat{\text{j}}$. Therefore,

$$\vec{\ell} = m\left(-xv_z + zv_x\right)\hat{\text{j}} = (0.25 \text{ kg})\left(-(2.0 \text{ m})(5.0 \text{ m/s}) + (-2.0 \text{ m})(-5.0 \text{ m/s})\right)\hat{\text{j}} = 0.$$

(b) If we write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$\left(yF_z - zF_y\right)\hat{\text{i}} + \left(zF_x - xF_z\right)\hat{\text{j}} + \left(xF_y - yF_x\right)\hat{\text{k}}.$$

With $x = 2.0$, $z = -2.0$, $F_y = 4.0$ and all other components zero (and SI units understood) the expression above yields

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(8.0\,\hat{\text{i}} + 8.0\,\hat{\text{k}}\right) \text{N} \cdot \text{m}.$$

30. (a) The acceleration vector is obtained by dividing the force vector by the (scalar) mass:

$$\vec{a} = \vec{F}/m = (3.00\ \text{m/s}^2)\hat{\text{i}} - (4.00\ \text{m/s}^2)\hat{\text{j}} + (2.00\ \text{m/s}^2)\hat{\text{k}}.$$

(b) Use of Eq. 11-18 leads directly to

$$\vec{L} = (42.0\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{i}} + (24.0\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{j}} + (60.0\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{k}}.$$

(c) Similarly, the torque is

$$\vec{\tau} = \vec{r} \times \vec{F} = (-8.00\ \text{N}\cdot\text{m})\hat{\text{i}} - (26.0\ \text{N}\cdot\text{m})\hat{\text{j}} - (40.0\ \text{N}\cdot\text{m})\hat{\text{k}}.$$

(d) We note (using the Pythagorean theorem) that the magnitude of the velocity vector is 7.35 m/s and that of the force is 10.8 N. The dot product of these two vectors is $\vec{v}\cdot\vec{F} = -48$ (in SI units). Thus, Eq. 3-20 yields

$$\theta = \cos^{-1}[-48.0/(7.35 \times 10.8)] = 127°.$$

31. (a) Since the speed is (momentarily) zero when it reaches maximum height, the angular momentum is zero then.

(b) With the convention (used in several places in the book) that clockwise sense is to be associated with the negative sign, we have $L = -\, r_{\perp} m\, v$ where $r_{\perp} = 2.00$ m, $m = 0.400$ kg, and v is given by free-fall considerations (as in chapter 2). Specifically, y_{max} is determined by Eq. 2-16 with the speed at max height set to zero; we find $y_{max} = v_o^2/2g$ where $v_o = 40.0$ m/s. Then with $y = \frac{1}{2} y_{max}$, Eq. 2-16 can be used to give $v = v_o/\sqrt{2}$. In this way we arrive at $L = -22.6$ kg·m^2/s.

(c) As mentioned in the previous part, we use the minus sign in writing $\tau = -\, r_{\perp} F$ with the force F being equal (in magnitude) to mg. Thus, $\tau = -7.84$ N·m.

(d) Due to the way $r_{\perp}$ is defined it does not matter how far up the ball is. The answer is the same as in part (c), $\tau = -7.84$ N·m.

32. We use a right-handed coordinate system with $\hat{k}$ directed out of the *xy* plane so as to be consistent with counterclockwise rotation (and the right-hand rule). Thus, all the angular momenta being considered are along the $-\hat{k}$ direction; for example, in part (b) $\vec{\ell} = -4.0t^2\,\hat{k}$ in SI units. We use Eq. 11-23.

(a) The angular momentum is constant so its derivative is zero. There is no torque in this instance.

(b) Taking the derivative with respect to time, we obtain the torque:

$$\vec{\tau} = \frac{d\vec{\ell}}{dt} = \left(-4.0\hat{k}\right)\frac{dt^2}{dt} = (-8.0t \text{ N}\cdot\text{m})\hat{k}\,.$$

This vector points in the $-\hat{k}$ direction (causing the clockwise motion to speed up) for all $t > 0$.

(c) With $\vec{\ell} = (-4.0\sqrt{t})\hat{k}$ in SI units, the torque is

$$\vec{\tau} = \left(-4.0\hat{k}\right)\frac{d\sqrt{t}}{dt} = \left(-4.0\hat{k}\right)\left(\frac{1}{2\sqrt{t}}\right) = \left(-\frac{2.0}{\sqrt{t}}\hat{k}\right)\text{N}\cdot\text{m}\,.$$

This vector points in the $-\hat{k}$ direction (causing the clockwise motion to speed up) for all $t > 0$ (and it is undefined for $t < 0$).

(d) Finally, we have

$$\vec{\tau} = \left(-4.0\hat{k}\right)\frac{dt^{-2}}{dt} = \left(-4.0\hat{k}\right)\left(\frac{-2}{t^3}\right) = \left(\frac{8.0}{t^3}\hat{k}\right)\text{N}\cdot\text{m}.$$

This vector points in the $+\hat{k}$ direction (causing the initially clockwise motion to slow down) for all $t > 0$.

33. If we write (for the general case) $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r}\times\vec{v}$ is equal to

$$\left(yv_z - zv_y\right)\hat{\mathrm{i}} + \left(zv_x - xv_z\right)\hat{\mathrm{j}} + \left(xv_y - yv_x\right)\hat{\mathrm{k}}.$$

(a) The angular momentum is given by the vector product $\vec{\ell} = m\vec{r}\times\vec{v}$, where $\vec{r}$ is the position vector of the particle, $\vec{v}$ is its velocity, and m = 3.0 kg is its mass. Substituting (with SI units understood) $x = 3$, $y = 8$, $z = 0$, $v_x = 5$, $v_y = -6$ and $v_z = 0$ into the above expression, we obtain

$$\vec{\ell} = (3.0)\,[(3.0)(-6.0) - (8.0)(5.0)]\hat{\mathrm{k}} = (-1.7\times10^2\ \mathrm{kg\cdot m^2/s})\hat{\mathrm{k}}.$$

(b) The torque is given by Eq. 11-14, $\vec{\tau} = \vec{r}\times\vec{F}$. We write $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}}$ and $\vec{F} = F_x\hat{\mathrm{i}}$ and obtain

$$\vec{\tau} = \left(x\hat{\mathrm{i}} + y\hat{\mathrm{j}}\right)\times\left(F_x\hat{\mathrm{i}}\right) = -yF_x\hat{\mathrm{k}}$$

since $\hat{\mathrm{i}}\times\hat{\mathrm{i}} = 0$ and $\hat{\mathrm{j}}\times\hat{\mathrm{i}} = -\hat{\mathrm{k}}$. Thus, we find

$$\vec{\tau} = -\left(8.0\,\mathrm{m}\right)\left(-7.0\,\mathrm{N}\right)\hat{\mathrm{k}} = (56\,\mathrm{N\cdot m})\hat{\mathrm{k}}.$$

(c) According to Newton's second law $\vec{\tau} = d\vec{\ell}/dt$, so the rate of change of the angular momentum is 56 $\mathrm{kg\cdot m^2/s^2}$, in the positive z direction.

34. The rate of change of the angular momentum is

$$\frac{d\vec{\ell}}{dt} = \vec{\tau}_1 + \vec{\tau}_2 = (2.0\ \text{N}\cdot\text{m})\hat{\text{i}} - (4.0\ \text{N}\cdot\text{m})\hat{\text{j}}.$$

Consequently, the vector $d\vec{\ell}/dt$ has a magnitude $\sqrt{(2.0\ \text{N}\cdot\text{m})^2 + (-4.0\ \text{N}\cdot\text{m})^2} = 4.5\,\text{N}\cdot\text{m}$ and is at an angle θ (in the *xy* plane, or a plane parallel to it) measured from the positive x axis, where

$$\theta = \tan^{-1}\left(\frac{-4.0\ \text{N}\cdot\text{m}}{2.0\ \text{N}\cdot\text{m}}\right) = -63°\,,$$

the negative sign indicating that the angle is measured clockwise as viewed "from above" (by a person on the $+z$ axis).

35. (a) We note that

$$\vec{v} = \frac{d\,\vec{r}}{dt} = 8.0t\,\hat{\mathrm{i}} - (2.0 + 12t)\hat{\mathrm{j}}$$

with SI units understood. From Eq. 11-18 (for the angular momentum) and Eq. 3-30, we find the particle's angular momentum is $8t^2\,\hat{\mathrm{k}}$. Using Eq. 11-23 (relating its time-derivative to the (single) torque) then yields $\vec{\tau} = (48t\,\hat{\mathrm{k}})\ \mathrm{N\cdot m}$.

(b) From our (intermediate) result in part (a), we see the angular momentum increases in proportion to t^2.

36. (a) Eq. 10-34 gives $\alpha = \tau/I$ and Eq. 10-12 leads to $\omega = \alpha t = \tau t/I$. Therefore, the angular momentum at $t = 0.033$ s is

$$I\omega = \tau\, t = (16\,\mathrm{N\cdot m})(0.033\,\mathrm{s}) = 0.53\,\mathrm{kg\cdot m^2/s}$$

where this is essentially a derivation of the angular version of the impulse-momentum theorem.

(b) We find

$$\omega = \frac{\tau\, t}{I} = \frac{(16\,\mathrm{N\cdot m})(0.033\,\mathrm{s})}{1.2\times10^{-3}\,\mathrm{kg\cdot m^2}} = 440\,\mathrm{rad/s}$$

which we convert as follows: ω = (440 rad/s)(60 s/min)(1 rev/2π rad) ≈ 4.2×10^3 rev/min.

37. (a) Since $\tau = dL/dt$, the average torque acting during any interval Δt is given by $\tau_{\text{avg}} = \left(L_f - L_i\right)/\Delta t$, where L_i is the initial angular momentum and L_f is the final angular momentum. Thus,

$$\tau_{\text{avg}} = \frac{0.800\ \text{kg}\cdot\text{m}^2/\text{s} - 3.00\ \text{kg}\cdot\text{m}^2/\text{s}}{1.50\,\text{s}} = -1.47\ \text{N}\cdot\text{m},$$

or $|\tau_{\text{avg}}| = 1.47\ \text{N}\cdot\text{m}$. In this case the negative sign indicates that the direction of the torque is opposite the direction of the initial angular momentum, implicitly taken to be positive.

(b) The angle turned is $\theta = \omega_0 t + \alpha t^2/2$. If the angular acceleration α is uniform, then so is the torque and $\alpha = \tau/I$. Furthermore, $\omega_0 = L_i/I$, and we obtain

$$\theta = \frac{L_i t + \tau t^2/2}{I} = \frac{\left(3.00\,\text{kg}\cdot\text{m}^2/\text{s}\right)\left(1.50\,\text{s}\right) + \left(-1.467\,\text{N}\cdot\text{m}\right)\left(1.50\,\text{s}\right)^2/2}{0.140\,\text{kg}\cdot\text{m}^2} = 20.4\,\text{rad}.$$

(c) The work done on the wheel is

$$W = \tau\theta = \left(-1.47\,\text{N}\cdot\text{m}\right)\left(20.4\,\text{rad}\right) = -29.9\ \text{J}$$

where more precise values are used in the calculation than what is shown here. An equally good method for finding W is Eq. 10-52, which, if desired, can be rewritten as $W = \left(L_f^2 - L_i^2\right)/2I$.

(d) The average power is the work done by the flywheel (the negative of the work done on the flywheel) divided by the time interval:

$$P_{\text{avg}} = -\frac{W}{\Delta t} = -\frac{-29.8\ \text{J}}{1.50\,\text{s}} = 19.9\,\text{W}.$$

38. We relate the motions of the various disks by examining their linear speeds (using Eq. 10-18). The fact that the linear speed at the rim of disk A must equal the linear speed at the rim of disk C leads to $\omega_A = 2\omega_C$. The fact that the linear speed at the hub of disk A must equal the linear speed at the rim of disk B leads to $\omega_A = \frac{1}{2}\omega_B$. Thus, $\omega_B = 4\omega_C$. The ratio of their angular momenta depend on these angular velocities as well as their rotational inertias (see item (c) in Table 11-2), which themselves depend on their masses. If h is the thickness and ρ is the density of each disk, then each mass is $\rho\pi R^2 h$. Therefore,

$$\frac{L_C}{L_B} = \frac{(½)\rho\pi R_C^2\, h\, R_C^2\, \omega_C}{(½)\rho\pi R_B^2\, h\, R_B^2\, \omega_B} = 1024 .$$

39. (a) A particle contributes mr_2 to the rotational inertia. Here r is the distance from the origin O to the particle. The total rotational inertia is

$$I = m(3d)^2 + m(2d)^2 + m(d)^2 = 14md^2 = 14(2.3\times10^{-2}\,\text{kg})(0.12\text{ m})^2$$
$$= 4.6\times10^{-3}\ \text{kg}\cdot\text{m}^2.$$

(b) The angular momentum of the middle particle is given by $L_m = I_m\omega$, where $I_m = 4md^2$ is its rotational inertia. Thus

$$L_m = 4md^2\omega = 4(2.3\times10^{-2}\,\text{kg})(0.12\text{ m})^2(0.85\text{ rad/s}) = 1.1\times10^{-3}\ \text{kg}\cdot\text{m}^2/\text{s}.$$

(c) The total angular momentum is

$$I\omega = 14md^2\omega = 14(2.3\times10^{-2}\,\text{kg})(0.12\text{ m})^2(0.85\text{ rad/s}) = 3.9\times10^{-3}\ \text{kg}\cdot\text{m}^2/\text{s}.$$

40. The results may be found by integrating Eq. 11-29 with respect to time, keeping in mind that $\vec{L}_i = 0$ and that the integration may be thought of as "adding the areas" under the line-segments (in the plot of the torque versus time – with "areas" under the time axis contributing negatively). It is helpful to keep in mind, also, that the area of a triangle is $\frac{1}{2}$ (base)(height).

(a) We find that $\vec{L} = 24\ \text{kg·m}^2/\text{s}$ at $t = 7.0$ s.

(b) Similarly, $\vec{L} = 1.5\ \text{kg·m}^2/\text{s}$ at $t = 20$ s.

41. (a) For the hoop, we use Table 10-2(h) and the parallel-axis theorem to obtain

$$I_1 = I_{\text{com}} + mh^2 = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2 .$$

Of the thin bars (in the form of a square), the member along the rotation axis has (approximately) no rotational inertia about that axis (since it is thin), and the member farthest from it is very much like it (by being parallel to it) except that it is displaced by a distance *h*; it has rotational inertia given by the parallel axis theorem:

$$I_2 = I_{\text{com}} + mh^2 = 0 + mR^2 = mR^2 .$$

Now the two members of the square perpendicular to the axis have the same rotational inertia (that is $I_3 = I_4$). We find I_3 using Table 10-2(e) and the parallel-axis theorem:

$$I_3 = I_{\text{com}} + mh^2 = \frac{1}{12}mR^2 + m\left(\frac{R}{2}\right)^2 = \frac{1}{3}mR^2 .$$

Therefore, the total rotational inertia is

$$I_1 + I_2 + I_3 + I_4 = \frac{19}{6}mR^2 = 1.6\,\text{kg}\cdot\text{m}^2 .$$

(b) The angular speed is constant:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{2.5} = 2.5\,\text{rad/s}.$$

Thus, $L = I_{\text{total}}\,\omega = 4.0\,\text{kg}\cdot\text{m}^2/\text{s}$.

42. Torque is the time derivative of the angular momentum. Thus, the change in the angular momentum is equal to the time integral of the torque. With $\tau = (5.00 + 2.00t)\ \mathrm{N \cdot m}$, the angular momentum as a function of time is (in units $\mathrm{kg \cdot m^2/s}$)

$$L(t) = \int \tau dt = \int (5.00 + 2.00t) dt = L_0 + 5.00t + 1.00t^2$$

Since $L = 5.00\ \mathrm{kg \cdot m^2/s}$ when $t = 1.00\ \mathrm{s}$, the integration constant is $L_0 = -1$. Thus, the complete expression of the angular momentum is

$$L(t) = -1 + 5.00t + 1.00t^2 .$$

At $t = 3.00\ \mathrm{s}$, we have $L(t = 3.00) = -1 + 5.00(3.00) + 1.00(3.00)^2 = 23.0\ \mathrm{kg \cdot m^2/s}$.

43. (a) No external torques act on the system consisting of the man, bricks, and platform, so the total angular momentum of the system is conserved. Let I_i be the initial rotational inertia of the system and let I_f be the final rotational inertia. Then $I_i\omega_i = I_f\omega_f$ and

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{6.0\,\text{kg}\cdot\text{m}^2}{2.0\,\text{kg}\cdot\text{m}^2}\right)(1.2\,\text{rev/s}) = 3.6\,\text{rev/s}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}I_i\omega_i^2$, the final kinetic energy is $K_f = \frac{1}{2}I_f\omega_f^2$, and their ratio is

$$\frac{K_f}{K_i} = \frac{I_f\omega_f^2/2}{I_i\omega_i^2/2} = \frac{\left(2.0\,\text{kg}\cdot\text{m}^2\right)(3.6\,\text{rev/s})^2/2}{\left(6.0\,\text{kg}\cdot\text{m}^2\right)(1.2\,\text{rev/s})^2/2} = 3.0.$$

(c) The man did work in decreasing the rotational inertia by pulling the bricks closer to his body. This energy came from the man's store of internal energy.

44. We use conservation of angular momentum:

$$I_m \omega_m = I_p \omega_p.$$

The respective angles θ_m and θ_p by which the motor and probe rotate are therefore related by

$$\int I_m \omega_m dt = I_m \theta_m = \int I_p \omega_p dt = I_p \theta_p$$

which gives

$$\theta_m = \frac{I_p \theta_p}{I_m} = \frac{\left(12\,\text{kg}\cdot\text{m}^2\right)\left(30°\right)}{2.0\times10^{-3}\,\text{kg}\cdot\text{m}^2} = 180000°.$$

The number of revolutions for the rotor is then (1.8×10^5)º/(360º/rev) = 5.0×10^2 rev.

45. (a) No external torques act on the system consisting of the two wheels, so its total angular momentum is conserved. Let I_1 be the rotational inertia of the wheel that is originally spinning $(\text{at } \omega_i)$ and I_2 be the rotational inertia of the wheel that is initially at rest. Then $I_1\,\omega_i = (I_1 + I_2)\,\omega_f$ and

$$\omega_f = \frac{I_1}{I_1 + I_2}\omega_i$$

where ω_f is the common final angular velocity of the wheels. Substituting $I_2 = 2I_1$ and $\omega_i = 800 \text{ rev/min}$, we obtain $\omega_f = 267 \text{ rev/min}$.

(b) The initial kinetic energy is $K_i = \frac{1}{2}I_1\omega_i^2$ and the final kinetic energy is $K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2$. We rewrite this as

$$K_f = \frac{1}{2}(I_1 + 2I_1)\left(\frac{I_1\omega_i}{I_1 + 2I_1}\right)^2 = \frac{1}{6}I\omega_i^2.$$

Therefore, the fraction lost, $(K_i - K_f)/K_i$ is

$$1 - \frac{K_f}{K_i} = 1 - \frac{I\omega_i^2/6}{I\omega_i^2/2} = \frac{2}{3} = 0.667.$$

46. Using Eq. 11-31 with angular momentum conservation, $\vec{L}_i = \vec{L}_f$ (Eq. 11-33) leads to the ratio of rotational inertias being inversely proportional to the ratio of angular velocities. Thus, I_f/I_i = 6/5 = 1.0 + 0.2. We interpret the "1.0" as the ratio of disk rotational inertias (which does not change in this problem) and the "0.2" as the ratio of the roach rotational inertial to that of the disk. Thus, the answer is 0.20.

47. (a) We apply conservation of angular momentum: $I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$. The angular speed after coupling is therefore

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{\left(3.3\,\text{kg}\cdot\text{m}^2\right)\left(450\text{ rev/min}\right) + \left(6.6\,\text{kg}\cdot\text{m}^2\right)\left(900\text{ rev/min}\right)}{3.3\,\text{kg}\cdot\text{m}^2 + 6.6\,\text{kg}\cdot\text{m}^2}$$
$$= 750\text{ rev/min}.$$

(b) In this case, we obtain

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{\left(3.3\text{ kg}\cdot\text{m}^2\right)\left(450\text{ rev/min}\right) + \left(6.6\text{ kg}\cdot\text{m}^2\right)\left(-900\text{ rev/min}\right)}{3.3\text{ kg}\cdot\text{m}^2 + 6.6\text{ kg}\cdot\text{m}^2}$$
$$= -450\ \text{ rev/min}$$

or $|\,\omega| = 450\ \text{ rev/min}$.

(c) The minus sign indicates that $\vec{\omega}$ is in the direction of the second disk's initial angular velocity - clockwise.

48. Angular momentum conservation $I_i\omega_i = I_f\omega_f$ leads to

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f}\omega_i = 3$$

which implies

$$\frac{K_f}{K_i} = \frac{I_f\omega_f^2/2}{I_i\omega_i^2/2} = \frac{I_f}{I_i}\left(\frac{\omega_f}{\omega_i}\right)^2 = 3.$$

49. No external torques act on the system consisting of the train and wheel, so the total angular momentum of the system (which is initially zero) remains zero. Let $I = MR^2$ be the rotational inertia of the wheel. Its final angular momentum is

$$\vec{L}_f = I\omega\hat{k} = -M\,R^2|\omega|\,\hat{k},$$

where $\hat{k}$ is *up* in Fig. 11-47 and that last step (with the minus sign) is done in recognition that the wheel's clockwise rotation implies a negative value for ω. The linear speed of a point on the track is ωR and the speed of the train (going counterclockwise in Fig. 11-47 with speed v' relative to an outside observer) is therefore $v' = v - |\omega|\,R$ where v is its speed relative to the tracks. Consequently, the angular momentum of the train is $m\left(v - |\omega|\,R\right)R\,\hat{k}$. Conservation of angular momentum yields

$$0 = -MR^2|\omega|\hat{k} + m\left(v - |\omega|R\right)R\hat{k}.$$

When this equation is solved for the angular speed, the result is

$$|\omega| = \frac{mvR}{(M+m)R^2} = \frac{v}{(M/m+1)R} = \frac{(0.15\text{ m/s})}{(1.1+1)(0.43\text{ m})} = 0.17\text{ rad/s}.$$

50. So that we don't get confused about ± signs, we write the angular *speed* to the lazy Susan as $|\omega|$ and reserve the ω symbol for the angular velocity (which, using a common convention, is negative-valued when the rotation is clockwise). When the roach "stops" we recognize that it comes to rest relative to the lazy Susan (not relative to the ground).

(a) Angular momentum conservation leads to

$$mvR + I\omega_0 = \left(mR^2 + I\right)\omega_f$$

which we can write (recalling our discussion about angular speed versus angular velocity) as

$$mvR - I|\omega_0| = -\left(mR^2 + I\right)|\omega_f|.$$

We solve for the final angular speed of the system:

$$|\omega_f| = \frac{mvR - I|\omega_0|}{mR^2 + I} = \frac{(0.17\text{ kg})(2.0\text{ m/s})(0.15\text{ m}) - (5.0\times10^{-3}\text{ kg}\cdot\text{m}^2)(2.8\text{ rad/s})}{(5.0\times10^{-3}\text{ kg}\cdot\text{m}^2) + (0.17\text{ kg})(0.15\text{ m})^2}$$
$$= 4.2\text{ rad/s}.$$

(b) No, $K_f \neq K_i$ and — if desired — we can solve for the difference:

$$K_i - K_f = \frac{mI}{2}\frac{v^2 + \omega_0^2 R^2 + 2Rv|\omega_0|}{mR^2 + I}$$

which is clearly positive. Thus, some of the initial kinetic energy is "lost" — that is, transferred to another form. And the culprit is the roach, who must find it difficult to stop (and "internalize" that energy).

51. We assume that from the moment of grabbing the stick onward, they maintain rigid postures so that the system can be analyzed as a symmetrical rigid body with center of mass midway between the skaters.

(a) The total linear momentum is zero (the skaters have the same mass and equal-and-opposite velocities). Thus, their center of mass (the middle of the 3.0 m long stick) remains fixed and they execute circular motion (of radius r = 1.5 m) about it.

(b) Using Eq. 10-18, their angular velocity (counterclockwise as seen in Fig. 11-48) is

$$\omega = \frac{v}{r} = \frac{1.4 \text{ m/s}}{1.5 \text{ m}} = 0.93 \text{ rad/s}.$$

(c) Their rotational inertia is that of two particles in circular motion at r = 1.5 m, so Eq. 10-33 yields

$$I = \sum mr^2 = 2(50 \text{ kg})(1.5 \text{ m})^2 = 225 \text{ kg}\cdot\text{m}^2.$$

Therefore, Eq. 10-34 leads to

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(225 \text{ kg}\cdot\text{m}^2\right)(0.93 \text{ rad/s})^2 = 98 \text{ J}.$$

(d) Angular momentum is conserved in this process. If we label the angular velocity found in part (a) ω_i and the rotational inertia of part (b) as I_i, we have

$$I_i\omega_i = \left(225 \text{ kg}\cdot\text{m}^2\right)(0.93 \text{ rad/s}) = I_f\omega_f.$$

The final rotational inertia is $\sum mr_f^2$ where r_f = 0.5 m so $I_f = 25 \text{ kg}\cdot\text{m}^2$. Using this value, the above expression gives ω_f = 8.4 rad/s.

(e) We find

$$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}\left(25 \text{ kg}\cdot\text{m}^2\right)(8.4 \text{ rad/s})^2 = 8.8\times10^2 \text{ J}.$$

(f) We account for the large increase in kinetic energy (part (e) minus part (c)) by noting that the skaters do a great deal of work (converting their internal energy into mechanical energy) as they pull themselves closer — "fighting" what appears to them to be large "centrifugal forces" trying to keep them apart.

52. The gravitational force acts at the center of mass and cannot provide a torque to change the bola's angular momentum during the flight. So, the angular momentum before and after the configuration change must be equal. We treat both configurations as a rigid object rotating around a fixed point. The initial and final rotational inertias are

$$I_i = m(2\ell)^2 + m(2\ell)^2 + m(0)^2 = 8m\ell^2$$
$$I_f = m\ell^2 + m\ell^2 + m\ell^2 = 3m\ell^2.$$

(a) Since angular momentum is conserved, $L_i = L_f$, or $I_i\omega_i = I_f\omega_f$. Thus,

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f} = \frac{8m\ell^2}{3m\ell^2} = \frac{8}{3} = 2.7.$$

(b) The initial and final kinetic energies are $K_i = I_i\omega_i^2/2$ and $K_f = I_f\omega_f^2/2$, respectively. Thus, we find the ratio to be

$$\frac{K_f}{K_i} = \frac{I_f\omega_f^2/2}{I_i\omega_i^2/2} = \frac{I_f}{I_i}\left(\frac{\omega_f}{\omega_i}\right)^2 = \frac{I_f}{I_i}\left(\frac{I_i}{I_f}\right)^2 = \frac{I_i}{I_f} = \frac{8}{3} = 2.7.$$

53. For simplicity, we assume the record is turning freely, without any work being done by its motor (and without any friction at the bearings or at the stylus trying to slow it down). Before the collision, the angular momentum of the system (presumed positive) is $I_i\omega_i$ where $I_i = 5.0\times10^{-4}\ \text{kg}\cdot\text{m}^2$ and $\omega_i = 4.7\ \text{rad/s}$. The rotational inertia afterwards is

$$I_f = I_i + mR^2$$

where m = 0.020 kg and R = 0.10 m. The mass of the record (0.10 kg), although given in the problem, is not used in the solution. Angular momentum conservation leads to

$$I_i\omega_i = I_f\omega_f \Rightarrow \omega_f = \frac{I_i\omega_i}{I_i + mR^2} = 3.4\ \text{rad / s}.$$

54. Table 10-2 gives the rotational inertia of a thin rod rotating about a perpendicular axis through its center. The angular speeds of the two arms are, respectively,

$$\omega_1 = \frac{(0.500 \text{ rev})(2\pi \text{ rad/rev})}{0.700 \text{ s}} = 4.49 \text{ rad/s}$$
$$\omega_2 = \frac{(1.00 \text{ rev})(2\pi \text{ rad/rev})}{0.700 \text{ s}} = 8.98 \text{ rad/s}.$$

Treating each arm as a thin rod of mass 4.0 kg and length 0.60 m, the angular momenta of the two arms are

$$L_1 = I\omega_1 = mr^2\omega_1 = (4.0 \text{ kg})(0.60 \text{ m})^2(4.49 \text{ rad/s}) = 6.46 \text{ kg}\cdot\text{m}^2/\text{s}$$
$$L_2 = I\omega_2 = mr^2\omega_2 = (4.0 \text{ kg})(0.60 \text{ m})^2(8.98 \text{ rad/s}) = 12.92 \text{ kg}\cdot\text{m}^2/\text{s}.$$

From the athlete's reference frame, one arm rotates clockwise, while the other rotates counterclockwise. Thus, the total angular momentum about the common rotation axis though the shoulders is

$$L = L_2 - L_1 = 12.92 \text{ kg}\cdot\text{m}^2/\text{s} - 6.46 \text{ kg}\cdot\text{m}^2/\text{s} = 6.46 \text{ kg}\cdot\text{m}^2/\text{s}.$$

55. The axis of rotation is in the middle of the rod, with r = 0.25 m from either end. By Eq. 11-19, the initial angular momentum of the system (which is just that of the bullet, before impact) is $rmv \sin\theta$ where m = 0.003 kg and θ = 60°. Relative to the axis, this is counterclockwise and thus (by the common convention) positive. After the collision, the moment of inertia of the system is

$$I = I_{\text{rod}} + mr^2$$

where $I_{\text{rod}} = ML^2/12$ by Table 10-2(e), with M = 4.0 kg and L = 0.5 m. Angular momentum conservation leads to

$$rmv \sin\theta = \left(\frac{1}{12}ML^2 + mr^2\right)\omega.$$

Thus, with ω = 10 rad/s, we obtain

$$v = \frac{\left(\frac{1}{12}(4.0\text{ kg})(0.5\text{ m})^2 + (0.003\text{ kg})(0.25\text{ m})^2\right)(10\text{ rad/s})}{(0.25\text{ m})(0.003\text{ kg})\sin 60°} = 1.3\times10^3\text{ m/s}.$$

56. We denote the cockroach with subscript 1 and the disk with subscript 2. The cockroach has a mass $m_1 = m$, while the mass of the disk is $m_2 = 4.00\ m$.

(a) Initially the angular momentum of the system consisting of the cockroach and the disk is

$$L_i = m_1 v_{1i} r_{1i} + I_2 \omega_{2i} = m_1 \omega_0 R^2 + \frac{1}{2} m_2 \omega_0 R^2 .$$

After the cockroach has completed its walk, its position (relative to the axis) is $r_{1f} = R/2$ so the final angular momentum of the system is

$$L_f = m_1 \omega_f \left(\frac{R}{2}\right)^2 + \frac{1}{2} m_2 \omega_f R^2 .$$

Then from $L_f = L_i$ we obtain

$$\omega_f \left(\frac{1}{4} m_1 R^2 + \frac{1}{2} m_2 R\right) = \omega_0 \left(m_1 R^2 + \frac{1}{2} m_2 R^2\right).$$

Thus,

$$\omega_f = \left(\frac{m_1 R^2 + m_2 R^2/2}{m_1 R^2/4 + m_2 R^2/2}\right)\omega_0 = \left(\frac{1 + (m_2/m_1)/2}{1/4 + (m_2/m_1)/2}\right)\omega_0 = \left(\frac{1+2}{1/4\ +\ 2}\right)\omega_0 = 1.33\omega_0 .$$

With $\omega_0 = 0.260$ rad/s, we have $\omega_f = 0.347$ rad/s.

(b) We substitute $I = L/\omega$ into $K = \frac{1}{2} I \omega^2$ and obtain $K = \frac{1}{2} L\omega$. Since we have $L_i = L_f$, the kinetic energy ratio becomes

$$\frac{K}{K_0} = \frac{L_f \omega_f / 2}{L_i \omega_i / 2} = \frac{\omega_f}{\omega_0} = 1.33 .$$

(c) The cockroach does positive work while walking toward the center of the disk, increasing the total kinetic energy of the system.

57. By angular momentum conservation (Eq. 11-33), the total angular momentum after the explosion must be equal to before the explosion:

$$L'_p + L'_r = L_p + L_r$$

$$\left(\frac{L}{2}\right) m v_p + \frac{1}{12} M L^2 \, \omega' = I_p \, \omega + \frac{1}{12} M L^2 \, \omega$$

where one must be careful to avoid confusing the length of the rod (L = 0.800 m) with the angular momentum symbol. Note that $I_p = m(L/2)^2$ by Eq.10-33, and

$$\omega' = v_{end}/r = (v_p - 6)/(L/2),$$

where the latter relation follows from the penultimate sentence in the problem (and "6" stands for "6.00 m/s" here). Since $M = 3m$ and $\omega = 20$ rad/s, we end up with enough information to solve for the particle speed: $v_p = 11.0$ m/s.

58. (a) With $r = 0.60$ m, we obtain $I = 0.060 + (0.501)r^2 = 0.24 \text{ kg} \cdot \text{m}^2$.

(b) Invoking angular momentum conservation, with SI units understood,

$$\ell_0 = L_f \quad \Rightarrow \quad mv_0 r = I\omega \quad \Rightarrow \quad (0.001)\, v_0\, (0.60) = (0.24)(4.5)$$

which leads to $v_0 = 1.8 \times 10^3$ m/s.

59. Their angular velocities, when they are stuck to each other, are equal, regardless of whether they share the same central axis. The initial rotational inertia of the system is

$$I_0 = I_{\text{big disk}} + I_{\text{small disk}} \qquad \text{where } I_{\text{big disk}} = \frac{1}{2}MR^2$$

using Table 10-2(c). Similarly, since the small disk is initially concentric with the big one, $I_{\text{small disk}} = \frac{1}{2}mr^2$. After it slides, the rotational inertia of the small disk is found from the parallel axis theorem (using $h = R - r$). Thus, the new rotational inertia of the system is

$$I = \frac{1}{2}MR^2 + \frac{1}{2}mr^2 + m\left(R - r\right)^2.$$

(a) Angular momentum conservation, $I_0\omega_0 = I\omega$, leads to the new angular velocity:

$$\omega = \omega_0 \frac{(MR^2/2) + (mr^2/2)}{(MR^2/2) + (mr^2/2) + m\left(R - r\right)^2}.$$

Substituting $M = 10m$ and $R = 3r$, this becomes $\omega = \omega_0(91/99)$. Thus, with $\omega_0 = 20$ rad/s, we find $\omega = 18$ rad/s.

(b) From the previous part, we know that

$$\frac{I_0}{I} = \frac{91}{99} \qquad \text{and} \qquad \frac{\omega}{\omega_0} = \frac{91}{99}.$$

Plugging these into the ratio of kinetic energies, we have

$$\frac{K}{K_0} = \frac{I\omega^2/2}{I_0\omega_0^2/2} = \frac{I}{I_0}\left(\frac{\omega}{\omega_0}\right)^2 = \frac{99}{91}\left(\frac{91}{99}\right)^2 = 0.92.$$

60. The initial rotational inertia of the system is $I_i = I_{\text{disk}} + I_{\text{student}}$, where $I_{\text{disk}} = 300$ kg$\cdot$m^2 (which, incidentally, does agree with Table 10-2(c)) and $I_{\text{student}} = mR^2$ where $m =$ 60 kg and $R = 2.0$ m.

The rotational inertia when the student reaches $r = 0.5$ m is $I_f = I_{\text{disk}} + mr^2$. Angular momentum conservation leads to

$$I_i\omega_i = I_f\omega_f \Rightarrow \omega_f = \omega_i \frac{I_{\text{disk}} + mR^2}{I_{\text{disk}} + mr^2}$$

which yields, for $\omega_i = 1.5$ rad/s, a final angular velocity of $\omega_f = 2.6$ rad/s.

61. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocities in this problem are positive. With $r = 0.60$ m and $I_0 = 0.12$ kg · m^2, the rotational inertia of the putty-rod system (after the collision) is

$$I = I_0 + (0.20)r^2 = 0.19 \text{ kg} \cdot \text{m}^2.$$

Invoking angular momentum conservation $L_0 = L_f$ or $I_0\omega_0 = I\omega$, we have

$$\omega = \frac{I_0}{I}\omega_0 = \frac{0.12 \text{ kg} \cdot \text{m}^2}{0.19 \text{ kg} \cdot \text{m}^2}(2.4 \text{ rad/s}) = 1.5 \text{ rad/s}.$$

62. We treat the ballerina as a rigid object rotating around a fixed axis, initially and then again near maximum height. Her initial rotational inertia (trunk and one leg extending outward at a $90°$ angle) is

$$I_i = I_{\text{trunk}} + I_{\text{leg}} = 0.660\ \text{kg}\cdot\text{m}^2 + 1.44\ \text{kg}\cdot\text{m}^2 = 2.10\ \text{kg}\cdot\text{m}^2.$$

Similarly, her final rotational inertia (trunk and *both* legs extending outward at a $\theta = 30°$ angle) is

$$I_f = I_{\text{trunk}} + 2I_{\text{leg}}\sin^2\theta = 0.660\ \text{kg}\cdot\text{m}^2 + 2(1.44\ \text{kg}\cdot\text{m}^2)\sin^2 30° = 1.38\ \text{kg}\cdot\text{m}^2,$$

where we have used the fact that the effective length of the extended leg at an angle θ is $L_\perp = L\sin\theta$ and $I \sim L_\perp^2$. Once air-borne, there is no external torque about the ballerina's center of mass and her angular momentum cannot change. Therefore, $L_i = L_f$ or $I_i\omega_i = I_f\omega_f$, and the ratio of the angular speeds is

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f} = \frac{2.10\ \text{kg}\cdot\text{m}^2}{1.38\ \text{kg}\cdot\text{m}^2} = 1.52.$$

63. (a) We consider conservation of angular momentum (Eq. 11-33) about the center of the rod:

$$L_i = L_f \;\Rightarrow\; -dmv + \frac{1}{12}ML^2\omega = 0$$

where negative is used for "clockwise." Item (e) in Table 11-2 and Eq. 11-21 (with $r_\perp = d$) have also been used. This leads to

$$d = \frac{ML^2\,\omega}{12\,m\,\text{v}} = \frac{M(0.60\text{ m})^2\,(80\text{ rad/s})}{12(M/3)(40\text{ m/s})} = 0.180\text{ m}.$$

(b) Increasing d causes the magnitude of the negative (clockwise) term in the above equation to increase. This would make the total angular momentum negative before the collision, and (by Eq. 11-33) also negative afterwards. Thus, the system would rotate clockwise if d were greater.

64. The aerialist is in extended position with $I_1 = 19.9\ \mathrm{kg\cdot m^2}$ during the first and last quarter of the turn, so the total angle rotated in t_1 is $\theta_1 = 0.500$ rev. In t_2 he is in a tuck position with $I_2 = 3.93\ \mathrm{kg\cdot m^2}$, and the total angle rotated is $\theta_2 = 3.500$ rev. Since there is no external torque about his center of mass, angular momentum is conserved, $I_1\omega_1 = I_2\omega_2$. Therefore, the total flight time can be written as

$$t = t_1 + t_2 = \frac{\theta_1}{\omega_1} + \frac{\theta_2}{\omega_2} = \frac{\theta_1}{I_2\omega_2 / I_1} + \frac{\theta_2}{\omega_2} = \frac{1}{\omega_2}\left(\frac{I_1}{I_2}\theta_1 + \theta_2\right).$$

Substituting the values given, we find ω_2 to be

$$\omega_2 = \frac{1}{t}\left(\frac{I_1}{I_2}\theta_1 + \theta_2\right) = \frac{1}{1.87\ \mathrm{s}}\left(\frac{19.9\ \mathrm{kg\cdot m^2}}{3.93\ \mathrm{kg\cdot m^2}}(0.500\ \mathrm{rev}) + 3.50\ \mathrm{rev}\right) = 3.23\ \mathrm{rev/s}.$$

65. This is a completely inelastic collision which we analyze using angular momentum conservation. Let m and v_0 be the mass and initial speed of the ball and R the radius of the merry-go-round. The initial angular momentum is

$$\vec{\ell}_0 = \vec{r}_0 \times \vec{p}_0 \Rightarrow \ell_0 = R\left(mv_0\right)\cos 37°$$

where ϕ=37° is the angle between $\vec{v}_0$ and the line tangent to the outer edge of the merry-go-around. Thus, $\ell_0 = 19\,\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}$. Now, with SI units understood,

$$\ell_0 = L_f \quad \Rightarrow \quad 19\ \mathrm{kg}\cdot\mathrm{m}^2 = I\omega = \left(150 + (30)R^2 + (1.0)R^2\right)\omega$$

so that ω= 0.070 rad/s.

66. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocities (and angles) in this problem are positive. Mechanical energy conservation applied to the particle (before impact) leads to

$$mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}$$

for its speed right before undergoing the completely inelastic collision with the rod. The collision is described by angular momentum conservation:

$$mvd = \left(I_{\text{rod}} + md^2\right)\omega$$

where I_{rod} is found using Table 10-2(e) and the parallel axis theorem:

$$I_{\text{rod}} = \frac{1}{12}Md^2 + M\left(\frac{d}{2}\right)^2 = \frac{1}{3}Md^2.$$

Thus, we obtain the angular velocity of the system immediately after the collision:

$$\omega = \frac{md\sqrt{2gh}}{(Md^2/3) + md^2}$$

which means the system has kinetic energy $\left(I_{\text{rod}} + md^2\right)\omega^2/2$ which will turn into potential energy in the final position, where the block has reached a height H (relative to the lowest point) and the center of mass of the stick has increased its height by $H/2$. From trigonometric considerations, we note that $H = d(1 - \cos\theta)$, so we have

$$\frac{1}{2}\left(I_{\text{rod}} + md^2\right)\omega^2 = mgH + Mg\frac{H}{2} \;\;\Rightarrow\;\; \frac{1}{2}\frac{m^2d^2\left(2gh\right)}{(Md^2/3) + md^2} = \left(m + \frac{M}{2}\right)gd\left(1 - \cos\theta\right)$$

from which we obtain

$$\begin{aligned}
\theta &= \cos^{-1}\left(1 - \frac{m^2h}{\left(m + M/2\right)\left(m + M/3\right)}\right) = \cos^{-1}\left(1 - \frac{h/d}{\left(1 + M/2m\right)\left(1 + M/3m\right)}\right) \\
&= \cos^{-1}\left(1 - \frac{(20\text{ cm}/\,40\text{ cm})}{(1+1)(1+2/3)}\right) = \cos^{-1}(0.85) \\
&= 32^\circ.
\end{aligned}$$

67. (a) If we consider a short time interval from just before the wad hits to just after it hits and sticks, we may use the principle of conservation of angular momentum. The initial angular momentum is the angular momentum of the falling putty wad. The wad initially moves along a line that is $d/2$ distant from the axis of rotation, where d = 0.500 m is the length of the rod. The angular momentum of the wad is $mvd/2$ where m = 0.0500 kg and v = 3.00 m/s are the mass and initial speed of the wad. After the wad sticks, the rod has angular velocity ω and angular momentum $I\omega$, where I is the rotational inertia of the system consisting of the rod with the two balls and the wad at its end. Conservation of angular momentum yields $mvd/2 = I\omega$ where

$$I = (2M + m)(d/2)^2$$

and M = 2.00 kg is the mass of each of the balls. We solve

$$mvd/2 = (2M + m)(d/2)^2\,\omega$$

for the angular speed:

$$\omega = \frac{2mv}{(2M + m)d} = \frac{2(0.0500\text{ kg})(3.00\text{ m/s})}{(2(2.00\text{ kg}) + 0.0500\text{ kg})(0.500\text{ m})} = 0.148\text{ rad/s}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}mv^2$, the final kinetic energy is $K_f = \frac{1}{2}I\omega^2$, and their ratio is $K_f/K_i = I\omega^2/mv^2$. When $I = (2M + m)d^2/4$ and $\omega = 2mv/(2M + m)d$ are substituted, this becomes

$$\frac{K_f}{K_i} = \frac{m}{2M + m} = \frac{0.0500\text{ kg}}{2(2.00\text{ kg}) + 0.0500\text{ kg}} = 0.0123.$$

(c) As the rod rotates, the sum of its kinetic and potential energies is conserved. If one of the balls is lowered a distance h, the other is raised the same distance and the sum of the potential energies of the balls does not change. We need consider only the potential energy of the putty wad. It moves through a 90° arc to reach the lowest point on its path, gaining kinetic energy and losing gravitational potential energy as it goes. It then swings up through an angle θ, losing kinetic energy and gaining potential energy, until it momentarily comes to rest. Take the lowest point on the path to be the zero of potential energy. It starts a distance $d/2$ above this point, so its initial potential energy is $U_i = mgd/2$. If it swings up to the angular position θ, as measured from its lowest point, then its final height is $(d/2)(1 - \cos\theta)$ above the lowest point and its final potential energy is

$$U_f = mg(d/2)(1 - \cos\theta).$$

The initial kinetic energy is the sum of that of the balls and wad:

$$K_i = \frac{1}{2} I\omega^2 = \frac{1}{2}(2M+m)(d/2)^2\,\omega^2.$$

At its final position, we have $K_f = 0$. Conservation of energy provides the relation:

$$mg\frac{d}{2} + \frac{1}{2}(2M+m)\left(\frac{d}{2}\right)^2 \omega^2 = mg\frac{d}{2}(1-\cos\theta).$$

When this equation is solved for cos θ, the result is

$$\cos\theta = -\frac{1}{2}\left(\frac{2M+m}{mg}\right)\left(\frac{d}{2}\right)\omega^2 = -\frac{1}{2}\left(\frac{2(2.00\text{ kg})+0.0500\text{ kg}}{(0.0500\text{ kg})(9.8\text{ m/s}^2)}\right)\left(\frac{0.500\text{ m}}{2}\right)(0.148\text{ rad/s})^2$$
$$= -0.0226.$$

Consequently, the result for θ is 91.3°. The total angle through which it has swung is 90° + 91.3° = 181°.

68. (a) The angular speed of the top is $\omega = 30 \text{ rev/s} = 30(2\pi) \text{ rad/s}$. The precession rate of the top can be obtained by using Eq. 11-46:

$$\Omega = \frac{Mgr}{I\omega} = \frac{(0.50 \text{ kg})(9.8 \text{ m/s}^2)(0.040 \text{ m})}{(5.0\times10^{-4} \text{ kg}\cdot\text{m}^2)(60\pi \text{ rad/s})} = 2.08 \text{ rad/s} \approx 0.33 \text{ rev/s}.$$

(b) The direction of the precession is clockwise as viewed from overhead.

69. The precession rate can be obtained by using Eq. 11-46 with r =(11/2) cm = 0.055 m. Noting that $I_{\text{disk}}=MR^2/2$ and its angular speed is

$$\omega = 1000 \text{ rev/min} = \frac{2\pi(1000)}{60} \text{ rad/s} \approx 1.0\times10^2 \text{ rad/s},$$

we have

$$\Omega = \frac{Mgr}{(MR^2/2)\omega} = \frac{2gr}{R^2\omega} = \frac{2(9.8 \text{ m/s}^2)(0.055 \text{ m})}{(0.50 \text{ m})^2(1.0\times10^2 \text{ rad/s})} \approx 0.041 \text{ rad/s}.$$

70. Item (*i*) in Table 10-2 gives the moment of inertia about the center of mass in terms of width a (0.15 m) and length b (0.20 m). In using the parallel axis theorem, the distance from the center to the point about which it spins (as described in the problem) is $\sqrt{(a/4)^2 + (b/4)^2}$. If we denote the thickness as h (0.012 m) then the volume is abh, which means the mass is ρabh (where ρ = 2640 kg/m^3 is the density). We can write the kinetic energy in terms of the angular momentum by substituting $\omega = L/I$ into Eq. 10-34:

$$K = \frac{1}{2}\frac{L^2}{I} = \frac{1}{2}\frac{(0.104)^2}{\rho abh((a^2 + b^2)/12 + (a/4)^2 + (b/4)^2)} = 0.62 \text{ J} .$$

71. We denote the cat with subscript 1 and the ring with subscript 2. The cat has a mass $m_1 = M/4$, while the mass of the ring is $m_2 = M = 8.00$ kg. The moment of inertia of the ring is $I_2 = m_2(R_1^2 + R_2^2)/2$ (Table 10-2), and $I_1 = m_1 r^2$ for the cat, where r is the perpendicular distance from the axis of rotation.

Initially the angular momentum of the system consisting of the cat (at $r = R_2$) and the ring is

$$L_i = m_1 v_{1i} r_{1i} + I_2 \omega_{2i} = m_1 \omega_0 R_2^2 + \frac{1}{2} m_2 (R_1^2 + R_2^2)\omega_0 = m_1 R_2^2\, \omega_0 \left[1 + \frac{1}{2}\frac{m_2}{m_1}\left(\frac{R_1^2}{R_2^2} + 1\right)\right].$$

After the cat has crawled to the inner edge at $r = R_1$ the final angular momentum of the system is

$$L_f = m_1 \omega_f R_1^2 + \frac{1}{2} m_2 (R_1^2 + R_2^2)\omega_f = m_1 R_1^2\, \omega_f \left[1 + \frac{1}{2}\frac{m_2}{m_1}\left(1 + \frac{R_2^2}{R_1^2}\right)\right].$$

Then from $L_f = L_i$ we obtain

$$\frac{\omega_f}{\omega_0} = \left(\frac{R_2}{R_1}\right)^2 \frac{1 + \frac{1}{2}\frac{m_2}{m_1}\left(\frac{R_1^2}{R_2^2} + 1\right)}{1 + \frac{1}{2}\frac{m_2}{m_1}\left(1 + \frac{R_2^2}{R_1^2}\right)} = (2.0)^2 \frac{1 + 2(0.25 + 1)}{1 + 2(1 + 4)} = 1.273.$$

Thus, $\omega_f = 1.273\omega_0$. Using $\omega_0 = 8.00$ rad/s, we have $\omega_f = 10.2$ rad/s. By substituting $I = L/\omega$ into $K = I\omega^2/2$, we obtain $K = L\omega/2$. Since $L_i = L_f$, the kinetic energy ratio becomes

$$\frac{K_f}{K_i} = \frac{L_f \omega_f / 2}{L_i \omega_i / 2} = \frac{\omega_f}{\omega_0} = 1.273.$$

which implies $\Delta K = K_f - K_i = 0.273 K_i$. The cat does positive work while walking toward the center of the ring, increasing the total kinetic energy of the system.

Since the initial kinetic energy is given by

$$\begin{aligned} K_i &= \frac{1}{2}\left[m_1 R_2^2 + \frac{1}{2} m_2 (R_1^2 + R_2^2)\right]\omega_0^2 = \frac{1}{2} m_1 R_2^2 \omega_0^2 \left[1 + \frac{1}{2}\frac{m_2}{m_1}\left(\frac{R_1^2}{R_2^2} + 1\right)\right] \\ &= \frac{1}{2}(2.00\text{ kg})(0.800\text{ m})^2 (8.00\text{ rad/s})^2 [1 + (1/2)(4)(0.5^2 + 1)] \\ &= 143.36\text{ J}, \end{aligned}$$

the increase in kinetic energy is

$$\Delta K = (0.273)(143.36\text{ J}) = 39.1\text{ J}.$$

72. The total angular momentum (about the origin) before the collision (using Eq. 11-18 and Eq. 3-30 for each particle and then adding the terms) is

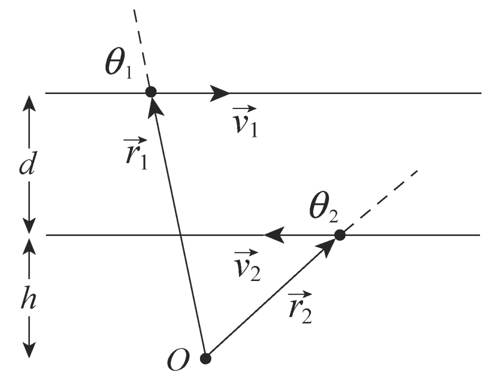
$$\vec{L}_i = [(0.5\text{ m})(2.5\text{ kg})(3.0\text{ m/s}) + (0.1\text{ m})(4.0\text{ kg})(4.5\text{ m/s})]\hat{k}.$$

The final angular momentum of the stuck-together particles (after the collision) measured relative to the origin is (using Eq. 11-33)

$$\vec{L}_f = \vec{L}_i = (5.55\text{ kg}\cdot\text{m}^2/\text{s})\hat{k}.$$

73. (a) The diagram below shows the particles and their lines of motion. The origin is marked O and may be anywhere. The angular momentum of particle 1 has magnitude

$$\ell_1 = mvr_1 \sin\theta_1 = mv(d+h)$$



and it is into the page. The angular momentum of particle 2 has magnitude

$$\ell_2 = mvr_2 \sin\theta_2 = mvh$$

and it is out of the page. The net angular momentum has magnitude

$$\begin{aligned} L &= mv(d+h) - mvh = mvd \\ &= (2.90\times10^{-4}\ \text{kg})(5.46\ \text{m/s})(0.042\ \text{m}) \\ &= 6.65\times10^{-5}\ \text{kg}\cdot\text{m}^2/\text{s}. \end{aligned}$$

and is into the page. This result is independent of the location of the origin.

(b) As indicated above, the expression does not change.

(c) Suppose particle 2 is traveling to the right. Then

$$L = mv(d+h) + mvh = mv(d+2h).$$

This result depends on h, the distance from the origin to one of the lines of motion. If the origin is midway between the lines of motion, then $h = -d/2$ and $L = 0$.

(d) As we have seen in part (c), the result depends on the choice of origin.

74. (a) We use Table 10-2(e) and the parallel-axis theorem to obtain the rod's rotational inertia about an axis through one end:

$$I = I_{\text{com}} + Mh^2 = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3}ML^2$$

where L = 6.00 m and M = 10.0/9.8 = 1.02 kg. Thus, the inertia is $I = 12.2\ \text{kg}\cdot\text{m}^2$.

(b) Using ω = (240)(2π/60) = 25.1 rad/s, Eq. 11-31 gives the magnitude of the angular momentum as

$$I\omega = \left(12.2\ \text{kg}\cdot\text{m}^2\right)\left(25.1\,\text{rad/s}\right) = 308\ \ \text{kg}\cdot\text{m}^2/\text{s}\ .$$

Since it is rotating clockwise as viewed from above, then the right-hand rule indicates that its direction is down.

75. We use $L = I\omega$ and $K = \frac{1}{2} I\omega^2$ and observe that the speed of points on the rim (corresponding to the speed of points on the belt) of wheels A and B must be the same (so $\omega_A R_A = \omega_B r_B$).

(a) If $L_A = L_B$ (call it L) then the ratio of rotational inertias is

$$\frac{I_A}{I_B} = \frac{L/\omega_A}{L/\omega_B} = \frac{\omega_A}{\omega_B} = \frac{R_A}{R_B} = \frac{1}{3} = 0.333.$$

(b) If we have $K_A = K_B$ (call it K) then the ratio of rotational inertias becomes

$$\frac{I_A}{I_B} = \frac{2K/\omega_A^2}{2K/\omega_B^2} = \left(\frac{\omega_B}{\omega_A}\right)^2 = \left(\frac{R_A}{R_B}\right)^2 = \frac{1}{9} = 0.111.$$

76. Both $\vec{r}$ and $\vec{v}$ lie in the *xy* plane. The position vector $\vec{r}$ has an *x* component that is a function of time (being the integral of the *x* component of velocity, which is itself time-dependent) and a *y* component that is constant ($y = -2.0$ m). In the cross product $\vec{r} \times \vec{v}$, all that matters is the *y* component of $\vec{r}$ since $v_x \neq 0$ but $v_y = 0$:

$$\vec{r} \times \vec{v} = -y v_x \hat{\mathrm{k}}.$$

(a) The angular momentum is $\vec{\ell} = m(\vec{r} \times \vec{v})$ where the mass is $m = 2.0$ kg in this case. With SI units understood and using the above cross-product expression, we have

$$\vec{\ell} = (2.0)\left(-(-2.0)\left(-6.0t^2\right)\right)\hat{\mathrm{k}} = -24t^2\hat{\mathrm{k}}$$

in $\mathrm{kg \cdot m^2/s}$. This implies the particle is moving clockwise (as observed by someone on the $+z$ axis) for $t > 0$.

(b) The torque is caused by the (net) force $\vec{F} = m\vec{a}$ where

$$\vec{a} = \frac{d\vec{v}}{dt} = (-12t\,\hat{\mathrm{i}})\mathrm{m/s^2}.$$

The remark above that only the *y* component of $\vec{r}$ still applies, since $a_y = 0$. We use $\vec{\tau} = \vec{r} \times \vec{F} = m(\vec{r} \times \vec{a})$ and obtain

$$\vec{\tau} = (2.0)\left(-(-2.0)(-12t)\right)\hat{\mathrm{k}} = \left(-48t\,\hat{\mathrm{k}}\right)\mathrm{N \cdot m}.$$

The torque on the particle (as observed by someone on the $+z$ axis) is clockwise, causing the particle motion (which was clockwise to begin with) to increase.

(c) We replace $\vec{r}$ with $\vec{r}\,'$ (measured relative to the new reference point) and note (again) that only its *y* component matters in these calculations. Thus, with $y' = -2.0 - (-3.0) = 1.0$ m, we find

$$\vec{\ell}\,' = (2.0)\left(-(1.0)\left(-6.0t^2\right)\right)\hat{\mathrm{k}} = (12t^2\,\hat{\mathrm{k}})\ \mathrm{kg \cdot m^2/s}..$$

The fact that this is positive implies that the particle is moving counterclockwise relative to the new reference point.

(d) Using $\vec{\tau}\,' = \vec{r}\,' \times \vec{F} = m(\vec{r}\,' \times \vec{a})$, we obtain

$$\vec{\tau} = (2.0)\left(-(1.0)(-12t)\right)\hat{\mathrm{k}} = (24t\,\hat{\mathrm{k}})\,\mathrm{N \cdot m}..$$

The torque on the particle (as observed by someone on the $+z$ axis) is counterclockwise, relative to the new reference point.

77. As the wheel-axel system rolls down the inclined plane by a distance *d*, the decrease in potential energy is $\Delta U = mgd\sin\theta$. This must be equal to the total kinetic energy gained:

$$mgd\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 .$$

Since the axel rolls without slipping, the angular speed is given by $\omega = v/r$, where r is the radius of the axel. The above equation then becomes

$$mgd\sin\theta = \frac{1}{2}I\omega^2\left(\frac{mr^2}{I}+1\right) = K_{\text{rot}}\left(\frac{mr^2}{I}+1\right)$$

(a) With m=10.0 kg, d = 2.00 m, r = 0.200 m, and $I = 0.600\ \text{kg}\cdot\text{m}^2$, mr^2/I =2/3, the rotational kinetic energy may be obtained as $98\ \text{J} = K_{\text{rot}}(5/3)$, or $K_{\text{rot}} = 58.8\ \text{J}$.

(b) The translational kinetic energy is $K_{\text{trans}} = (98 - 58.8)\text{J} = 39.2\ \text{J}$.

78. (a) The acceleration is given by Eq. 11-13:

$$a_{\text{com}} = \frac{g}{1 + I_{\text{com}} / MR_0^2}$$

where upward is the positive translational direction. Taking the coordinate origin at the initial position, Eq. 2-15 leads to

$$y_{\text{com}} = v_{\text{com},0}t + \frac{1}{2}a_{\text{com}}t^2 = v_{\text{com},0}t - \frac{\frac{1}{2}gt^2}{1 + I_{\text{com}} / MR_0^2}$$

where $y_{\text{com}} = -1.2$ m and $v_{\text{com},0} = -1.3$ m/s. Substituting $I_{\text{com}} = 0.000095\ \text{kg}\cdot\text{m}^2$, M = 0.12 kg, R_0 = 0.0032 m and g = 9.8 m/s^2, we use the quadratic formula and find

$$\begin{aligned}
t &= \frac{\left(1 + \frac{I_{\text{com}}}{MR_0^2}\right)\left(v_{\text{com},0} \mp \sqrt{v_{\text{com},0}^2 - \frac{2gy_{\text{com}}}{1+I_{\text{com}}/MR_0^2}}\right)}{g} \\
&= \frac{\left(1 + \frac{0.000095}{(0.12)(0.0032)^2}\right)\left(-1.3 \mp \sqrt{(1.3)^2 - \frac{2(9.8)(-1.2)}{1+0.000095/(0.12)(0.0032)^2}}\right)}{9.8} \\
&= -21.7 \text{ or } 0.885
\end{aligned}$$

where we choose t = 0.89 s as the answer.

(b) We note that the initial potential energy is $U_i = Mgh$ and h = 1.2 m (using the bottom as the reference level for computing U). The initial kinetic energy is as shown in Eq. 11-5, where the initial angular and linear speeds are related by Eq. 11-2. Energy conservation leads to

$$\begin{aligned}
K_f &= K_i + U_i = \frac{1}{2}mv_{\text{com},0}^2 + \frac{1}{2}I\left(\frac{v_{\text{com},0}}{R_0}\right)^2 + Mgh \\
&= \frac{1}{2}(0.12\ \text{kg})(1.3\ \text{m/s})^2 + \frac{1}{2}\left(9.5\times10^{-5}\ \text{kg}\cdot\text{m}^2\right)\left(\frac{1.3\ \text{m/s}}{0.0032\ \text{m}}\right)^2 + (0.12\ \text{kg})\left(9.8\ \text{m/s}^2\right)(1.2\ \text{m}) \\
&= 9.4\ \text{J}.
\end{aligned}$$

(c) As it reaches the end of the string, its center of mass velocity is given by Eq. 2-11:

$$v_{\text{com}} = v_{\text{com},0} + a_{\text{com}}t = v_{\text{com},0} - \frac{gt}{1 + I_{\text{com}} / MR_0^2}.$$

Thus, we obtain

$$v_{\text{com}} = -1.3\ \text{m/s} - \frac{\left(9.8\ \text{m/s}^2\right)\left(0.885\ \text{s}\right)}{1+\dfrac{0.000095\ \text{kg}\cdot\text{m}^2}{\left(0.12\ \text{kg}\right)\left(0.0032\ \text{m}\right)^2}} = -1.41\ \ \text{m/s}$$

so its linear speed at that moment is approximately $1.4\ \text{m/s}$.

(d) The translational kinetic energy is $\frac{1}{2}mv_{\text{com}}^2 = \frac{1}{2}\left(0.12\ \text{kg}\right)\left(-1.41\ \text{m/s}\right)^2 = 0.12\ \text{J}$.

(e) The angular velocity at that moment is given by

$$\omega = -\frac{v_{\text{com}}}{R_0} = -\frac{-1.41\ \text{m/s}}{0.0032\ \text{m}} = 441\,\text{rad/s} \approx 4.4\times10^2\ \text{rad/s}\,.$$

(f) And the rotational kinetic energy is

$$\frac{1}{2}I_{\text{com}}\omega^2 = \frac{1}{2}\left(9.50\times10^{-5}\ \text{kg}\cdot\text{m}^2\right)\left(441\ \text{rad/s}\right)^2 = 9.2\ \text{J}.$$

79. (a) When the small sphere is released at the edge of the large "bowl" (the hemisphere of radius R), its center of mass is at the same height at that edge, but when it is at the bottom of the "bowl" its center of mass is a distance r above the bottom surface of the hemisphere. Since the small sphere descends by $R - r$, its loss in gravitational potential energy is $mg(R- r)$, which, by conservation of mechanical energy, is equal to its kinetic energy at the bottom of the track. Thus,

$$K = mg(R-r) = (5.6\times10^{-4}\,\text{kg})(9.8\ \text{m/s}^2)(0.15\ \text{m} - 0.0025\ \text{m}) = 8.1\times10^{-4}\ \text{J}.$$

(b) Using Eq. 11-5 for K, the asked-for fraction becomes

$$\frac{K_{\text{rot}}}{K} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}I\omega^2 + \frac{1}{2}Mv_{\text{com}}^2} = \frac{1}{1+\left(\frac{M}{I}\right)\left(\frac{v_{\text{com}}}{\omega}\right)^2}.$$

Substituting $v_{\text{com}} = R\omega$ (Eq. 11-2) and $I = \frac{2}{5}MR^2$ (Table 10-2(f)), we obtain

$$\frac{K_{\text{rot}}}{K} = \frac{1}{1+\left(\frac{5}{2R^2}\right)R^2} = \frac{2}{7} \approx 0.29.$$

(c) The small sphere is executing circular motion so that when it reaches the bottom, it experiences a radial acceleration upward (in the direction of the normal force which the "bowl" exerts on it). From Newton's second law along the vertical axis, the normal force F_N satisfies $F_N - mg = ma_{\text{com}}$ where

$$a_{\text{com}} = v_{\text{com}}^2/(R-r).$$

Therefore,

$$F_N = mg + \frac{mv_{\text{com}}^2}{R-r} = \frac{mg(R-r) + mv_{\text{com}}^2}{R-r}.$$

But from part (a), $mg(R - r) = K$, and from Eq. 11-5, $\frac{1}{2}mv_{\text{com}}^2 = K - K_{\text{rot}}$. Thus,

$$F_N = \frac{K + 2(K - K_{\text{rot}})}{R-r} = 3\left(\frac{K}{R-r}\right) - 2\left(\frac{K_{\text{rot}}}{R-r}\right).$$

We now plug in $R - r = K/mg$ and use the result of part (b):

$$F_N = 3mg - 2mg\left(\frac{2}{7}\right) = \frac{17}{7}mg = \frac{17}{7}(5.6\times10^{-4}\ \text{kg})(9.8\ \text{m/s}^2) = 1.3\times10^{-2}\ \text{N}.$$

80. Conservation of energy implies that mechanical energy at maximum height up the ramp is equal to the mechanical energy on the floor. Thus, using Eq. 11-5, we have

$$\frac{1}{2}mv_f^2 + \frac{1}{2}I_{\text{com}}\omega_f^2 + mgh = \frac{1}{2}mv^2 + \frac{1}{2}I_{\text{com}}\omega^2$$

where $v_f = \omega_f = 0$ at the point on the ramp where it (momentarily) stops. We note that the height h relates to the distance traveled along the ramp d by $h = d\sin(15^\circ)$. Using item (*f*) in Table 10-2 and Eq. 11-2, we obtain

$$mgd\sin 15^\circ = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2.$$

After canceling m and plugging in $d = 1.5$ m, we find $v = 2.33$ m/s.

81. (a) Interpreting h as the height increase for the center of mass of the body, then (using Eq. 11-5) mechanical energy conservation, $K_i = U_f$, leads to

$$\frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}I\omega^2 = mgh \quad \Rightarrow \quad \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = mg\left(\frac{3v^2}{4g}\right)$$

from which v cancels and we obtain $I = \frac{1}{2}mR^2$.

(b) From Table 10-2(c), we see that the body could be a solid cylinder.

82. (a) Using Eq. 2-16 for the translational (center-of-mass) motion, we find

$$v^2 = v_0^2 + 2a\Delta x \Rightarrow a = -\frac{v_0^2}{2\Delta x}$$

which yields $a = -4.11$ for $v_0 = 43$ and $\Delta x = 225$ (SI units understood). The magnitude of the linear acceleration of the center of mass is therefore 4.11 m/s^2.

(b) With $R = 0.250$ m, Eq. 11-6 gives

$$|\alpha| = |a|/R = 16.4\,\text{rad/s}^2.$$

If the wheel is going rightward, it is rotating in a clockwise sense. Since it is slowing down, this angular acceleration is counterclockwise (opposite to ω) so (with the usual convention that counterclockwise is positive) there is no need for the absolute value signs for α.

(c) Eq. 11-8 applies with Rf_s representing the magnitude of the frictional torque. Thus,

$$Rf_s = I\alpha = (0.155\text{ kg}\cdot\text{m}^2)\,(16.4\text{ rad/s}^2) = 2.55\text{ N}\cdot\text{m}.$$

83. If the polar cap melts, the resulting body of water will effectively increase the equatorial radius of the Earth from R_e to $R_e' = R_e + \Delta R$, thereby increasing the moment of inertia of the Earth and slowing its rotation (by conservation of angular momentum), causing the duration T of a day to increase by ΔT. We note that (in rad/s) $\omega = 2\pi/T$ so

$$\frac{\omega'}{\omega} = \frac{2\pi/T'}{2\pi/T} = \frac{T}{T'}$$

from which it follows that

$$\frac{\Delta\omega}{\omega} = \frac{\omega'}{\omega} - 1 = \frac{T}{T'} - 1 = -\frac{\Delta T}{T'}.$$

We can approximate that last denominator as T so that we end up with the simple relationship $|\Delta\omega|/\omega = \Delta T/T$. Now, conservation of angular momentum gives us

$$\Delta L = 0 = \Delta(I\omega) \approx I(\Delta\omega) + \omega(\Delta I)$$

so that $|\Delta\omega|/\omega = \Delta I/I$. Thus, using our expectation that rotational inertia is proportional to the equatorial radius squared (supported by Table 10-2(f) for a perfect uniform sphere, but then this isn't a perfect uniform sphere) we have

$$\frac{\Delta T}{T} = \frac{\Delta I}{I} = \frac{\Delta\left(R_e^2\right)}{R_e^2} \approx \frac{2\Delta R_e}{R_e} = \frac{2(30\text{m})}{6.37\times10^6\,\text{m}}$$

so with T = 86400s we find (approximately) that ΔT = 0.8 s. The radius of the earth can be found in Appendix C or on the inside front cover of the textbook.

84. With $r_\perp = 1300\,\text{m}$, Eq. 11-21 gives

$$\ell = r_\perp m v = (1300\,\text{m})(1200\,\text{kg})(80\,\text{m/s}) = 1.2\times 10^8\,\text{kg}\cdot\text{m}^2/\text{s}.$$

85. (a) In terms of the radius of gyration k, the rotational inertia of the merry-go-round is $I = Mk^2$. We obtain

$$I = (180 \text{ kg})\,(0.910 \text{ m})^2 = 149 \text{ kg}\cdot\text{m}^2.$$

(b) An object moving along a straight line has angular momentum about any point that is not on the line. The magnitude of the angular momentum of the child about the center of the merry-go-round is given by Eq. 11-21, mvR, where R is the radius of the merry-go-round. Therefore,

$$\left|\vec{L}_{\text{child}}\right| = (44.0 \text{ kg})(3.00 \text{ m/s})(1.20 \text{ m}) = 158 \text{ kg}\cdot\text{m}^2/\text{s}.$$

(c) No external torques act on the system consisting of the child and the merry-go-round, so the total angular momentum of the system is conserved. The initial angular momentum is given by mvR; the final angular momentum is given by $(I + mR^2)\,\omega$, where ω is the final common angular velocity of the merry-go-round and child. Thus $mvR = \left(I + mR^2\right)\omega$ and

$$\omega = \frac{mvR}{I + mR^2} = \frac{158 \text{ kg}\cdot\text{m}^2/\text{s}}{149 \text{ kg}\cdot\text{m}^2 + (44.0 \text{ kg})(1.20 \text{ m})^2} = 0.744 \text{ rad/s}.$$

86. For a constant (single) torque, Eq. 11-29 becomes $\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{\Delta\vec{L}}{\Delta t}$. Thus, we obtain Δt = 600/50 = 12 s.

87. This problem involves the vector cross product of vectors lying in the *xy* plane. For such vectors, if we write $\vec{r}\,' = x'\hat{\text{i}} + y'\hat{\text{j}}$, then (using Eq. 3-30) we find

$$\vec{r}\,' \times \vec{v} = \left(x'v_y - y'v_x\right)\hat{\text{k}}.$$

(a) Here, $\vec{r}\,'$ points in either the $+\hat{\text{i}}$ or the $-\hat{\text{i}}$ direction (since the particle moves along the x axis). It has no y' or z' components, and neither does $\vec{v}$, so it is clear from the above expression (or, more simply, from the fact that $\hat{\text{i}} \times \hat{\text{i}} = 0$) that $\vec{\ell} = m(\vec{r}\,' \times \vec{v}) = 0$ in this case.

(b) The net force is in the $-\hat{\text{i}}$ direction (as one finds from differentiating the velocity expression, yielding the acceleration), so, similar to what we found in part (a), we obtain $\tau = \vec{r}\,' \times \vec{F} = 0$.

(c) Now, $\vec{r}\,' = \vec{r} - \vec{r}_{\text{o}}$ where $\vec{r}_{\text{o}} = 2.0\hat{\text{i}} + 5.0\hat{\text{j}}$ (with SI units understood) and points from (2.0, 5.0, 0) to the instantaneous position of the car (indicated by $\vec{r}$ which points in either the $+x$ or $-x$ directions, or nowhere (if the car is passing through the origin)). Since $\vec{r} \times \vec{v} = 0$ we have (plugging into our general expression above)

$$\vec{\ell} = m\left(\vec{r}\,' \times \vec{v}\right) = -m\left(\vec{r}_{\text{o}} \times \vec{v}\right) = -(3.0)\Big((2.0)(0) - (5.0)\left(-2.0t^3\right)\Big)\hat{\text{k}}$$

which yields $\vec{\ell} = (-30t^3\hat{\text{k}})\ \text{kg}\cdot\text{m/s}^2$.

(d) The acceleration vector is given by $\vec{a} = \frac{d\vec{v}}{dt} = -6.0t^2\hat{\text{i}}$ in SI units, and the net force on the car is $m\vec{a}$. In a similar argument to that given in the previous part, we have

$$\vec{\tau} = m\left(\vec{r}\,' \times \vec{a}\right) = -m\left(\vec{r}_{\text{o}} \times \vec{a}\right) = -(3.0)\Big((2.0)(0) - (5.0)\left(-6.0t^2\right)\Big)\hat{\text{k}}$$

which yields $\vec{\tau} = (-90t^2\hat{\text{k}})\ \text{N}\cdot\text{m}$.

(e) In this situation, $\vec{r}\,' = \vec{r} - \vec{r}_{\text{o}}$ where $\vec{r}_{\text{o}} = 2.0\hat{\text{i}} - 5.0\hat{\text{j}}$ (with SI units understood) and points from (2.0, –5.0, 0) to the instantaneous position of the car (indicated by $\vec{r}$ which points in either the $+x$ or $-x$ directions, or nowhere (if the car is passing through the origin)). Since $\vec{r} \times \vec{v} = 0$ we have (plugging into our general expression above)

$$\vec{\ell} = m\left(\vec{r}\,' \times \vec{v}\right) = -m\left(\vec{r}_{\text{o}} \times \vec{v}\right) = -(3.0)\Big((2.0)(0) - (-5.0)\left(-2.0t^3\right)\Big)\hat{\text{k}}$$

which yields $\vec{\ell} = (30t^3\hat{\text{k}})\ \text{kg}\cdot\text{m}^2\text{/s}$.

(f) Again, the acceleration vector is given by $\vec{a} = -6.0t^2\hat{\text{i}}$ in SI units, and the net force on the car is $m\vec{a}$. In a similar argument to that given in the previous part, we have

$$\vec{\tau} = m\left(\vec{r}\,' \times \vec{a}\right) = -m\left(\vec{r}_{\text{o}} \times \vec{a}\right) = -(3.0)\Big((2.0)(0) - (-5.0)\left(-6.0t^2\right)\Big)\hat{\text{k}}$$

which yields $\vec{\tau} = (90t^2\hat{\text{k}})\ \text{N}\cdot\text{m}$.

88. The rotational kinetic energy is $K = \frac{1}{2} I\omega^2$, where $I = mR^2$ is its rotational inertia about the center of mass (Table 10-2(a)), m = 140 kg, and $\omega = v_{com}/R$ (Eq. 11-2). The ratio is

$$\frac{K_{\text{transl}}}{K_{\text{rot}}} = \frac{\frac{1}{2} m v_{\text{com}}^2}{\frac{1}{2}\left(mR^2\right)\left(v_{\text{com}}/R\right)^2} = 1.00.$$

89. We note that its mass is M = 36/9.8 = 3.67 kg and its rotational inertia is $I_{\text{com}} = \frac{2}{5}MR^2$ (Table 10-2(f)).

(a) Using Eq. 11-2, Eq. 11-5 becomes

$$K = \frac{1}{2}I_{\text{com}}\omega^2 + \frac{1}{2}Mv_{\text{com}}^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_{\text{com}}}{R}\right)^2 + \frac{1}{2}Mv_{\text{com}}^2 = \frac{7}{10}Mv_{\text{com}}^2$$

which yields K = 61.7 J for v_{com} = 4.9 m/s.

(b) This kinetic energy turns into potential energy Mgh at some height $h = d \sin\theta$ where the sphere comes to rest. Therefore, we find the distance traveled up the θ = 30° incline from energy conservation:

$$\frac{7}{10}Mv_{\text{com}}^2 = Mgd\sin\theta \quad \Rightarrow \quad d = \frac{7v_{\text{com}}^2}{10g\sin\theta} = 3.43\,\text{m}.$$

(c) As shown in the previous part, M cancels in the calculation for d. Since the answer is independent of mass, then, it is also independent of the sphere's weight.

90. The speed of the center of mass of the car is v = (40)(1000/3600) = 11 m/s. The angular speed of the wheels is given by Eq. 11-2: $\omega = v/R$ where the wheel radius R is not given (but will be seen to cancel in these calculations).

(a) For one wheel of mass M = 32 kg, Eq. 10-34 gives (using Table 10-2(c))

$$K_{\text{rot}} = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{4}Mv^2$$

which yields $K_{\text{rot}} = 9.9 \times 10^2$ J. The time given in the problem (10 s) is not used in the solution.

(b) Adding the above to the wheel's translational kinetic energy, $\frac{1}{2}Mv^2$, leads to

$$K_{\text{wheel}} = \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2 = \frac{3}{4}(32\ \text{kg})(11\ \text{m/s})^2 = 3.0\times 10^3\ \text{J}.$$

(c) With M_{car} = 1700 kg and the fact that there are four wheels, we have

$$\frac{1}{2}M_{\text{car}}v^2 + 4\left(\frac{3}{4}Mv^2\right) = 1.2\times 10^5\ \text{J}.$$

91. We denote the wheel with subscript 1 and the whole system with subscript 2. We take clockwise as the negative sense for rotation (as is the usual convention).

(a) Conservation of angular momentum gives $L = I_1\omega_1 = I_2\omega_2$, where $I_1 = m_1R_1^2$. Thus

$$\omega_2 = \omega_1 \frac{I_1}{I_2} = (-57.7\,\text{rad/s})\frac{\left(37\,\text{N}/9.8\,\text{m/s}^2\right)(0.35\,\text{m})^2}{2.1\,\text{kg}\cdot\text{m}^2} = -12.7\ \text{rad/s},$$

or $|\omega_2| = 12.7$ rad/s .

(b) The system rotates clockwise (as seen from above) at the rate of 12.7 rad/s.

92. Information relevant to this calculation can be found in Appendix C or on the inside front cover of the textbook. The angular speed is constant so

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{86400} = 7.3\times10^{-5}\ \text{rad/s}.$$

Thus, with m = 84 kg and $R = 6.37 \times 10^6$ m, we find $\ell = mR^2\omega = 2.5\times10^{11}\ \text{kg}\cdot\text{m}^2/\text{s}$.

93. The initial angular momentum of the system is zero. The final angular momentum of the girl-plus-merry-go-round is $(I + MR^2)\ \omega$ which we will take to be positive. The final angular momentum we associate with the thrown rock is negative: $-mRv$, where v is the speed (positive, by definition) of the rock relative to the ground.

(a) Angular momentum conservation leads to

$$0 = \left(I + MR^2\right)\omega - mRv \Rightarrow \omega = \frac{mRv}{I + MR^2}.$$

(b) The girl's linear speed is given by Eq. 10-18:

$$R\omega = \frac{mvR^2}{I + MR^2}.$$

94. (a) With $\vec{p} = m\vec{v} = -16\,\hat{\text{j}}$ kg·m/s, we take the vector cross product (using either Eq. 3-30 or, more simply, Eq. 11-20 and the right-hand rule): $\vec{\ell} = \vec{r} \times \vec{p} = (-32\,\text{kg}\cdot\text{m}^2/\text{s})\hat{\text{k}}$.

(b) Now the axis passes through the point $\vec{R} = 4.0\,\hat{\text{j}}$ m, parallel with the z axis. With $\vec{r}\,' = \vec{r} - \vec{R} = 2.0\,\hat{\text{i}}$ m, we again take the cross product and arrive at the same result as before:

$$\vec{\ell}\,' = \vec{r}\,' \times \vec{p} = (-32\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{k}}.$$

(c) Torque is defined in Eq. 11-14: $\vec{\tau} = \vec{r} \times \vec{F} = (12\ \text{N}\cdot\text{m})\hat{\text{k}}$.

(d) Using the notation from part (b), $\vec{\tau}\,' = \vec{r}\,' \times \vec{F} = 0$.

95. We make the unconventional choice of *clockwise* sense as positive, so that the angular acceleration is positive (as is the linear acceleration of the center of mass, since we take rightwards as positive).

(a) We approach this in the manner of Eq. 11-3 (*pure rotation* about point *P*) but use torques instead of energy. The torque (relative to point *P*) is $\tau = I_P\alpha$, where

$$I_P = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

with the use of the parallel-axis theorem and Table 10-2(c). The torque is due to the $F_{\text{app}} = 12\text{ N}$ force and can be written as $\tau = F_{\text{app}}(2R)$. In this way, we find

$$\tau = I_P\alpha = \left(\frac{3}{2}MR^2\right)\alpha = 2RF_{\text{app}}$$

which leads to

$$\alpha = \frac{2RF_{\text{app}}}{3MR^2/2} = \frac{4F_{\text{app}}}{3MR} = \frac{4(12\text{ N})}{3(10\text{ kg})(0.10\text{ m})} = 16\,\text{rad/s}^2.$$

Hence, $a_{\text{com}} = R\alpha = 1.6\text{ m/s}^2$.

(b) As shown above, $\alpha = 16\text{ rad/s}^2$.

(c) Applying Newton's second law in its linear form yields $(12\text{ N}) - f = Ma_{\text{com}}$. Therefore, $f = -4.0$ N. Contradicting what we assumed in setting up our force equation, the friction force is found to point *rightward* with magnitude 4.0 N, i.e., $\vec{f} = (4.0\text{ N})\hat{\text{i}}$.

96. (a) Sample Problem 10-8 gives I = 19.64 kg·m^2 and ω = 1466 rad/s. Thus, the angular momentum is

$$L = I\omega = (19.64\ \text{kg}\cdot\text{m}^2)(1466\ \text{rad/s}) \approx 2.9\times 10^4\ \text{kg}\cdot\text{m}^2/\text{s}.$$

(b) We rewrite Eq. 11-29 as $|\vec{\tau}_{\text{avg}}| = |\Delta\vec{L}|/\Delta t$ and plug in $|\Delta\vec{L}| = 2.9\times 10^4\ \text{kg}\cdot\text{m}^2/\text{s}$ and $\Delta t = 0.025\ \text{s}$, which leads to $|\vec{\tau}_{\text{avg}}| = 1.2\times 10^6\ \text{N}\cdot\text{m}$.

97. Since we will be taking the vector cross product in the course of our calculations, below, we note first that when the two vectors in a cross product $\vec{A}\times\vec{B}$ are in the *xy* plane, we have $\vec{A} = A_x\hat{\mathrm{i}} + A_y\hat{\mathrm{j}}$ and $\vec{B} = B_x\hat{\mathrm{i}} + B_y\hat{\mathrm{j}}$, and Eq. 3-30 leads to

$$\vec{A}\times\vec{B} = \left(A_xB_y - A_yB_x\right)\hat{\mathrm{k}}.$$

Now, we choose coordinates centered on point *O*, with +*x* rightwards and +*y* upwards. In unit-vector notation, the initial position of the particle, then, is $\vec{r}_0 = s\,\hat{\mathrm{i}}$ and its later position (halfway to the ground) is $\vec{r} = s\,\hat{\mathrm{i}} - \frac{1}{2}h\,\hat{\mathrm{j}}$. Using either the free-fall equations of Ch. 2 or the energy techniques of Ch. 8, we find the speed at its later position to be $v = \sqrt{2g|\Delta y|} = \sqrt{gh}$. Its momentum there is $\vec{p} = -M\sqrt{gh}\,\hat{\mathrm{j}}$. We find the angular momentum using Eq. 11-18 and our observation, above, about the cross product of two vectors in the *xy* plane:

$$\vec{\ell} = \vec{r}\times\vec{p} = -sM\sqrt{gh}\,\hat{\mathrm{k}}$$

Therefore, its magnitude is

$$|\vec{\ell}| = sM\sqrt{gh} = (0.45\text{ m})(0.25\text{ kg})\sqrt{(9.8\text{ m/s}^2)(1.8\text{ m})} = 0.47\text{ kg}\cdot\text{m}^2/\text{s}\,.$$

98. This problem involves the vector cross product of vectors lying in the *xy* plane. For such vectors, if we write $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}}$, then (using Eq. 3-30) we find

$$\vec{r} \times \vec{p} = \left(\Delta x p_y - \Delta y p_x\right)\hat{\mathrm{k}}.$$

The momentum components are

$$p_x = p \cos \theta$$
$$p_y = p \sin \theta$$

where p = 2.4 (SI units understood) and θ = 115°. The mass (0.80 kg) given in the problem is not used in the solution. Thus, with x = 2.0, y = 3.0 and the momentum components described above, we obtain

$$\vec{\ell} = \vec{r} \times \vec{p} = (7.4\,\mathrm{kg \cdot m^2/s})\hat{\mathrm{k}}.$$